**Institut für
Volkswirtschaftslehre
und Statistik**

No. 552-97

Strategic Trade Policy under International Price and Quantitiy Competition

Klaus Conrad

Beiträge zur angewandten Wirtschaftsforschung

**Universität Mannheim
A5, 6
D-68131 Mannheim**

Strategic Trade Policy under International Price and Quantity Competition

Klaus Conrad

Mannheim University and

University of Toronto

1. Introduction

A standard critique of the strategic, two-stage industrial and trade policy models is that trade policy recommendations depend on the nature of competition between firms. Brander and Spencer (1985) have shown that under quantity competition in an international market governments opt for export subsidies to maximize national welfare. However, as shown by Eaton and Grossman (1986), trade policy conclusions are different if there is Bertrand price competition instead of Cournot quantity competition. Then the optimal trade policy is to tax exports. In these two papers production is for a third country and the governments act strategically to distort free trading. The interesting question is therefore, whether their propositions on opposite trade policy recommendations do also hold if the assumption of no home consumption is dropped. Since the case of no home consumption is anyway not very realistic, we will check the robustness of the propositions under price or quantity competition when goods are consumed domestically with intra-industry trade between the two competing countries. The purpose of this paper is to investigate whether trade policy always switches from export subsidy to export tax if firms engage in Bertrand competition instead of Cournot competition with trade between the two countries. In section 2 we will show that an export subsidy is optimal also under intra-industry trade. In section 3 we will characterize market structures and types of goods where an export subsidy is even optimal under Bertrand price competition. In Section 4 we will use the same model to derive inducements for import tariffs (or subsidies) if the nature of competition is Bertrand Section 5 concludes the paper.

2. An optimal export subsidy under intra-industry trade

We choose the same simple model of firm behavior as in Brander and Spencer, i.e. with one domestic firm and one foreign firm which produce identical products. I begin by analyzing the last stage of the two stage game, that is, the choice of output for the domestic as well as for the foreign market. The domestic firm produces output x for the domestic market and output x_E for the foreign market at cost $c(x + x_E)$. Let us denote by lower case letters the domestic variables and by upper case letters the corresponding foreign variables. Then profit π of the domestic firm is:

$$\pi(x, x_E, X, X_E; s) = p(x + X_E)x + P(X + x_E)x_E - c(x + x_E) + s \cdot x_E \tag{1}$$

Revenue is the sum of revenue in the domestic market and of revenue in the foreign market. $x+X_E$ is total sales in the domestic country's market and $p(x + X_E)$ is the inverse domestic demand function. $X + x_E$ is total sales in the foreign country's market and $P(X + x_E)$ is the inverse foreign demand function. Similarly, profit Π of the foreign firm is given by:[1]

$$\Pi(X, X_E, x, x_E; S) = P(X + x_E)X + p(x + X_E)X_E - C(X + X_E) + S \cdot X_E \tag{2}$$

The Nash equilibrium in the four outputs is characterized by the first-order conditions:

$$\pi_x = p + xp' - c_{\bar{x}} = 0 \tag{3}$$

$$\pi_{x_E} = P + x_E P' - c_{\bar{x}} + s = 0 \tag{4}$$

$$\Pi_X = P + XP' - C_{\bar{X}} = 0 \tag{5}$$

$$\Pi_{X_E} = p + X_E p' - C_{\bar{X}} + S = 0 \tag{6}$$

where $\bar{x} = x + x_E$ and $\bar{X} = X + X_E$ is total supply. The second order conditions are:

[1] In Brander (1991) the two countries are assumed to be identical (same price, same cost functions), but there are transport costs and increasing returns to scale due to fixed costs.

$\pi_{xx} < 0$, $\pi_{x_E x_E} < 0$, $\Pi_{XX} < 0$, $\Pi_{X_E X_E} < 0$.

We assume that own effects of output on marginal profit dominate cross effects, which ensures uniqueness and global stability of the equilibrium in non-cooperative models. Since we will assume linear demand functions and constant returns to scale (CRTS) to bring out the main points, the equilibrium is unique and globally stable. The solutions of (3) to (6) depend on domestic and foreign subsidies *s* and *S* and can be written as

(7) $x = x(s,S)$, $x_E = x_E(s,S)$, $X = X(s,S)$, $X_E = X_E(s,S)$

Given our assumption of CRTS, the reaction functions derived from (3) to (6) are:

(8) $x = r(X_E)$, $x_E = r_E(X,s)$, $X = R(x_E)$, $X_E = R_E(x,S)$

If in the first stage of the game the national governments change *s*, or *S* respectively, then the reaction functions[2] of export supply will shift and outputs in (7), and hence foreign market shares, will change. If the domestic government raises the subsidy rate *s*, then the domestic firm will export more, the foreign firm will react by supplying less for its market, and therefore the domestic firm will increase its market share in the foreign country, i.e. $x_E / (X + x_E)$. An algebraic analysis requires total differentiation of (3) to (6) with respect to $x, x_E X, X_E, s$ and *S*. Under the assumptions made (linear demand and CRTS), we obtain:

(9) $\frac{dx}{ds} = 0$, $\frac{dx_E}{ds} = -\frac{2}{3P'} > 0$, $\frac{dX}{ds} = \frac{1}{3P'} < 0$, $\frac{dX_E}{ds} = 0$

(10) $\frac{dx}{dS} = \frac{1}{3p'} < 0$, $\frac{dx_E}{dS} = 0$, $\frac{dX}{dS} = 0$, $\frac{dX_E}{dS} = -\frac{2}{3p'} > 0$.

The relationship between export and domestic supply of the competing firm is:

(11) $dx = -\frac{1}{2} dX_E$, $dX = -\frac{1}{2} dx_E$.

---

[2] It is $\frac{dX}{dx_E} = R'(x_E) < 0$

From (9) and (10) follows that the supply of goods on the other market is increasing in the subsidy rate, i.e.

(12) $\frac{d}{ds}(X+x_E)>0\,,\quad \frac{d}{ds}(x+X_E)=0$

(13) $\frac{d}{dS}(x+X_E)>0\,,\quad \frac{d}{dS}(X+x_E)=0$

Thus the governments know that subsidies will lower prices in the competing country and will not change prices in the own country. Their goal is to maximize national welfare in terms of home consumer surplus and industry profit. They are aware of the effect of subsidies on the terms of trade and on rent-shifting in foreign trade. The domestic government maximizes:

(14) $\max_s w(s;S)=\left[\int_0^{x+X_E} p(\xi)d\xi - p(x+X_E)\,(x+X_E)\right]+\pi\,(x,x_E;X,X_E,s)-s\cdot x_E$

A similar objective function can be written down for the foreign government. The first order condition for (14) is an implicit reaction function of the domestic government. Since the price on the domestic market will not change (see (12)), consumer surplus will not change. Since $\pi_x=\pi_{x_E}=0$ and $\frac{dX_E}{ds}=0$, the f.o.c. of (14) reduces to

$$\pi_X\,\frac{dX}{ds}+\pi_s-s\,\frac{dx_E}{ds}-x_E=0 \quad \text{or, as} \quad \pi_s=x_E \quad \text{and} \quad \pi_X=P'\cdot x_E \text{ , to}$$

(15) $s=P'\cdot x_E\,\frac{\frac{dX}{ds}}{\frac{dx_E}{ds}}>0$

This is exactly the export subsidy, derived in Brander and Spencer (1985, p. 89) when export is for a third country market. Since under our assumption of CRTS consumer surplus in the domestic market does not change, rent shifting concentrate solely on profit as in Brander and Spencer. Using (9), the subsidy is $s=-\frac{P'x_E}{2}$, or, in terms of market structure and conduct:

(16) $$s = \frac{1}{2}\ \frac{x_E}{X + x_E}\ \frac{P}{\left|\varepsilon_p\right|}$$

where

$$\varepsilon_P = \frac{d(X + x_E)}{dP}\ \frac{P}{X + x_E} < 0$$

is the foreign price elasticity of demand. The subsidy increases with the share of export in foreign demand and is the higher the less elastic the foreign price elasticity is.

The foreign government can act in a similar way by imposing a subsidy *S*, similar to (15), given the other government is passive. A non-cooperate Nash equilibrium in subsidies occurs if each government is assumed to choose its subsidy level given the subsidy level of the other government. The optimal values of the subsidy rates in equilibrium are the solution of the two implicit reaction functions $\frac{\partial w}{\partial s} = 0$ and $\frac{\partial W}{\partial S} = 0$. Since the subsidy of one government affects welfare of the other country only indirect through its impact on output levels, the solution will always be a subsidy and not a tax.

To complete the analysis of quantity competition under trade subsidies we compare the intra-industry trade model with the model by Brander and Spencer (1985) where export is for a third market and where consumer surplus is included in the government's welfare function. Then the domestic firm maximizes

$$\max_{x, x_E} \pi = p(x)x + P_E(x_E + X_E)x_E - c(x + x_E) + s \cdot x_E$$

and the foreign firm maximizes

$$\max_{X, X_E} \Pi = P(X)X + P_E(x_E + X_E)X_E - C(X_E + X) + S \cdot X_E$$

where $P_E(x_E + X_E)$ is the third country's demand function. The government maximizes

$$\max_{s} w(s, S) = \int_0^x p(\xi)d\xi - p(x)x + \pi - s \cdot x_E$$

Again, under constant marginal costs, the markets are separated in terms of output decision making, and home consumer surplus is not affected by the subsidy. It is easy to show that the subsidy is

$$s = P_E' \, x_E \, \frac{\frac{dX_E}{ds}}{\frac{dx_E}{ds}} > 0$$

In the intra-industry trade model it is $\frac{dX_E}{ds} = 0$, i.e. the domestic subsidy does not affect foreign export decision if *S* is kept unchanged. In the model with export for a third country, the competitor reduces his export quantity, i.e. $\frac{dX_E}{ds} < 0$. A comparative static analysis shows that $\frac{dX_E}{ds} = \frac{1}{3P_E'} < 0$ *and* $\frac{dx_E}{ds} = -\frac{2}{3P_E'} > 0$. Hence $s = -P_E' \, (x_E + X_E) \, \frac{x_E}{2}$ or

$$\text{(16')} \quad s = \frac{1}{2} \frac{P_E}{|\varepsilon_{P_E}|} \frac{x_E}{x_E + X_E}$$

where

$$\varepsilon_{P_E} = \frac{d(X_E + x_E)}{dP_E} \frac{P_E}{X_E + x_E} \ .$$

If we compare the difference in the subsidy under intra-industry trade (16) and under third market exports, we recognize that in (16') only variables, affecting the third country, enter the subsidy formula. In (16), however, price and quantity on the competitor's home market enters the subsidy. Its export volumn X_E is not relevant for the size of the subsidy.

3. Subsidizing exports under price competition

The main purpose of the previous section has been to present the basic structure of our intra-industry trade model which will also be used when firms engage in price competition. Making the same assumption in either models (CRTS and linear demand functions), it is easier to understand strategic trade policy under price and quantity competition. For each firm there are prices p, and P respectively, and quantities x, x_E, X and X_E which are jointly related by the demand functions $x = x(p, P)$, etc. In this heterogeneous duopoly each national monopolist produces for the domestic as well as for the foreign market. We start by analyzing again the last stage of the game, the choices of prices, given the subsidy rates. Profit for the domestic firm is:

$$(17) \qquad \max_{p} \; \pi(p,P;s) = p \cdot x(p,P) + p \cdot x_E(p,P) - c(\bar{x}) + s \cdot x_E(p,P)$$

where $\bar{x} = x(p,P) + x_E(p,P)$ is total domestic production. Similarly, profit of the foreign firm is:

$$(18) \qquad \max_{P} \; \Pi(p,P;S) = P \cdot X(p,P) + P \cdot X_E(p,P) - C(\bar{X}) + S \cdot X_E(p,P)$$

The Bertrand equilibrium in the two prices is characterized by the two f.o.c.:

$$(19) \qquad \bar{x} + (p - c_{\bar{x}}) \frac{\partial \bar{x}}{\partial p} + s \frac{\partial x_E}{\partial p} = 0$$

$$(20) \qquad \bar{X} + (P - C_{\bar{X}}) \frac{\partial \bar{X}}{\partial P} + S \frac{\partial X_E}{\partial P} = 0$$

The second order conditions and the condition for uniqueness and global stability of the equilibrium are satisfied under our assumption of CRTS and linear demand functions. Linear demand functions are derived from a quadratic indirect utility function for the domestic consumer.[3] Then the home and the export demand functions are:

$$(21) \qquad x = \alpha_{\circ} - \alpha \, p + \beta \, P$$

$$(22) \qquad X_E = \alpha_{\circ}^{E} - \alpha_E \, P + \beta \, p$$

with $\alpha > 0, \alpha_E > 0$ and $\beta \geq 0$ since we assume goods to be ordinary substitutes.

---

[3] See the Appendix for details

Preferences of the foreign consumers are assumed to be identical, so that home and export demand are:

(23) $X = \alpha_{\circ} - \alpha\, P + \beta\, p$

(24) $x_E = \alpha_{\circ}^{E} - \alpha_E\, p + \beta\, P$

Finally, since an indirect utility function must be convex in the prices, this property implies the following inequality on the parameters:

(25) $\alpha\, \alpha_E - \beta^2 > 0$

The solutions to (19) and (20) depend on domestic and foreign subsidies *s* and *S* and can be written as:

(26) $p = p(s,S) \quad ; \quad P = P(s,S)$

If in the first stage of the game the governments change *s*, or *S* respectively, then the reaction functions $p = r(P,s)$ and $P = R(p,S)$, derived from (19) and (20), will shift and prices in (26), and hence market shares will change. To derive the direction of the price changes under export subsidies we have to do comparative statics by totally differentiating (19) and (20) with respect to *p*, *P*, *s*, and *S*. The results are:[4]

(27) $\dfrac{dp}{ds} = \dfrac{dP}{dS} = -\dfrac{2\alpha_E(\alpha + \alpha_E)}{D} < 0\,, \quad \dfrac{dP}{ds} = \dfrac{dp}{dS} = \dfrac{-2\,\alpha_E \cdot \beta}{D} < 0$

where *D* is the determinant with $D = 4\,(\alpha + \alpha_E)^2 - 4\,\beta^2 > 0$. Both firms' Bertrand equilibrium levels of prices are decreasing under national as well as foreign subsidization. The reaction of prices in (27) on a subsidy implies for the quantities, using (21) - (24):

(28) $\dfrac{dx}{ds} > 0\,, \quad \dfrac{dx_E}{ds} > 0\,, \quad \dfrac{dX}{ds} < 0\,, \quad and \quad \dfrac{dX_E}{ds} < 0$

[4] See the Appendix for details

and similarly for *S*. Domestic total supply increases in the subsidy rate and foreign supply decreases.

Next we analyze the first stage of the game, the maximization of national welfare by the government. The objective function, given the foreign subsidy, consists of consumer surplus and of profit less the subsidy:

(29) $$\max_s w(s;S) = \int_p^{\bar{p}} x(\xi,P)d\xi + \int_{\bar{p}_E}^{p} X_E(\xi,P)d\xi + \pi\,(p,P;s) - s\cdot x_E$$

with profit and the linear demand functions as given in (17), and (21) - (24). $\bar{p}$ is the finite „choke“ price which leads to zero sales, i.e. $\bar{p}$ from $x(\bar{p},P)=0$, which implies

(30) $$\bar{p}(P) = \frac{(\beta\, P + \alpha_0)}{\alpha}$$

Similarly, if the domestic firm sets the price as low as $\bar{p}_E$, given *P*, the foreign firm will not sell anything on the domestic market. This implies

(31) $$\bar{p}_E(P) = \frac{(\alpha_E P - \alpha_0^E)}{\beta}\ .$$

The f.o.c. of (29) is:

(32) $$\frac{dw(s;S)}{ds} = \frac{dCS}{ds} + \frac{dCS_E}{ds} + \pi_p \frac{dp}{ds} + \pi_P \frac{dP}{ds} + \pi_S - x_E - s\frac{dx_E}{ds} = 0$$

where *CS* and CS_E are the consumer surpluses, derived from the integrals in (29):

$$CS = \frac{x\cdot(\bar{p}-p)}{2}\,,\quad CS_E = \frac{X_E\cdot(p-\bar{p}_E)}{2}$$

The derivatives of the consumer surplus with respect to *s* are:[5]

[5] The proofs are given in the Appendix

(33) $$\frac{dCS}{ds} = \frac{x}{\alpha} \quad \frac{dx}{ds} > 0 \ ,$$

(34) $$\frac{dCS_E}{ds} = \frac{X_E}{\beta} \quad \frac{dX_E}{ds} < 0 \ .$$

A subsidy increases home consumer surplus as it increases x and it decreases consumer surplus from imports as X_E decreases. The next terms in (32) are $\pi_p = 0$ because of (19), $\pi_s = x_E$, and

(35) $$\pi_P = (p - c_{\bar{x}}) \frac{\partial \, \bar{x}}{\partial \, P} + s \cdot \frac{\partial \, x_E}{\partial \, P}$$

and finally,

(36) $$s \, \frac{dx_E}{ds} = s \left(\frac{\partial \, x_E}{\partial \, p} \, \frac{dp}{ds} + \frac{\partial \, x_E}{\partial \, P} \, \frac{dP}{ds} \right)$$

Solving (32) for *s*, using (35) and (36) yields:

(37) $$-s \, \alpha_E \, \frac{dp}{ds} = \frac{dCS}{ds} + \frac{dCS_E}{ds} + \pi_P^0 \, \frac{dP}{ds}$$

where

$$\pi_P^0 = 2 \, \beta (p - c_{\bar{x}}) \geq 0$$

is π_P in (35) but without $s \cdot \frac{\partial \, x_E}{\partial \, P}$ which cancels out if π_P is inserted in (32). Next, we divide both sides by $-\frac{dp}{ds}$:

(38) $$s = \frac{1}{\alpha_E \left| \frac{dp}{ds} \right|} \left[\frac{dCS}{ds} + \frac{dCS_E}{ds} + \pi_P^0 \, \frac{dP}{ds} \right]$$

Since not all three terms in the bracket are positive, there is an incentive for an unilateral subsidization in some market and for a taxation of exports in other market. The first term, $\frac{dCS}{ds} > 0$, supports a subsidization. Although a lower P shifts the demand curve for the domestic output x downwards, the lower own price p dominates the consumers' surplus reducing effect as $\frac{dx}{ds}$ increases. However, $\frac{dCS_E}{ds} < 0$; again, a lower P increases import demand X_E, but the effect of an even lower price cut of p reduces import demand. Since $\frac{dX_E}{ds} < 0$, consumers lose surplus from consuming less imports X_E and they gain surplus from consuming more domestic output x. Therefore, a necessary condition for $s > 0$ is that the positive CS effect dominates the negative CS_E effect. A sufficient condition is that this positive net effect dominates the negative effect of a subsidy on profit as the foreign firm responds to a lower p by also lowering its price P. Our central question will be to characterize markets where this total net effect will become positive.

Let us first consider the case that the goods are independent; i.e. $\beta = 0$. Then is $\frac{dP}{ds} = 0$ and CS_E is a constant, i.e. $\frac{dCS_E}{ds} = 0$. Furthermore, profit will not change either as the competitor does not respond to s. In that case, the subsidy is $s = \frac{x}{\alpha_E}$, or

(39) $$s = \frac{p}{\left|\varepsilon_{x_E,p}\right| \cdot \frac{x_E}{x}} > 0$$

where $\left|\varepsilon_{x_E,p}\right| = \frac{\alpha_E p}{x_E}$ is the export price elasticity. If the export price elasticity is high, the subsidy is low, and it increases if $\frac{x_E}{x}$ is low. The inducement to introduce a subsidy is to raise the share of domestic production $\bar{x}$ to foreign production $\bar{X}$ as $\frac{d\bar{x}}{ds} > 0$ and $\frac{d\bar{X}}{ds} = 0$. The other extreme case is that the goods are perfect substitutes, i.e. $\alpha = \beta = \alpha_E$. Now the inducement is to tax exports:

$$s = -\frac{X_E + x_E}{\alpha} = -\frac{2\alpha_0^E}{\alpha} < 0$$

It is $\frac{dx}{ds} = \frac{dx_E}{ds} > 0$, but $\frac{dX_E}{ds} = -\frac{dx_E}{ds}$ and $\frac{d\overline{x}}{ds} = -\frac{d\overline{X}}{ds}$. There is no way to change the balance of trade and no need to pay a subsidy. The higher the trade volume, the higher the tax. It is s = S and there is no tax competition between the governments since the tax is a constant.

A standard case in oligopoly theory is to assume symmetry, i.e. $\alpha_0^E = \alpha_0$, $c_{\overline{x}} = C_{\overline{X}}$ and $\alpha = \alpha_E$. In that case it is $x = x_E = X = X_E$. The following proposition can be proven:

Proposition 1: If the firms are symmetric, then there is a unilateral incentive to introduce an export subsidy $s > 0$ if $\alpha^2 - 3\beta^2 > 0$.

If we use $\frac{\beta^2}{\alpha^2}$ as an index of product differentiation since this expression uniformly increases from zero to unity as the goods range from being independent ($\beta = 0$) to perfect substitutes ($\alpha = \beta$), then this index should be less than 1/3. We will call those goods which satisfy this condition, weak substitutes. The lower β, the lower π_P^0 and $\frac{dP}{ds}$, and hence the negative impact on profit from the price cut of the competitor. In that case, the positive consumer surplus effect dominates the other two negative effects, and s is a subsidy. Examples of weak substitutes are goods with a wellknown brand name like French champagne or German luxury cars. It is of national interest for some cuntries to expand or to keep the export share of those products. In this category belong also goods which seem to be good substitutes but are not in reality. The French auto makers, for example, are strong only in their own market. On the back of a government incentive scheme that helped small-car buyers sales climbed rapidly in the French market in 1996. There is a fight for market shares in Europe's auto market in which the weakest, those with high costs and a narrow market base, will find themselves in real trouble. National pride, always linked to car manufacture, stands in the way of cross-border mergers and the closing of plants.

Another special case of our model could be a multinational firm located in one country. If it is located in the domestic country, then $X_E = 0$ and $x = \alpha_0 - \alpha\, p$. The subsidy then is determined by the following formula:[6]

Proposition 2: The incentive of a government to subsidize its multinational firm is:

$$s = \frac{1}{A}(2x\alpha\,(\alpha + \alpha_E) - \beta^2\,\overline{x})$$

[6] For a proof see the Appendix.

where $A = \alpha_E \, (2\alpha\,(\alpha + \alpha_E) - \beta^2) > 0$. A sufficient condition for $s > 0$ is that x_E is not greater than $3x$. Since exports x_E are normally not greater than three times production for the domestic market, a subsidy $s > 0$ and not a tax $s < 0$ is very likely.

We finally consider the case that the goods are complements, i.e. $\beta < 0$.

Proposition 3: The sign of the subsidy s will not change with the sign of β. The inducement to subsidize does not depend on whether the goods are substitutes ($\beta > 0$) or complements ($\beta < 0$).

In order to prove this statement, we show that the expression for s does only depend on β^2. The formula for s is:

$$s = \frac{1}{B}\left[x\,(\alpha + \alpha_E) - \beta^2 \, \frac{x}{\alpha} - X_E \, \alpha - \frac{2\beta^2 \, \bar{x}}{(\alpha + \alpha_E)} \right]$$

where $B = \left[(\alpha + \alpha_E)^2 - 2\beta^2)\right] \dfrac{\alpha_E}{(\alpha + \alpha_E)} > 0$.

We finally turn to the general case. For that purpose we use (33) and (34) to rewrite (38):

$$(40) \qquad s = \frac{-1}{\alpha_E \dfrac{dp}{ds}} \left[\frac{x}{\alpha} \, \frac{dx}{ds} + \frac{X_E}{\beta} \, \frac{dX_E}{ds} + \pi_P^0 \, \frac{dP}{ds} \right]$$

or, by introducing price elasticities:

$$(41) \qquad s = \frac{1}{\alpha_E \left| \dfrac{dp}{ds} \right|} \left[\frac{p}{\left| \varepsilon_{x,p} \right|} \, \frac{dx}{ds} + \frac{p}{\varepsilon_{X_E,p}} \, \frac{dX_E}{ds} + \pi_P^0 \, \frac{dP}{ds} \right]$$

where $\left| \varepsilon_{x,p} \right| = \alpha \cdot \dfrac{p}{x}$ and $\varepsilon_{X_E,p} = \beta \cdot \dfrac{p}{X_E}$. An incentive for a subsidy $s > 0$ can be derived, if the price elasticity $\left| \varepsilon_{x,p} \right|$ for domestic goods is low and the price elasticity for imported goods $\varepsilon_{X_E,p}$ is rather high at the Cournot-Nash equilibrium. An inducement for an export tax $s < 0$ is possible if the reverse holds; the price elasticity $\left| \varepsilon_{x,p} \right|$ is high and the price elasticity for imported goods $\varepsilon_{X_E,p}$ is rather low. How could such a market be characterized where taxation of exports is a reasonable strategy? It could be a market where domestic marginal cost are rather high and therefore p is high. Due to the high price elasticity for domestic goods, consumers buy low quantities x and prefer the imported good which is cheaper due to lower marginal costs. If the government puts the same weight on consumer and on producer surplus, then a tax on exports is a collectively rational trade policy. In reality, however, there will be no incentive to introduce such a tax, because governments do not wish to increase the flow of imported goods just to raise consumer surplus from imported goods. They are interested in the shift of profit to domestic firms, represented by the term Π_P^0 , and in raising consumer surplus due to lower domestic prices, i.e. because of $\frac{dp}{ds} < 0$. A government would therefore put higher weights on these two aspects in its welfare function and would give the consumer surplus effect from imports a low weight.

Since there is subsidy competition for shifting market shares, the action of both governments must be considered. The Nash equilibrium which occurs if each government acts independently given the subsidy rate set by the other government, can be obtained by solving two first order conditions with respect to s and S; i.e $\frac{dw}{ds} = 0 \quad and \quad \frac{dW}{dS} = 0$. Since the subsidy program of each government affects welfare of the other government only indirectly, i.e. through ist impact on prices, the subsidy rates in the Nash equilibrium will have the same structure as in (40).

4. Import tariffs under price competition

Since the role of export subsidies as an instrument for interventionist trade or industrial policy is quite limited, we will analyze in this section the inducement to introduce taxes on imports. Before we try to find cases for strategic import taxes by governments, we state the problem of the firm, given a tax on imports:

$$(42) \qquad \max_p \quad \pi(p,P;t) = p \cdot x(p,P+t) + p \cdot x_E(p+T,P) - c(\bar{x})$$

where $\bar{x} = x(p,P+t) + x_E(p+T,P)$ is total domestic production. Similarly, profit of the foreign firm is:

(43) $$\max_{P} \quad \Pi\,(p,P;T) = P\cdot X\,(p+T,P) + P\cdot X_E\,(p,P+t) - C\,(\overline{X}).$$

The f.o.c. are:

(44) $$-(\alpha+\alpha_E)\,(2p-c_{\bar{x}}) + 2\beta P + \beta t + \alpha_0 - \alpha_E T + \alpha_0^E = 0$$

(45) $$-(\alpha+\alpha_E)\,(2P-C_{\overline{X}}) + 2\beta p + \beta T + \alpha_0 - \alpha_E t + \alpha_0^E = 0$$

where we used again the linear demand functions as specified in (21) - (24).

In order to derive the direction of the price changes if in the first stage of the game the governments change the import tax rates, we do again comparative statics. The outcome is:

(46) $$\frac{dp}{dt} = \frac{dP}{dT} = \frac{2\alpha\beta}{D} > 0 \quad , \quad \frac{dp}{dT} = \frac{dP}{dt} = \frac{-2\,(\alpha_E(\alpha+\alpha_E)-\beta^2)}{D} < 0$$

where $D > 0$ as in (27). The equilibrium level of prices increases if the own government taxes imports, and it decreases if the other government raises import taxes. This reaction implies for the quantities:[7]

(47) $$\frac{dx}{dt} > 0\,,\ \frac{dx_E}{dt} < 0\,,\ \frac{dX}{dt} > 0\,,\ \textit{and}\ \ \frac{dX_E}{dt} < 0\,.$$

A tax on imports will promote domestic production *x*, of course, but will lower export demand x_E as the domestic firm raises its price whereas the foreign firm lowers its price. The demand for foreign export X_E will be lower, but the foreign firm can partly compensate this loss in output by a higher supply on its own market.

Taxes on imports can be justified in cases of negative externalities, which must be reduced by taxing the polluting activities. For instance, if a nation suffers from transborder pollution from foreign production. Examples are foreign producers of domestic demand for steel causing acid rain in the domestic country, or upstream paper mills in the foreign country causing costs of waste dumping in the downstream (domestic) country which affects the

[7] To check the signs, the demand functions (21) - (24) have to be differentiated and then (46) has to be used.

quality of the river services (scenery, swimming, fishing).[8] In such a situation the only way in which the domestic country can discourage the foreign pollution is by taxing imports of the product made by a polluting process. Imports can also be taxed if they emit CFCs or contribute to the carbon dioxide building up from fossil fuel, provided the domestic country has banned CFCs or wishes to reduce carbon dioxide (energy tax, gasoline tax). In the latter case there is in reality no intra-industry trade so that the foreign country can not respond by initiating a tax competition.

To show that an inducement to tax imports exists we analyze the first stage of the game. The objective function of the domestic government, given a foreign tax on imports (which can be zero, of course), is:

$$(48) \qquad \max_t \quad w(t,T) = \int_p^{\bar{p}} x(\xi, P+t)\, d\xi + \int_{\bar{p}_E}^{p} X_E\ (\xi, P+t)\, d\xi + \pi\ (p, P; t, T) + t \cdot X_E\ (p, P+t)$$

where the „choke“ prices are

$$(49) \qquad \bar{p}\ (P+t) = \frac{\beta\,(P+t) + \alpha_0}{\alpha} \quad , \quad \bar{p}_E\ (P+t) = \frac{\alpha_E\ (P+t) - \alpha_E^0}{\beta} \ .$$

The f.o.c. of (48) is:

$$(50) \qquad \frac{d\,w\,(t;T)}{dt} = \frac{d\,CS}{dt} + \frac{d\,CS_E}{dt} + \pi_p\ \frac{dp}{dt} + \pi_P\ \frac{dP}{dt} + \pi_t + X_E + t\ \frac{d\,X_E}{dt} = 0 \ .$$

The formulas for the consumer surpluses are the same as derived in the previous section. Furthermore, it is $\pi_p = 0\,, \pi_t = \beta\,(p - c_{\bar{x}})$ and $\pi_P = 2\,\beta\,(p - c_{\bar{x}})$. The tax rate,or optimal response function to a foreign import tax, is therefore

$$(51) \qquad t = \frac{-1}{\dfrac{d\,X_E}{dt}} \left[\frac{d\,CS}{dt} + \frac{CS_E}{dt} + \beta\,(p - c_{\bar{x}}) \left(2\,\frac{dP}{dt} + 1 \right) + X_E \right]$$

where

[8] See Lindert and Pugel (1996), International Economics, Chapter 12, 9th edition.

$$\frac{d\,CS}{dt} = \frac{x}{\alpha}\,\frac{dx}{dt} > 0 \quad , \quad \frac{d\,CS_E}{dt} = \frac{X_E}{\beta}\,\frac{d\,X_E}{dt} < 0$$

can be derived in a similar way as done for (33) and (34). Since[9] $\frac{dP}{dt} > -\frac{1}{2}$, the third term in the brackets is positive. If the three positive terms dominate the only negative term, the loss in consumer surplus from imports X_E, then t is an import tax and not a subsidy. Since in reality $x > X_E$, we expect that the consumer surplus effect $\frac{d\,CS}{dt}$, the positive net effect on profit from the import tax $\left(\pi_P \frac{dP}{dt} + \pi_t\right)$ and the tariff revenue effect $\left(\frac{d\,(t \cdot X_E)}{dt} = X_E \text{ , } given\ X_E\right)$ dominate the negative consumer surplus effect from reduced imports that must be set against it.

Again, import tax competition between government could be considered resulting in a Nash equilibrium in import tax rates. If, for example, one government taxes imported coal on environmental grounds, the other government might tax imported wine or cognac for reasons of health.

5. Concluding remarks

The purpose of our analysis has been to resume the question how comparative advantages can be created by subsidies or taxes if firms compete in prices. Our point of departure has been that strategic trade models are supposed to be not robust with respect to their policy recommendation which depends on whether there is price or quantity competition. Since price competition prevails in high-tech industries as semiconductors, computers, telecommunications or aircrafts, it is of interest to show whether strategic trade policy suggests to encourage such industries as has been shown if there is quantity competition. These high-tech industries are characterized by intra-industry trade, require large-scale production to achieve economies of scale, and give rise to extensive external economies when successful. Most nations encourage development in these industries, although not by taxing imports or subsidizing exports, but by financing research and development, granting tax advantages for investment in the industry, while protecting the domestic market from foreign

---

[9] For a proof see the Appendix

competition. It is, however, no problem to reformulate our intra-industry price competition model in order to analyze international R&D rivalry and industrial strategy.[10] While strategic trade policy can theoretically improve the market outcome in oligopolistic markets subject to extensive external economies and increase the country's growth and welfare, we are fully aware of the serious difficulties in implementing it. However, although the role of taxes and subsidies in international trade is quite limited, a stringent analysis of an inducement to introduce these instruments and of their effects on domestic production and trade flours should be part of the theory on strategic trade under imperfect competition. Our objective was only to weaken the argument that strategic trade models are of limited use, because the particular policy recommendation depends critically on the nature of competition.

[10] See Brander and Spencer (1983) where competition is in terms of quantities.

References

Brander, J.A. (1981), „Intra-Industry Trade in Identical Commodities“, *Journal of International Economics*, 11, 1-14.

Brander, J.A. and B.J. Spencer (1985), „Export Subsidies and International Market Share Rivalry“, *Journal of International Economics*, 18, 83-100.

Brander, J.A. and B.J. Spencer (1983), „International R&D Rivalry and Industrial Strategy“, *Review of Economic Studies*, 707-722.

Dixit, A. (1984), „International Trade Policy for Oligopolistic Industries“, *Economic Journal*, supplement, 94, 1-16.

Eaton, J. and G.M. Grossman (1986), „Optimal Trade and Industrial Policy under Oligopoly“, *The Quarterly Journal of Economics*, 101, 383-406.

Krugman, P.R. (1989), „International Organization and International Trade“, in: R. Schmalensee and R.D. Willig, *Handbook of Industrial Organization*, Vol.II, Chapter 20, Elsevier Science Publishers B.V.

Appendix

The demand functions in (21) and (22) are derived from an indirect quadratic utility function where some parameters have been set equal to zero a priori (*m* is income):

$$v(p,P,m) = v_0 + \tilde{\alpha}_0 p + \tilde{\alpha}_0^E \; P + \tilde{\alpha}_m \cdot m + 0.5\,\tilde{\alpha}\; p^2 + 0.5\,\tilde{\alpha}_E \; P^2 + \tilde{\alpha}_{12} \; p\,P$$

Using Roy's identity, we obtain linear demand functions:

$$x = \frac{-\dfrac{\partial v}{\partial p}}{\dfrac{\partial v}{\partial m}} = -\frac{(\tilde{\alpha}_0 + \tilde{\alpha}\; p + \tilde{\alpha}_{12}\; P)}{\tilde{\alpha}_m}$$

$$X_E = \frac{-\dfrac{\partial v}{\partial p}}{\dfrac{\partial v}{\partial m}} = -\frac{(\tilde{\alpha}_0^E + \tilde{\alpha}_E \cdot P + \tilde{\alpha}_{12}\; p)}{\tilde{\alpha}_m}$$

Defining the parameters $\beta = -\dfrac{\tilde{\alpha}_{12}}{\tilde{\alpha}_m}$, $\alpha = \dfrac{\tilde{\alpha}}{\tilde{\alpha}_m}$, $\alpha_E = \dfrac{\tilde{\alpha}_E}{\tilde{\alpha}_m}$, $\alpha_0^E = -\dfrac{\alpha_0^E}{\tilde{\alpha}_m}$, and $-\dfrac{\tilde{\alpha}_0}{\tilde{\alpha}_m} = \tilde{\alpha}_0$ yields the demand functions in (21) and (22). Of course, neither the utility function nor the demand functions are homogeneous of degree zero in *p*, *P* and *m*.

Proof of (27)

The f.o.c. (19) and (20) with linear demand function and CRTS are:

(A1) $\quad \bar{x} - (p - c_{\bar{x}})\,(\alpha + \alpha_E) = s\,\alpha_E$

(A2) $\quad \bar{X} - (P - C_{\bar{X}})\,(\alpha + \alpha_E) = S\,\alpha_E$

Total differentials yields:

(A3) $$\begin{aligned} -2\,(\alpha+\alpha_E)\,dp + 2\,\beta\,dP &= \alpha_E\,ds \\ -2\,(\alpha+\alpha_E)\,dP + 2\beta\,dp &= \alpha_E\,dS \end{aligned}$$

To derive (27) is standard algebra.

Proof of (33) and (34)

$$\frac{dCS}{ds} = \frac{1}{2}\,(\bar{p}-p)\left(-\alpha\,\frac{dp}{ds}+\beta\,\frac{dP}{ds}\right)+\frac{x}{2}\left(\frac{\beta}{\alpha}\,\frac{dP}{ds}-\frac{dp}{ds}\right)$$

$$= \frac{1}{2}\,\frac{dp}{ds}\left[-\alpha\,(\bar{p}-p)-x+\beta\,\frac{\frac{dP}{ds}}{\frac{dp}{ds}}\left(\bar{p}-p+\frac{x}{\alpha}\right)\right]$$

Next, it is:

$$-\alpha\,(\bar{p}-p)-x = -\alpha\left(\frac{\beta}{\alpha}\,P+\frac{\alpha_0}{\alpha}-p\right)-x = -2x$$

and

$$\bar{p}-p+\frac{x}{\alpha} = \frac{\beta}{\alpha}P+\frac{\alpha_0}{\alpha}-p+\frac{x}{\alpha} = 2\,\frac{x}{\alpha}$$

Therefore,

$$\frac{dCS}{ds} = -x\,\frac{dp}{ds}+\beta\,\frac{dP}{ds}\,\frac{x}{\alpha} = \frac{x}{\alpha}\left(-\alpha\,\frac{dp}{ds}+\beta\,\frac{dP}{ds}\right) \Rightarrow (33)$$

Furthermore,

$$\frac{dCS_E}{ds} = \frac{1}{2}\,(p-\bar{p}_E)\left(-\alpha_E\,\frac{dP}{ds}+\beta\,\frac{dp}{ds}\right)+\frac{X_E}{2}\left(\frac{dp}{ds}-\frac{\alpha_E}{\beta}\,\frac{dP}{ds}\right)$$

$$= \frac{1}{2}\,\frac{dp}{ds}\left[(p-\bar{p}_E)\beta + X_E + \frac{\frac{dP}{ds}}{\frac{dp}{ds}}\,(-\alpha_E\,(p-\bar{p}_E)-\frac{\alpha_E}{\beta}\,X_E\right]$$

It is:

$$(p-\overline{p}_E)\beta + X_E = \beta\, p - \alpha_E\, P + \alpha_0^E + X_E = 2\, X_E$$

and

$$-\alpha_E (p-\overline{p}_E) - \frac{\alpha_E}{\beta}\, X_E = -\alpha_E\;\, p + \frac{\alpha_E}{\beta}\,(\alpha_E\;\, P - \alpha_0^E) - \frac{\alpha_E}{\beta}\, X_E = -\frac{\alpha_E}{\beta}\, 2\, X_E$$

Therefore:

$$\frac{dCS_E}{ds} = X_E\,\frac{dp}{ds} - \frac{\alpha_E}{\beta}\, X_E\,\frac{dP}{ds} = \frac{X_E}{\beta}\left(\beta\,\frac{dp}{ds} - \alpha_E\,\frac{dP}{ds}\right) \Rightarrow (34)$$

Multinational firm:

$$X_E = 0\,, \qquad x = \alpha_0 - \alpha\, p$$

Profit of the multinational firm:

$$\pi = p\cdot(\alpha_0 - \alpha\, p) + p\cdot x_E - c(\overline{x}) + s\, x_E$$

Profit for the foreign firm:

$$\Pi = P\cdot X(p,P) - C(X)$$

The f.o. conditions are:

(19') $\overline{x} - (\alpha + \alpha_E)(p - c_{\overline{x}}) - s\,\alpha_E = 0$

$X - \alpha\, P - \alpha\, C_X = 0$

For the comparative statics we obtain:

(20') $-2\,(\alpha + \alpha_E)\, dp + \beta\, dP = \alpha_E\;\, ds$

$\beta\; dp - 2\;\alpha\;\, dP = 0$

It is

$$D = 4\,\alpha\,(\alpha + \alpha_E) - \beta^2$$

and

$$\frac{dp}{ds} = \frac{-2\,\alpha_E\,\alpha}{D} < 0 \qquad \text{and} \qquad \frac{dP}{ds} = -\frac{\beta\,\alpha_E}{D}$$

It is

$$\frac{dx}{ds} = -\alpha\,\frac{dp}{ds} = \frac{2\,\alpha^2\,\alpha_E}{D}$$

The formula for the subsidy is now:

$$(38') \qquad s = \frac{1}{\alpha_E \left|\frac{dp}{ds}\right|}\left[\frac{dCS}{ds} + \pi_P^0\,\frac{dP}{ds}\right]$$

where

$$\pi_P^0 = \beta\,(p - c_{\bar{x}}) \;, \qquad \text{and} \qquad p - c_{\bar{x}} = \frac{(-s\,\alpha_E + \bar{x})}{(\alpha + \alpha_E)}$$

follows from (19'). Replacing π_P^0 in (38') yields:

$$s\,\alpha_E\left[\left|\frac{dp}{ds}\right| + \frac{\beta}{\alpha + \alpha_E}\,\frac{dP}{ds}\right] = \frac{x}{\alpha}\,\frac{dx}{ds} + \frac{\beta\,\bar{x}}{\alpha + \alpha_E}\,\frac{dP}{ds}$$

By using the expressions, derived for $\frac{dp}{ds}, \frac{dP}{ds}$, and $\frac{dx}{ds}$, we obtain:

$$s\,\alpha_E\,\frac{\left[2\,\alpha\,(\alpha + \alpha_E) - \beta^2\right]}{\alpha + \alpha_E} = 2\,x\,\alpha - \frac{\beta^2\,\bar{x}}{\alpha + \alpha_E}$$

or

$$s = \frac{1}{\alpha_E \left[2\,\alpha\,(\alpha + \alpha_E) - \beta^2\right]}\;(2\,x\,\alpha\,(\alpha + \alpha_E) - \beta^2\;\bar{x})$$

If we use the inequalities $\alpha > \beta$, $\alpha_E > \beta$, then a sufficient condition for $s > 0$ is that x_E is not greater than $3x$.

Proof of $\frac{dP}{dt} > -\frac{1}{2}$ in (51):

It is

$$\frac{dP}{dt} = \frac{-2}{D}(\alpha_E\;(\alpha + \alpha_E) - \beta^2) = -\frac{2}{4}\,\frac{(\alpha_E\;(\alpha + \alpha_E) - \beta^2)}{(\alpha + \alpha_E)^2 - \beta^2} > -\frac{1}{2} \qquad \text{Q.e.d.}$$