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# A Theory of Production with Waste and Recycling

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## Abstract

The management of solid waste has become an urgent problem. Product responsibility means that a product will accompany its producer from cradle to grave; prevention, recycling and disposal of waste are part of a theory of the firm which we develop under solid residual management. We assume that the government stimulates firms to enhance recycling of resources by a fee on waste. A comparative statics analysis shows the impact of a fee on waste reduction, on the structure of the production process, on recycling, on input demand, material saving effort, number of firms, and on the amount of waste disposal.

# A Theory of Production with Waste and Recycling

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## 1. Introduction

Given increasing rates of GDP growth and of population in most countries of the world, the management of solid waste becomes more and more an urgent problem. Recycling of depletable natural resources is one way to mitigate this problem. Recycling means the re-processing and re-utilization of solid (or liquid) waste as a resource at the wrong place in the production line and in a wrong consistency. In a law in Germany on resource circular flow and waste economics enacted in October 1996, the new element is the product responsibility of the pollutor or of the producer of waste. Producers of waste are personally responsible for abating their waste. Contrary to the former law, now private abatement is the rule and state or urban abatement activities are the exception. The producer has to prove that the required treatment technologies and pollution prevention measures have been installed and do function. This means that a product will accompany its producer from cradle to grave. It should be multiple usable, technically long lasting and easy to dispose. The order of priorities is waste prevention, waste recycling and waste dispose.

As the title of this paper signals there is to our knowledge no theory of the firm which deals explicitly with waste prevention, waste recycling, and waste disposal. There are models which analyse these aspects but not a neoclassical model which integrates all three aspects. Although the economics of exhaustible resources deals with models of recycling and pollution control, these models mostly analyse the aspect of intertemporal allocation using highly aggregated optimization models. This strand of the literature looks into the interaction of stocks and flows of natural resources including those of recycleable resources.[2] Discounted social welfare is maximized and optimal time paths for consumption and resource utilization are derived. These paths are based on the interdependency between extraction of the resource and the piling up or reduction of the stock of recycleable waste. A second strand of the literature deals with linear programming models.[3] These models take into account the costs and benefits of alternative recycling processes for different sorts of waste, the different possibilities to use waste-inputs, the cost of transportation, and standard for air and water quality. These models are useful for applications but they can not be used to characterize producers' behavior in terms of responses to changes in relative prices in a general way. A third strand of the literature advances public policies that encourage recycling in order to reduce environmental costs from waste disposal. Policies that have been proposed are taxes on the use of virgin materials, deposit/refund programs, subsidies to recycled material production, and recycled content standards (see Sigman (1995), Niel and Foshee (1992), Anderson and Spiegelman (1977), Rousso and Sha (1994), and Dinan (1992)). These papers analyse inefficiencies related to solid waste management and promote policies of efficient waste disposal and recycling practices. These questions are often addressed by building models of household solid waste behavior (Fullerton and Kinnaman (1995), Dobbs (1991), Morris and Holthausen (1994), Wertz (1976)).

The point of departure for our model is the standard argument of the opponents to the neoclassical theory of production that this theory assumes production to be efficient. There can be no waste of resources, because all processes characterized lie exactly on the frontier of the production function. But there exist no production activities which transform material / energy inputs completely in desirable goods; waste will always be a by-product. This waste can be reduced during the production process by (labour-) cost increasing efforts or it can be partly recycled at the "end of the pipe" by using labour as an input. Waste not completely recycleable or not recycled will leave the firm as disposal which also entails cost to the firm. We will derive efficient production functions from production functions with waste as a by-product and will use the corresponding cost function to develop a theory of the firm under solid residuals management.

The goal of a waste intensive firm is to remain competitive in the market, whereas the concern of the government is about waste accumulation in an economy. Aggregated over all firms (and consumers), the economic system leaves behind a huge amount of refuse which in turn has a negative impact on the economic system. If this process of environmental deterioration in quality and quantity continues, the conditions for future economic activities (regional and world-wide) will be seriously endangered. In our paper we assume that the government aims at stimuling the firms to enhance recycling of resources. This activity should yield a “double dividend“ in terms of preservation of depletable resources as well as in preventing the degradation of the quality of the environment. An instrument to stimulate recycling activities could be a fee on waste (solid or fluid waste). Such a fee is an indirect way to increase the price of material and hence will induce the firm to substitute other inputs for material, to promote material saving technical progress, and to encourage recycling processes. The tax system can also be used to promote recycling by taxing virgin materials and by subsidizing recycling activities.

The paper is organized as follows. The next section is devoted to a model of the firm which includes material saving efforts, recycling and the use of waste inputs. In section 3 a comparative statics analysis shows the impact of a fee on waste on the effort to reduce waste by restructuring the production process. In section 4 and 5 we carry out a comparative statics analysis of waste intensive firms operating in different market structures. In section 4 we assume a perfectly competitive market, and in section 5 a market under imperfect competition. Section 6 concludes the paper.

## 2. A simple model of production with waste as a byproduct and with recycling

We begin with a production technology which produces a good of quantity x with two inputs, labor ($L$) and gross material ($GM$):

**(1)** $$x = F\ (L, GM)$$

By gross (or raw) material we mean material input with a byproduct waste which reduces the efficiency of the production process. We could think of wood or iron where some percentage is waste and does not enter into the final output. We therefore distinguish between gross or crude material and net or pure material ($M$) input where

**(2)** $$M = (1-\alpha)\,GM \quad , \quad 0 \le \alpha \le 1 \quad , \text{ and } \; GW = \alpha \cdot GM$$

with *GW* as gross waste and α as the waste coefficient. By gross waste we mean that there is a potential to reduce it to net waste using recycling processes. The problem with the function in (1) is that it is not a production function because it does not characterize efficient production. The net material input has to be the appropriate argument in $F(\cdot)$ for $F$ to be a production function:

**(4)** $$x = f(L,M)$$

Our figure shows the relationship between inefficient production and the production function where inefficiency increases with α. As can be seen from the production function in the fourth orthand, the firm with the higher α is more productive. In order to produce four units of output, the firm with α = 0.2 requires 3.2 units of *M*, i.e. it has to purchase 4 units of *GM*. If a firm with α = 0.4 has also produced 4 units of output by using 4 units of *GM*, then it has produced those four units of *x* with only 2.4 units of *M*. In other words: if the more productive firm would reduce α from α = 0.4 to α = 0.2, then it could produce with four units of *GM* more than four units of output (about 4.6 in the Fig.).

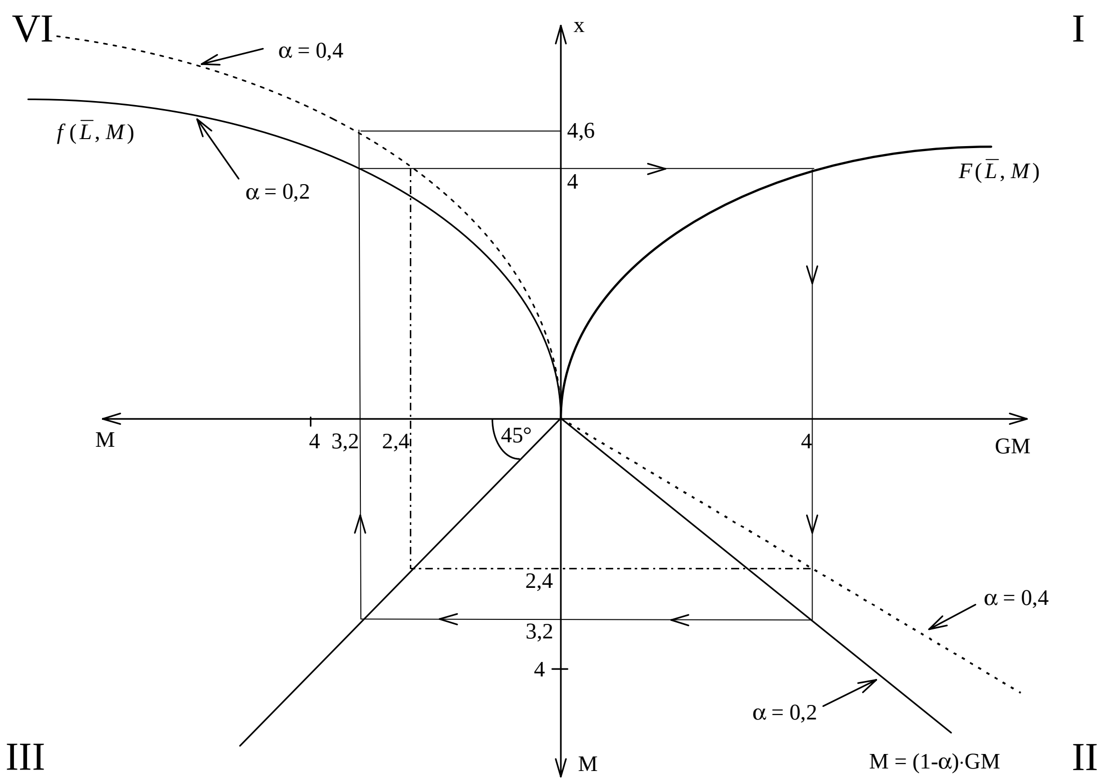


Fig. 1 Inefficient production $F(\bar{L}, GM)$ and the production function under different waste coefficients α, given $\bar{L}$.

However, to reduce the waste coefficient is costly because it requires time and hence cost of labor. Also machinery equipment has to be more precise and gross material has to be of higher quality. We denote with $e$ the effort to reduce waste emission and assume $\alpha(e)$ to be a function of this effort ($\alpha'(e) < 0$, $\alpha''(e) > 0$). In addition, we assume that waste entails costs, $PW \cdot GW$, where $PW$ is the price of waste and could be a tax or fee on waste, the cost of waste collection and of transporting a unit of waste to the waste site, the cost of the incinerator or the cost of waste treatment per unit of waste. We include $e$ also as an argument in the production function in order to represent the aspect that a higher effort in avoiding waste will reduce productivity. Using (2) and (3), we cast the standard cost minimization problem $PL \cdot L + PGM \cdot GM + PW \cdot GW$ s.t. $x = F(L, GM)$ such that the quantity of net material $M$ and not of gross material $GM$ is the input the firm focuses on:

**(5)** $$\min_{L,M,e} \quad PL \cdot L + PGM \cdot M / (1-\alpha(e)) + PW \frac{\alpha(e)}{1-\alpha(e)} M$$

s.t. $\qquad x = \mathrm{f}\,(L, M, e)$ with $f_e < 0$[4]

and $PGM$ $(PL)$ as the price of gross material (labor).

We next assume that gross waste can either be used directly (totally or partly) or after further reprocessing for use as waste-input. This transformation process from waste into waste-inputs characterizes a recycling process. It comprises activities like a) the re-use of goods (e.g. bottles); b) regaining of resources once used to make the product (direct recycling; e.g. ferrous scrap as a waste-input); c) further utilization of goods (indirect recycling; e.g. energy product from organic waste). The new German law on the responsibility of the firm for the circulation of its products supports our characterization of a production process using material and waste-inputs. A steel company has to produce steel from virgin ores as well as from ferrous-scrap. A tire company will produce new vehicle tires as well as round-renewed ones. The relationship $\beta$ between recycled waste (RW) and gross waste (GW) is

**(6)** $$RW = \beta \cdot GW = \beta \quad \frac{\alpha(e)}{1-\alpha(e)} \quad M$$

where β is the share of waste to be recycled. This share or degree of recycling is defined as waste to be recycled divided by maximum waste ($0 \leq \beta \leq 1$). Hence waste which can not be used anymore and which then entails the cost of disposal, is

**(7)** $$W = (1-\beta)\ \frac{\alpha(e)}{1-\alpha(e)}\ M$$

where *W* means the dust of waste. In modeling disposal decisions, it is important to recognize that potentially recyclable waste can be divided into two types of scrap: old scrap and new scrap. New scrap is composed of the residual materials generated during production. For example (see Tietenberg (1994) on the importance of new scrap and old scrap), as steel beams are formed, the small remnants of steel left over are new scrap. Old scrap is recovered from products used by consumers. In the US aluminum industry, a total of 1.1 billion metric tons of aluminum were recovered in 1987 from new scrap, and about the same amount were recovered from old scrap. Our model deals mainly with recycling new scrap which is significantly less difficult than recycling old scrap. New scrap is already at the place of production, and with most processes it can simply be reentered into the input stream without transportation cost. Also because new scrap never leaves the factory, it remains under the complete control of the manufacturer.

The quantity of recycled waste *RW* depends on the cost of recycling. We denote with $c(\beta)$ the cost of recycling measures per unit of waste to be recycled, i.e. cost of recycling is $c(\beta) \cdot RW$ or $c(\beta) \cdot \beta \cdot GW$. The costs per unit depend on the degree of recycling with $c'(\beta) > 0$ and $c''(\beta) \geq 0$, whereas the cost of recycling are proportional to the quantity of waste considered for recycling, i.e. to $\beta \cdot GW$. For some natural resources like minerals the process of recycling can be carried out repeatedly or for even an infinite number of times. In such a case marginal costs might increase and the quality of the waste to be recycled will decrease. For reason of simplicity we will restrict the recycling process to one re-processing process where waste from the recycling activity has then costly to be treated or removed. This assumption implies that the physical or chemical structure of the once recycled material has changed so much that it is too costly or even impossible to recycle it again.[4] If the firm has determined the quantity of waste *RW* to be considered for recycling, its objective is to produce net material input, *MW*, using *RW* and labor *LW*. We assume a Leontief production technology, i.e.

**(8)** $$MW = \min \quad [RW \cdot aw, LW \cdot al].$$

We finally can rewrite our objective function in terms of all cost components:

**(9)** $$\begin{aligned} &PL \cdot L + PGM \cdot GM + PW \cdot (1-\beta) GW + \\ &c(\beta) \cdot \beta \cdot GW + PL \cdot LW - PGM \cdot MW \end{aligned}$$

Total costs consist of costs for labor and gross material, cost of waste not being recycled, costs of recycling in order to avoid the costs of waste disposal, and the cost of labor employed by the recycling division. As recycled waste is assumed to be a perfect substitute for net material, total costs can be reduced by the value of net material recovered from recycling.

We next express *GM, GW, LW* and *MW* in terms of *M,* using (2), (3) and the production of net material from waste (*MW*) in fixed proportion (see (8)).Then the objective function is:

**(10)** $$\begin{aligned} &\min_{L,M,\beta,e} PL \cdot L + \frac{\alpha(e)}{1-\alpha(e)} [PGM / \alpha(e) + PW \cdot (1-\beta) \\ &+ c(\beta) \cdot \beta + PL \, \frac{aw}{al} \, \beta - PGM \cdot aw \cdot \beta] \cdot M \end{aligned}$$

s.t. $x = \mathrm{f}(L, M, e)$

where we used $LW = aw/al \cdot RW$ from (8). We recognize that this problem can be stated in terms of only two input prices *PL* and *PM*, where *PM* as the price of net material consists of all cost components given in the brackets in front of *M* in (10):

**(11)** $$\begin{aligned} &PM(e,\beta) = \frac{\alpha(e)}{1-\alpha(e)} [PGM / \alpha(e) + PW(1-\beta) \\ &+ c(\beta) \cdot \beta + PL \frac{aw}{al} \beta - PGM \cdot aw \cdot \beta] \end{aligned}$$

Since the cost function, dual to the production function, is more convenient for the analysis we have in mind, we state the problem as one of profit maximization under perfect competition by using a cost function:

$$\max_{x,\beta,e} \Pi = p \cdot x - C(x, PL, PM(e,\beta), e)$$

with *PM* as defined in (11). The decline in productivity from a higher effort *e* is now expressed in terms of $C_e > 0$ and $C_{ee} > 0$. A careful and accurate production process to produce *x* with less waste increases the cost for producing *x*. The f.o.c. with respect to $\beta$ is $C_{PM} \cdot PM_\beta = 0$; i.e.

**(12)** $$c'(\beta) \cdot \beta + c(\beta) + PL \cdot \frac{aw}{al} = PGM \cdot aw + PW$$

from $PM_\beta = 0$. Marginal cost of recycling plus the cost of labor to produce net material *MW* from one unit of recycled waste *RW* should be equal to marginal benefit of recycling in terms of marginal revenue of the recovered *M* plus the cost *PW* saved per unit of waste recycled. The f.o.c. for β can be solved for β to obtain β as a function of the prices *PGM, PL* and *PW*, i.e. $\hat{\beta} = \beta(PGM, PL, PW)$. If we hold *PGM* and *PL* constant, total differentiation of (12) yields:

**(13)** $$\frac{d\beta}{dPW} = (c''(\beta) \cdot \beta + 2c'(\beta))^{-1} > 0$$

If *PW* represents a tax on waste or higher disposal costs, the firm will raise its degree of recycling. Similarly, if we hold *PW* and *PL* constant, we can derive from (12):

**(14)** $$\frac{d\beta}{dPGM} = \frac{aw}{(c''(\beta) \cdot \beta + 2c'(\beta))} > 0 \ .$$

Since $aw = \frac{MW}{RW} < 1$ , the incentive to recycle is higher under a marginal increase in waste fees than under a marginal increase in *PGM*; for instance by a tax on virgin material.

The f.o.c. with respect to *x* is:

**(15)** $$p - C_X(x, PM, PL, e) = 0 \ ,$$

and the f.o.c. with respect to *e* is:

**(16)** $$-M \cdot PM_e - C_e(\cdot) = 0$$

because of Shephard's lemma *($M = C_{PM}$)*. It is $PM_e < 0$ because of $\alpha' < 0$ and $aw \leq 1$:

**(17)** $$PM_e = \frac{\alpha'}{(1-\alpha)^2}\left[PW(1-\beta) + c(\beta)\,\beta + PL\,\frac{aw}{al}\,\beta + PGM(1-aw\cdot\beta)\right] < 0$$

The economic meaning for the negative $PM_e$ is that effort reduces gross waste. $PM_e$ is negative because an increase in effort reduces gross waste. This entails a reduction in fees due to less non-recycled waste (first term in the bracket in (17)), a reduction in recycling costs due to less gross waste (second term), less labor cost for producing material from recycled waste (third term), and a reduction in costs due to more net material from gross material, adjusted by the quantity recovered formerly from recycled waste and which is now gained due to higher effort *e* (fourth term). According to (16), the level of effort *e* is optimal if marginal savings in the cost of net material input justifies exactly the increase in the cost of production from loss in productivity.

## 3. A comparative statics analysis for production with waste as a byproduct

In order to find out the effect of a change in the prices *PGM, PW, PL* on production, effort, and waste, we totally differentiate equations (15) (i.e. $\Pi_x = 0$) and (16) (i.e. $\Pi_e = 0$):

**(18)** $$\Pi_{xx}dx + \Pi_{xe}de = -\Pi_{xPW}dPW - \Pi_{xPGM}dPGM - \Pi_{x,PL}dPL$$

**(19)** $$\Pi_{ex}dx + \Pi_{ee}de = -\Pi_{ePW}dPW - \Pi_{ePGM}dPGM - \Pi_{e,PL}dPL$$

In order to get unambiguous qualitative results as well as for a much simpler analysis it is convenient to find an assumption which implies $\Pi_{xe} = \Pi_{ex} = 0$. Such an assumption is a homothetic production function[5] and we will make it in this section of the paper. The elasticity of output with respect to a higher fee on waste is[6]

**(20)** $$\frac{d \ln x}{d \ln PW} = -\frac{PW \cdot W}{p \cdot x} \; \frac{\eta_{M,x}}{\eta_{MC,x}} < 0$$

where $\eta_{M,x} > 0$ is the elasticity of material input with respect to output and $\eta_{MC,x} > 0$ is the elasticity of marginal cost with respect to output. The higher the cost share of waste, the higher the decline in output. The elasticity of output with respect to the price of gross material *PGM* is

**(21)** $$\frac{d \ln x}{d \ln PGM} = -\frac{PGM \cdot (GM - MW)}{p \cdot x} \; \frac{\eta_{M,x}}{\eta_{MC,x}}.$$

*GM* - *MW* is gross material less material recovered from waste recycling (*MW* in (8)). This is the material input which is affected by the increase in the price *PGM*. The higher this amount of material, the higher the negative impact on output. Since we can assume, that the cost of gross material is higher than the value of material, recovered from waste, including the fee on waste, i.e. $PGM \cdot GM > PGM \cdot MW + PW \cdot W$, the cost share in (21) is higher than in (20). Therefore, a tax on virgin material is inferior to a tax on waste because the negative impaction growth is higher and the positive impact on recycling is lower (see (13) and (14)).

Furthermore, the change in effort *e* with respect to a change in the waste fee is[7]

**(22)** $$\frac{de}{dPW} = \frac{W \cdot \left( \varepsilon_{M,e} + \varepsilon_{M,PM} \cdot \varepsilon_{PM,e} + \frac{1}{1-\alpha} \, \varepsilon_{\alpha,e} \right)}{e \cdot \Pi_{ee}} > 0$$

where $\Pi_{ee} < 0$ due to the strict concavity assumption of the profit function in *e* and *x*. As we expect $de / dPW > 0$, the numerator in (22) should turn out to be negative. First of all, all elasticities $\epsilon$ are negative. The elasticity of material input with respect to effort *e*, $\epsilon_{M,e}$, is negative by assumption because we assume that effort to reduce waste is material saving. $\epsilon_{M,PM}$ is negative since it is a price elasticity of input demand; the elasticity of the price of net material with respect to *e* is negative because of $PM_e < 0$ (see (17)), and the elasticity of the waste coefficient $\alpha(e)$ with respect to *e* is negative as $\alpha' < 0$. We will assume that the product of the two elasticities, which is a positive figure, will not dominate the two negative effects in the numerator; i.e. the effect of *e* on material demand via the price *PM* is weaker than the sum of the direct effects of *e* on material saving productivity as well as on lowering the waste coefficient. This seems to be a reasonable assumption and it explains the positive sign in (22).

For *de / dPGM* we obtain a similar expression as for *de / dPW* in (22):

**(23)**
$$\frac{de}{d\ PGM} = \frac{(GM - MW)(\varepsilon_{M,e} + \varepsilon_{M,PM} \cdot \varepsilon_{PM,e}) + (GW - MW) \cdot \dfrac{1}{1-\alpha}\, \varepsilon_{\alpha,e}}{e\, \Pi_{ee}} > 0$$

where we have assumed again that the product of the two elasticities is sufficiently small. If the numerator in (23) is larger in absolute terms than the numerator in (22), than a tax on virgin material is a better instrument to raise effort than a tax on waste. This is the case since $GM - MW > W$ and $GW - MW = \dfrac{(1-\beta \cdot aw)}{(1-\beta)\, W} > W$ as $aw < 1$. Waste awareness at the beginning of the production process is preferable to waste awareness at the end of the pipe. A tax on virgin material favours a production process which integrates environmental aspects right at the beginning whereas a tax on waste affects the production process at the end of the pipe.

It is of interest to decompose the impact of a fee PW on reducing waste *W*. By differentiating the equation for *W* in (7) totally, the following six aspects are captured by our model.

**(24)**
$$\frac{d \ln W}{d \ln PW} = \frac{\varepsilon_{\alpha,e}}{1-\alpha} \; \frac{d \ln e}{d \ln PW} + \frac{d \ln (1-\beta)}{d \ln PW} + \eta_{M,x} \; \frac{d \ln x}{d \ln PW}$$
$$+\varepsilon_{M,PM}\left(\frac{d \ln PM}{d \ln PW} + \varepsilon_{PM,e}\, \frac{d \ln e}{d \ln PW}\right) + \varepsilon_{M,e}\, \frac{d \ln e}{d \ln PW}\ .$$

The first term is negative and represents the effect of a higher fee on *e* (positive) which in turn lowers the waste coefficient α. The second term is also negative because a fee raises $\beta$, the degree of recycling (see (14)). The third term (also negative) captures the reduction in waste due to a lower production level. The fourth term (negative) represents the demand effect on *M* because *PW* increases the price of net material. The fifth term (positive) shows an offsetting effect of *PW* on *M* because a waste fee raises effort, effort in turn lowers *PM* via α‘(*e*) < 0, which raises demand for *M*. The sixth term (negative) finally shows the benefit of a fee in terms of a material saving bias of the effort. The model therefore captures all relevant aspects for an environmental policy which is aimed at reducing waste.

Comparative statics with respect to a change in the price of labor yields $dx/d\,PL < 0$. A higher wage rate makes labor in production as well as in the recycling process (i.e. the labor cost component in *PM*) more expensive. Next, it is

**(25)**
$$\frac{de}{d\,PL} = \frac{LW \cdot \left( \varepsilon_{M,e} + \varepsilon_{M,PM} \cdot \varepsilon_{PM,e} + \frac{1}{1-\alpha}\, \varepsilon_{\alpha,e} \right)}{e \cdot \Pi_{ee}} + \frac{L\left( \frac{PM \cdot M}{PL \cdot L} \quad \eta_{M,PL} \cdot \varepsilon_{PM,e} + \eta_{L,e} \right)}{e \cdot \Pi_{ee}} < 0 \,.$$

When deriving the sign of $de\,/\,d\,PW$ (see (22)), we concluded that the first term in (25) might be positive. It reflects the aspect that a higher wage increases effort to avoid waste for saving labor cost in recycling waste. The second term is ambigous in sign. Whereas in (22) we assumed effort to be material saving ($\varepsilon_{M,e} < 0$), for labor we assume that the effort to reduce waste is labor using (i.e. $\eta_{L,e} > 0$). Since labor and material are substitutes ($\eta_{M,PL} > 0$), the second fraction can be positive or negative. If $e$ can reduce *PM* significantly ($\left|\varepsilon_{PM,e}\right|$ large ), and *M* and *L* are good substitutes, then also the second term could be positive and a higher wage rate will raise effort. If $\varepsilon_{PM,e}$ is rather low and / or the inputs are not good substitutes then the second fraction is negative and hence in opposite sign to the first fraction. Given the high weight *L* in the second fraction compared to the low weight *LW* in the first fraction, it seems plausible to assume that in this case an increase in the wage rate lowers the effort, mainly in order to save cost of labor when separating waste from gross material.

We finally derive the total demand for labor (*TL*) and the impact on it from a higher fee. Labor demand *TL* comes from production and from recycling:

**(26)**
$$TL = \frac{\partial\, C(\cdot)}{\partial\, PL} = L(x, PM, PL, e) + LW$$

where $LW = \left(\frac{aw}{al}\right) \cdot RW = \left(\frac{aw}{al}\right)\left(\frac{\alpha}{1-\alpha}\,\beta \cdot M\right)$. The firm’s response in labor demand to a change in the waste fee is:

**(27)**

$$\frac{d \ln TL}{d \ln PW} = s_L \left[ \eta_{L,x} \frac{d \ln x}{d \ln PW} + \eta_{L,PM} \frac{d \ln PM}{d \ln PW} + \eta_{L,e} \frac{d \ln e}{d \ln PW} \right] +$$
$$s_{LW} \left[ \frac{d \ln \beta}{d \ln PW} + \eta_{M,x} \frac{d \ln x}{d \ln PW} + \varepsilon_{M,PM} \frac{d \ln PM}{d \ln PW} + \varepsilon_{M,e} \frac{d \ln e}{d \ln PW} + \varepsilon_{\alpha,e} \frac{1}{1-\alpha} \frac{d \ln e}{d \ln PW} \right]$$

where $s_L = \frac{L}{TL}$ and $s_{LW} = \frac{LW}{TL}$ are the labor input shares in the production and in the recycling division. If the fee is used as an instrument for another type of a double dividend policy, namely to reduce waste as well as unemployment, the following eight effects on labor demand, given in (27), have to be taken into account:

1. the effect from lower output has a negative impact
2. the effect from the relatively more expensive substitute material has a positive impact
3. waste awareness increases labor demand (positive impact)
4. raising the degree of recycling also raises the demand for labor in this division (positive impact)
5. the decline in output implies a decline also for labor in the recycling division (negative impact)
6. if material becomes more expensive, its demand decreases and hence less can be recycled (negative impact)
7. waste awareness reduces material, and this affects negatively the demand for labor to recycle (negative impact)
8. a lower waste coefficient implies less labor in the recycling division (negative impact).

There are three effects which have a positive impact on labor demand and five effects which have a negative one. The interesting feature of our simple model is that it discovers eight (and maybe all) aspects one can think of when the relation between a waste fee and the labor market is discussed.

## 4. Production, waste recycling and waste disposal in a perfectly competitive market

The market for our analysis in this section could be the aluminum, the steel or the plastics industry, or any other industry where material is only partly utilized, or where water is an important input. In such industries, all firms have to cope with waste disposal, recycling and the cost of getting rid of waste. We assume that *n* identical firms operate in such a perfectly competitive market. The market is in long-run equilibrium and is characterized by the zero-profit condition, by the price equal to marginal cost condition, and by the cost-benefit condition for effort:

**(28)** $$\Pi = p(n \cdot x)x - C(x, PM(PW, \beta, e), PL, e) = 0\,,$$

**(29)** $$\Pi_x = p(n \cdot x) - C_x(\cdot) = 0\,,$$

**(30)** $$\Pi_e = -M(\cdot) \cdot PM_e - C_e(\cdot) = 0\,.$$

Our purpose is to analyse the impact of a waste fee on industry's output, $X = n \cdot x$, on firm's output $x$, and on effort $e$. Again, all results can be summarized in terms of cost shares and elasticities.

Total differentiation of the system (28) to (30) yields:

**(31)** $$p' \cdot x\ dnx - M \cdot PM_{PW}\ dPW = 0$$

**(32)** $$p'\ dnx - C_{xx}\ dx - M_x(PM_{PW}\ dPW + PM_e\ de) - C_{xe}\ de = 0$$

**(33)** $$\Pi_{ee}\ de + \Pi_{e,x}\ dx - M_e\ PM_{PW}\ dPW - M_{PM}\ PM_e\ PM_{PW}\ dPW - M \cdot PM_{e,PW}\ dPW = 0$$

where we made use of (29) and (30) for reducing the system.[8] Since the equilibrium condition in the long run, $C_x = \frac{C}{x}$, holds as an identity in all factor prices and in *e*, we can differentiate both sides of this identity with respect to *PM* and *e*, respectively:

$$C_{PM,x} = \frac{C_{PM}}{x} \quad , \quad \text{i.e.} \quad M_x = \frac{M}{x} \qquad \text{and} \qquad C_{xe} = \frac{C_e}{x}\,.$$

This implies $\Pi_{e,x} = 0$ when using (30).[9] For (32), we obtain

**(32')** $$p'\ dnx - C_{xx}\ dx - M_x\ PM_{PW}\ dPW = 0$$

**Proposition 1:**
The elasticity of industry's output *x* with respect to the waste fee is:

**(34)** $$\frac{d\ \ln X}{d\ \ln PW} = \frac{PW \cdot W}{C} \quad \varepsilon_{X,p} \leq 0\,,$$

i.e. the price elasticity of demand, weighted by the cost share of waste disposal.

If the cost of waste really matters, then the industry's total output will shrink.[10] In that case the price will go up. If also the price elasticity is high, the response of output to a waste fee can be considerably.

**Proposition 2:**
A firm's output level will not change in the long-run equilibrium if waste fees do increase, i.e.

**(35)** $$\frac{dx}{d\ PW} = 0$$

Since total quantity *X* will fall according to (34), the number *n* of firms must have decreased. Because of (35) and

$$\frac{d\ \ln X}{d\ \ln PW} = \frac{d\ \ln n}{d\ \ln PW} + \frac{d\ \ln x}{d\ \ln PW}\,,$$

the percentage change in the number of firms is equal to the percentage change in total demand.

In (24) we had derived six effects which affect the elasticity of waste disposal with respect to a fee. For the industry, total waste is $TW = n \cdot W$, hence

**(36)** $$\frac{d\ \ln TW}{d\ \ln PW} = \frac{d\ \ln n}{d\ \ln PW} + \frac{d\ \ln W}{d\ \ln PW}\ .$$

A higher fee on waste causes a reduction in waste on the firm level (see (24)) and additionally a reduction through exit of some firms due to a lower market volume *X*.

Finally, the term in (22) which determined the sign of $\frac{de}{d\,PW}$ is the same in the perfectly competitive model. To derive this term from (33), the only difference is that now we do not need the assumption of a homothetic production function. Output at the efficient scale of production implies this time (22).

## 5. Production with waste under imperfect competition

We next consider a symmetric oligopoly with *n* equal firms each being confronted with the decision to recycle and to avoid the cost of waste management.[11] There will be no market exit or market entry after raising the waste fee, i.e. *n* is fixed. The Cournot-Nash equilibrium can be written as

**(37)** $$p(nx) + xp'(nx) - C_x(x, PM, PL, e) = 0$$

**(38)** $$-M \cdot PM_e - C_e = 0$$

where profit is

$$\Pi(PW) = p(nx)x - C(x, PM(PW, e), PL, e)$$

Total differentiation of (37) with respect to *PW* yields:

**(39)** $$\begin{gathered} p'\,dnx + p'\,dx + xp''\,dnx - C_{xx}\,dx \\ -C_{x,PM}(PM_{PW}\,d\,PW + PM_e\,de) - C_{xe}\,de = 0 \end{gathered}$$

To concentrate on the essential aspects, we assume a homothetic production function (which implies $\Pi_{xe} = 0$), and a linear demand function (i.e. *p*'' = 0). Next we use *dnx* = *n dx* to solve (39) for:

**(40)** $$\frac{d\,x}{d\,PW} = \frac{W \cdot \eta_{M,x}}{x((n+1)p' - C_{xx})} < 0$$

Output $x$ and the fee on waste are negatively related, of course. The change in profit is:

**(41)** $$\frac{d\,\Pi}{d\,PW} = (p + x \cdot n \cdot p' - C_x)\,\frac{dx}{d\,PW} - W < 0$$

Since the term in the brackets is zero (for a monopolist) or less than zero for $n > 2$ (see (37)), profit declines. If we replace $dx \,/\, d\,PW$ in (41) by (40), and assume constant returns to scale (i.e. $C_{xx} = 0$), then profit changes according to

**(42)** $$\frac{d\,\Pi}{d\,PW} = \left(\frac{n-1}{n+1} - 1\right) W$$

For a monopolist, profit will be lower by $W \cdot d\,PW$. He will reduce his output such that the higher price will cover exactly the increase in the cost of recycling (note that $d\,\beta \,/\, d\,PW > 0$). He has no intention to use his market power to let the consumer pay for part of the increase in the cost of waste disposal. For an oligopolist the term in the brackets in (41) is negative, however (see (37) and $n > 2$), i.e. the first term is positive. An oligopolist does not bear the full cost of $W \cdot d\,PM$. Oligopolists shift part of the higher fee onto the consumers. By reducing their output levels, the resulting higher price will not only cover the cost of recycling, but also part of the increase in the cost of waste disposal. As for the effect of a higher waste fee on waste disposal of an oligopolist, the elasticity $\varepsilon_{W,PW}$ in (24) as a sum of six weighted elasticities is the same, except that the effect on an oligopolist's output in (40) replaces (20), the change in output by a competitive firm.

Next we analyze a market structure with a dominant firm and a competitive fringe. This model is a mixture of the long-run perfectly competitive model and the oligopoly model. The dominant firm sets the price and $n$ identical small firms take this price as given and determine their profit-maximizing output levels. The fringe could be either exempted from paying waste fees (e.g. importers) or has higher costs of recycling per unit of waste than the dominant firm. The long-run equilibrium for the small firms is characterized by

**(43)** $$\Pi^s\ (PW) = x^s\ p\left(x^d + nx^s\right) - C^s\left(x^s, PM^s\left(PW, e^s\right), e^s\right) = 0\ ,$$

**(44)** $$p\left(x^d + nx^s\right) - C_x^s = 0\ ,$$

**(45)** $$-M\left(x^s, PM^s, e^s\right) \cdot PM_e^s - C_e^s = 0\ ,$$

where $x^d$ is the output of the dominant firm. It knows its residual demand function, given the supply of the small firms. It takes the relationship $x^s\ (x^d)$ into account when maximizing its profit with respect to $x^d$ and $e$:

**(46)** $$\Pi^d\ (PW) = x^d\ p\left(x^d + nx^s\left(x^d\right)\right) - C^d\left(x^d, PM^d\left(PW, e^d\right), e^d\right).$$

F.o.c. are:

**(47)** $$p\left(x^d + nx^s\left(x^d\right)\right) + x^d\ p'(\cdot)\left(1 + \frac{dnx^s}{dx^d}\right) - C_x^d = 0\ ,$$

**(48)** $$-M\left(x^d, PM^d, e^d\right) \cdot PM_e^d - C_e^d = 0\ .$$

As for the conjectural variation in (47), we postulate that it must be consistent with observed behaviour. A firm's conjecture is consistent if a rival behaves as predicted at the equilibrium and near the equilibrium. In order to derive such a consistent conjecture we totally differentiate the long-run zero profit condition (43):

$$pdx^s + x^s p'(\cdot)\left(dx^d + d\left(nx^s\right)\right) = C_x^s dx^s\ .$$

As $p = C_x^s$, we find

**(49)** $$\frac{dnx^s}{dx^d} = -1\,.$$

This is a consistent conjecture of the dominant firm because the competitive fringe has to respond with a reduction in $n$ and/or $x^s$ to keep the price constant if the dominant firm increases its output. Therefore (47) reduces to

**(50)** $$p\left(x^d + nx^s\right) = C_x^d\left(x^d, PM\left(PW, e^d\right), e^d\right).$$

For the comparative statics analysis we differentiate the system (43), (44), (45), (48) and (50) totally with respect to *PM* and *e*. From (43) (and using (45)) the response of total industry output with respect to a fee can be derived:

**Proposition 3:**
The elasticity of total industry output $X=x^d+nx^s$, with respect to a waste fee is

**(51)** $$\frac{d\ln X}{d\ln PW} = \frac{PW\cdot W^s}{C^s}\varepsilon_{X,p} < 0.$$

The output of a small firm from the competitive fringe does not change in response to a fee:

**(52)** $$\frac{dx^s}{dPW} = 0.$$

The result in (52) has been obtained by total differentiation of (44), using (45) and $\Pi_{x,e}=0$ due to the efficient scale output of firms of the competitive fringe. We observe that (51) and (52) are similar to (34) and (35). Waste is $W^s = \left(1-\beta^s\right)\frac{M^s\alpha\left(e^s\right)}{1-\alpha\left(e^s\right)}$ and $\beta^s$ depends on the unit cost of recycling $c^s$ $(\beta^s)$, which could be higher in small firms than in large firms, implying a lower degree of recycling. Furthermore, $\hat{e}^s$ as a solution of (45) could differ from $\hat{e}^d$, the solution of (48), because marginal cost of effort, $C_e^s$, is high in small firms with only a few people employed.

We finally proceed to the change in output of the dominant firm with respect to a waste fee.

**Proposition 4:**

Output of the dominant firm decreases by the rate:

**(53)** $$\frac{d\ln x^d}{d\ln PW} = -\frac{PW \cdot W^d}{p \cdot x^d}\frac{\eta^d_{M,x}}{\eta^d_{MC,x}} < 0$$

and the number of small firms changes by the rate

**(54)** $$\frac{d\ln n}{d\ln PW} = \frac{X}{n \cdot x_s}\frac{PW \cdot W^s}{C^s}\varepsilon_{X,p} + \frac{x^d}{n \cdot x^s}\frac{PW \cdot W^d}{p \cdot x^d}\frac{\eta^d_{M,x}}{\eta^d_{MC,x}}$$

where $\eta_{MC,x} > 0$ is the elasticity of marginal cost with respect to output.

According to (54), the number $n$ of firms can increase or decrease if $PW$ is raised. Total industry output $X$ and the dominant firm's output $x^d$ decline, but if the percentage decline in total output is higher than the percentage decline in the dominant firm's output, weighed by market share ratios, then there will be market exit of small firms. If the competitive fringe is a group of foreign exporters not faced with costs of waste disposal in their own countries, then total output $X$ as well as $x^s$ will not change. Total output will not change because (51) was based on total differentiation of the f.o.c. of small firms with respect to $PW^s$ which is now zero for these firms. According to (54) (now with $PW^s = 0$), the percentage increase in the number of foreign competitors corresponds with the percentage decline in output of the dominant firm. Finally, from (53) and (54) we recognize that in case of a rather step MC curve ($\eta_{MC,x}$ is rather larger), output of the dominant firm will not change much. Since total output will fall (see (51)), market exit by small firms will occur. This shifts their supply curve upwards and the residual demand curve of the dominant firm to the right. The price will increase and the remaining small firms can cover the higher cost of recycling and of the fee.

Finally, profit of the dominant firm will decline by the amount paid for waste disposal

**(55)** $$\frac{d\Pi^d}{dPW} = -W^d \ .$$

The dominant firm reduces its output and thus raises the price in order to cover the cost of recycling and the productivity loss from a higher effort. For the competitive fringe, operating on the zero profit restriction, this price increase must be high enough in order to cover the cost of recycling, the loss in productivity, and additionally the increase in the waste fee. If the price increase does not cover these costs, some firms must leave the market.

## 6. Concluding Remarks

Each production and consumption activity entails waste. In the theory of the firm, however, production is on the frontier of a transformation function, i.e. there is no inefficiency such as waste disposal. It is usually assumed away by the assumed of „free disposal". In reality, however, waste disposal becomes more and more expensive, waste disposal sites become scarce, and protests by the local community against waste incineration are nowadays common. Some countries defer the problem of waste accumulation by exporting part of it, but this sort of transfrontier pollution will soon become more and more unpopular, both in the exporting countries as well as in the recipient countries. One reason for the growth in waste was that producers did not have to care about what will be done with their durable products when durability ends. A new German law by which a producer is responsible for the circular flow of his product from virgin material to scrapping is supposed to raise awareness on the side of the producer. Therefore, waste prevention, waste recycling, and waste disposal will become an integral part of the production process. Our model of a waste intensive firm is a first step towards a neoclassical theory of the firm in present time. Part of a modern production process will be to achieve the efficient frontier by raising the effort to prevent waste. Another goal of efficient production is to recycle or re-use waste that is a by-product of the production process. These two strategies to achieve a de-coupling of material use and growth must be induced by price signals in the same way as the de-coupling between energy use and economic growth was achieved by higher energy prices. We therefore proposed a fee on waste which reflects the marginal social cost of waste disposal. Such a fee should include the cost of land fills, the degree of harmfulness and other damages which can not be prevented. If prices for waste disposal would reflect scarcity instead of cheap disposal, then prevention and recycling will be an integral part of a theory of the firm. Taxing virgin material can also be used to raise effort of resource conservation and to promote recycling. An example here is a tax on virgin lubricating oils in Europe where up to 65 percent of the available waste oil is recovered by the recycling industry.

# Endnotes

[1] This paper was written while I was visiting professor at the University of Toronto.

[2] Schulze (1974), Weinstein and Zeckhauser (1974), Smith (1972), D'Arge and Kogiku (1973), Lusky (1976).

[3] Schottmann (1977), Russel and Spofford (1972).

[4] See Kirchgaessner (1977) on that point.

[5] If a function is specified in several variables, a subscript indicates a derivative. If there is only one varialbe, a dash means a derivative.

[6] For a proof see the Appendix.

[7] All proofs are given in the Appendix.

[8] A $\epsilon$ is used if the elasticity is negative and $\eta$ is used if it is positive.

[9] Note that $C_{x,PM} = M_x$.

[10] t is $PM_e = -\frac{C_e}{M}$ according to (30).

[11] For a proof see the Appendix.

[12] There are several papers dealing with effluent fees and market structure. For references see Conrad and Wang (1993). There is, however, no one which looks into industries with waste disposal and recycling.

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