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Selling Reputation When Going out of Business

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Selling Reputation When Going out of Business[†]

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Abstract: Is the reputation of a firm tradeable when the previous owner has to retire even though ownership change is observable? We consider a competitive market in which a share of owners must retire in each period. New owners, observing only recent profits, bid for the firms that are for sale. Customers are concerned with the owners' type, which reflects the quality of the good or service provided. When a customer observes an ownership change, he may have an incentive to switch to a different firm, even if his past experience was good. However, we show that, in equilibrium, customers believe that the new owner is also good. Hence reputation is tradeable, although ownership change is observable. In our model, reputation is an intangible asset, embodied in an attractive customer base. Firms with good owners sell at a premium.

Keywords: Reputation, ownership change, intangible assets, theory of the firm.

JEL-Classification: D40, D82, L14, L15.

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1 Introduction

In many markets the customer side is only partially informed about the product's characteristics. This typically applies to experience goods. A firm's *address* is then valuable because customers, who have previously consumed a product or service provided by that firm, can easily recognize the product or service. For instance, if they learned that the product is of good quality, they can repurchase the good without risk in the future provided that the quality of the good cannot be easily downgraded over time. A different situation arises if customers observe an ownership change; that is, if the provider of the good or service changes while keeping the firm's name, address, or its customer records. Then new owners are keen to inherit the reputation for good quality, reliable service, or good treatment. However, it is a priori unclear why customers should trust the new owner, who may as well be of bad quality. If good names sell at a premium, the question is whether new good-quality providers will be willing to pay more for the firm's name than bad-quality providers. In other words, the question is whether a firm's reputation can be preserved and used by new owners. If this is the case, consumers may trust the new provider.

In this paper we formalize this story in a competitive, infinitely lived market with an inflow and outflow of market participants on both sides of the market. We consider a market in which reputation manifests itself in an attractive customer base. In other words, reputation is an intangible asset. Our argument applies to the sale of a business when the ownership change (but not the price paid for the intangible asset) is observable by customers. Observable ownership change implies here that customers can base their decision about whether to switch or to stay with the firm on the observation of whether an ownership change has occurred. In equilibrium, customers ignore this information in effect and reputation lives on, and supports a positive price for a firm with an attractive customer base. We show that new owners self-select because an attractive customer base is more valuable to new owners who do not exploit the intangible asset.

Our main result is that reputation can be a tradeable asset even if ownership change is observable. This complements the seminal work by Tadelis (1999), who shows that the firm's name is tradeable if ownership change is, at least partly, unobservable (see the literature review below). Our paper points to the reputation transfer from old to new owners for markets of physicians' and dentists' practices, pharmacies, and lawyers', notaries', and tax advisors' offices. In all these cases, clients typically observe any change in ownership. Another feature of these examples is that in many countries the price for market transactions is regulated. This allows us to focus on the pricing of the tradeable asset; we abstract from pricing issues for market transactions by assuming the price for market transactions as fixed. To guide our modelling efforts, we take a closer look at the market for ambulatory health care services in Section 2.¹ Because institutional arrangements differ across countries, we

¹Mailath and Samuelson (2001) informally discuss that a doctor's practice may sell at a higher

collect a number of facts for one specific country, namely Germany.² In Section 5 we argue that our results on reputation transfer are robust to a number of modifications and extensions of the model. In particular, they are robust to an endogenization of the price for market transactions. Therefore, our argument is more general and applies not only to price-regulated professions but also to a wide number of owner-run businesses in which customers are in direct contact with the owner and in which the good or service has attributes of experience goods. This includes repair services, owner-run shops (where the service is expertise), hairdressers and the like.

Related Literature. Our paper contributes to the literature on the firm as a bearer of reputation – this literature is reviewed in Bar-Isaac (2004). A first attempt to model reputation as a tradeable intangible asset, represented by the firm’s name, is the repeated game model presented by Kreps (1990). The basic idea of Kreps is that reputation can survive ownership change because owners have an interest in preserving the reputation and passing it on to new owners. A firm maintains its reputation if customers believe so. However, a shortcoming of his model is that there are multiple equilibria including equilibria in which the firm’s name has no value. In our model, such an ambiguity does not arise.

Our paper analyzes a competitive market for reputation with adverse selection. Closest in this spirit is the work by Tadelis (1999, 2003). He studies the market for the *name* of a firm. He also analyzes a large population of firms and customers under adverse selection. In his model, ownership change is, at least partly, *unobservable*. In his model the non-observability of ownership change is a necessary condition for the market for names to be active in all periods. In addition to the observability or non-observability of ownership change, there is another important difference between Tadelis’ and our framework. In Tadelis’ work the firm’s name summarizes the reputation of a firm; this name is known to all potential customers. In our framework the name of the firm has a meaning only for the (previous) customers of that firm; it is meaningless for all other customers. Tadelis shows that since a good new owner can build up reputation on his own, whereas a bad new owner cannot, bad new owners have an interest in buying good names. In his model, this effect is strong enough to destroy the complete sorting of types: in equilibrium, some bad new owners buy good names.³ In contrast, in our model, types are completely sorted.

Another related strand of literature treats umbrella branding and brand stretching. This refers to a situation of reputation transfer from one product to another. In a contribution by Wernerfelt (1988), brand stretching works because the owner wants

price if its reputation is good. They also observe that their model, where ownership is unobservable, does not apply to this example.

²We establish five facts. Two of those facts, namely facts 2 and 3 motivate our two-type model. Facts 1,4, and 5 are more general. As we show in remark 3, our main result also holds in a model with a single type of clients.

³Mailath and Samuelson (2001) and Tadelis (2002) present related analyses in a moral hazard environment. They show that reputation transfer alleviates the moral hazard problem.

to avoid losing the reputation of the original product. Closer to our work is the model Choi (1998), in which the quality of the established product is beyond doubt. Brand extension works here because an owner who uses the established products' brand name does not want to forego the earning possibilities from that brand for future products and therefore only uses the brand name for the current new product if the quality of this product is high. Here, the argument is reminiscent of the ones by Klein and Leffler (1981) and Shapiro (1983) where the firm does not deceive customers because otherwise it would suffer a long-term loss of reputation.⁴ Finally, Cabral (2000) considers an adverse selection environment which, in some respects, is similar to Tadelis (1999). In his model product quality is positively correlated across different products. He shows that brand stretching occurs if the quality is sufficiently high. The main difference from the work on brand stretching is that, in our work, there is an information problem between the old and new owner. In particular, old owners do not know the new owner's type. The market price for reputation then separates types. In the literature on brand stretching, there is no need to introduce a market for reputation.

With respect to consumer behavior, our setup is related to models with switching costs – for an overview of the literature see Klemperer (1995). In our model we have two kinds of switching costs. First, a customer who finds that a firm is good is inclined to stay because he is taking a gamble when switching companies. This means that such a customer behaves *as if* he faces switching costs.⁵ These switching costs are endogenous and may depend on the type of customer. Second, in addition to these switching costs arising from asymmetric information, there are exogenous switching costs which do not depend on the type of customer. This makes it costly for customers to leave the status quo, independent of their type.

The plan of the paper is as follows. In section 2 we present our illustrating example, the sale of a physician's practice. In section 3 we present the model. In section 4 we characterize an equilibrium in which there is a positive price for the sale of an intangible asset. In this equilibrium, new owners are sorted, that is, good new owners buy firms with a good reputation, and bad new owners buy firms with a bad reputation. Although a good new owner can build up a reputation over time on his own, this is less profitable than buying a reputation. We furthermore show that non-informative equilibria do not exist. In section 5 we discuss our results and possible extensions of the model. The proofs of the lemmas and the analysis under an alternative parameter constellation are relegated to the appendix.

⁴See also Andersson (2002) for an analysis in a moral hazard environment.

⁵Fishman and Rob (2002) consider a firm's decision to invest in quality over time, where only past realized qualities are observable. With respect to the customer behavior, our models are similar. Fishman and Rob postulate that a share of customers are experienced: these are customers who learn about the past quality of one of the firms. If a customer with information leaves that firm, he goes to a different firm at random. The remaining share of customers do not have previous knowledge; in our model these are new customers.

2 An Illustrative Example: The Sale of a Doctor's Practice

In this section, we present our exemplary story: the sale of a doctor's practice. For illustration, we first establish some facts for the ambulatory health care sector in Germany which guide our modelling effort.⁶ We then spell out our modelling strategy.

Patients and Practices in Germany: Some Facts. In the ambulatory health care sector in Germany, treatments are almost exclusively provided by office-based for-profit physicians – we call them doctors.⁷ Doctors typically run their own practices; group practices are less common except for some apparatus-intensive specializations.⁸ In the former case, a change in ownership is an abrupt shift for patients. It is thus clearly observable to patients.

Fact 1. Ownership change is observable.

This fact is the starting point of our modelling effort and the main distinguishing feature from related literature. A practice is typically sold when the old doctor retires, which can be seen as an exogenous event. A retiring doctor puts the practice up for sale, and it is acquired by a new doctor. Among other factors, the price of the practice depends on the number and type of treatments in a particular period and on the composition of the patient base.

Fact 2. There are two types of patients: high-value and low-value patients.

In Germany, health insurance is compulsory. Essentially, there are two types of patients: those insured by a statutory sickness fund (around 51 million members plus dependents in 1999) and those insured by a private health insurance company (around 7 million fully insured in 1999), see e. g. “Health Care Systems in Transition: Germany”, published in 2000 by the European Observatory on Health Care Systems. For any treatment a doctor charges a fixed fee, depending on the type of treatment. If the patient has private insurance, doctors charge a multiple of the fee which is charged to patients who are insured by a statutory sickness fund; this factor is regulated and depends on the type of treatment. In addition, patients with a private insurance typically have wider coverage for treatments. This means that a privately insured patient is of high value to a doctor. Doctors typically do not provide treatment exclusively for high-value patients: Among the 120,000 registered

⁶A good source on the institutional features of the German health care system for non-German speakers is “Health Care Systems in Transition: Germany”, published in 2000 by the European Observatory on Health Care Systems.

⁷Hospitals provide ambulatory treatments mainly in emergency cases.

⁸If the practice is run by a single doctor, the dominating channel of reputation transfer is from the retiring to the new doctor. In the case of group practices, reputation transfer can be achieved through additional channels (as in Tirole, 1996).

doctors in the ambulatory health care system, only 5,800 do not have agreements with the statutory sickness funds (data for the year 2000 from the German Medical Association). All doctors who have such agreements have the right to treat privately insured patients. Hence, the vast majority of practices have two types of patients.

Fact 3. High-value patients are more likely to change practices.

Guides for doctors caution new doctors not to take for granted that privately insured patients will stay with a practice after an ownership change. For instance, in his guide book on the sale and acquisition of a physician's practice, Klapp (1996, p. 31) writes that privately insured patients are more likely to switch practices than patients who are insured by a statutory sickness fund.⁹ In theory, the German ambulatory system is characterized by the patient's freedom to choose and the doctor's obligation to treat. In practice, however, some doctors reject low-value patients, whereas they accept high-value patients. This is often done in a subtle way. For instance, the next available date for a treatment of a new patient depends on whether he is privately insured or not. Also, a new patient calling up a practice is often told that unfortunately, a practice cannot take more patients. Matters change when the patient tells that he is privately insured. In effect, since low-value patients are more likely to be rejected (or to be treated less favorably), they have a lower incentive to switch practices.

Fact 4. Patients tend to have local information about the quality of doctors.

A patient-doctor relationship tends to be long-term. There appears to be little market transparency on the quality of doctors; for example, no ranking or outside advice is available which would provide a recommendation about the quality of a doctor. Advertising is severely restricted. Patients therefore typically have much better information about the quality of "their" doctor than of other doctors. Fact 4 implies that patients stay with a good doctor. The open question is whether new doctors can at least partly benefit from a large share of high-value patients of the retiring doctor. There is a tension: on the one hand it is commonly accepted that a large share of private patients can be seen as a negative factor for the value of a practice because of the risk that they will switch practices (see Klapp, 1996). On the other hand, privately insured patients are of high value to doctors.

Fact 5. Practices with a large share of high-value patients are sold at a premium.

In practitioners guides for German doctors it is recognized that in general a practice for sale with a large share of high-value patients is potentially more profitable – in spite of Fact 3. Klapp (1996, p. 31-32) states that every new doctor will appreciate a large share of privately insured patients. Checking Internet announcements for the

⁹More precisely, he states that after an ownership change an average of 30% of privately insured patients switch. While the equilibrium of our model does not feature the fact that switching rates depend on ownership change, this is easily accommodated in a modified model by introducing horizontal taste heterogeneity among patients; see the discussion.

sale of practices, we found that a large share of high-value patients is advertised as a selling point for practices. Hence, a retiring doctor with a large share of high-value patients can expect to receive a premium.

Fact 5 suggests that new doctors must be confident of retaining at least a fraction of the patients. Together with fact 4, this implies that a good doctor is more likely to enjoy long-term profit from these patients than a bad doctor. This makes a good doctor to bid higher than a bad doctor, *ceteris paribus*. However, a good doctor may buy a practice with a small share of high-value patients and gradually improve his base. Since new patients arrive in each period, and among experienced patients high-value patients are more likely to switch (fact 3), a practice with a small share of high-value patients is more profitable for a good than a bad doctor. This in effect reduces a new doctor's willingness to pay for a practice with a large share of high-value patients. Our analysis captures these two countervailing effects. Furthermore, our model accommodates the five facts stated above. Facts 1, 2, and 4 are reflected in our model assumptions. Facts 3 and 5 are equilibrium outcomes.

The Sale of a Doctor's Practice: Our Modelling Strategy. A doctor is interested in utilizing the full capacity of his practice and in attracting high-value patients. In our model total capacity equals demand, and there are no frictions that leave some patients without treatment.¹⁰ Hence, all practices operate under full capacity and only differ regarding the composition of patients.

Patients can be of high or of low value to doctors. They prefer good doctors, but can tell the quality of a doctor only from experience. Patients obtain only "local" information: they learn the quality of "their" doctor, but do not know the quality of the others. Clearly, if patients experience that their doctor is good and this doctor remains active in the next period, they have no reason to switch practices. If, however, patients observe an ownership change, they initially lack knowledge about the quality of the new doctor. Still, they can use their past experience with the old doctor to form beliefs about the quality of the new doctor.

Suppose that patients believe that the new doctor is as good as the old. In this case, a practice with a larger share of high-value patients is more valuable. This holds true for good and bad new doctors alike. However, the difference in profits for the two types of practices is greater for good than for bad new doctors. The reason is essentially that a bad new doctor loses many high-value patients after one period, whereas a good new doctor retains them and building up an attractive patient base takes a lot of time for a good doctor. Consequently, good new doctors are willing to pay a higher price for a practice with a large share of high-value patients than bad new doctors, and doctors self-select.

¹⁰We also consider the case that there is a single patient type and excess capacity. Our main findings are robust to this modification (see remark 3). See also the discussion section.

3 The Model

We consider a stationary environment where time is discrete, $t \in \mathbb{Z}$. There are two groups of agents, doctors and patients. Here, doctors are the providers of a service, and patients are the customers. Doctors are of either good or bad type, denoted by $d \in \{G, B\}$. Patients are of high or of low value for a doctor, denoted by $p \in \{H, L\}$.¹¹

Doctors. A fraction λ_G of doctors is of type G ; the remaining fraction $\lambda_B = 1 - \lambda_G$ is of type B .¹² The type d of a doctor is observable to the patient *after* a treatment. Hence the treatment is an experience good in the extreme form that goodwill has the maximum duration of one period. Doctors provide treatments, for which they charge fixed fees $f_p > 0$ that depend on the patient's type. Each doctor has a fixed capacity, which, without loss of generality, is set equal to 1. The opportunity cost of providing a treatment we set equal to zero. Suppose that a doctor has a share χ_d of high-value patients. Then his per-period profit is $\chi_d f_H + (1 - \chi_d) f_L$.

In each period, a doctor realizes that he is becoming too old to run his practice with probability $\delta_d > 0$, in which case he must sell and retire.¹³ Assuming stochastic independence, this results in a share of δ_d doctors retiring in each period. In each period, a continuum of mass δ_d of young doctors arrives; hence at each point in time there is a continuum of doctors of mass 1. Young doctors do not observe the type of the retiring doctor, but learn the composition of the patients at the practice for sale. The practices then are sold at some price T .

Doctors maximize the sum of the net present value of fees and the net present value of the sale of a practice. There is no discounting.

Patients. The continuum of patients has mass '1 × 1'. There is thus a continuum of mass 1 per practice. Patients frequent a doctor once in a period. Hence, the total capacity of all practices just covers the total demand for treatments. This means that even if a share of patients leaves a particular practice, that practice does not suffer from under- or overcapacity. As a result, we may focus on the composition of patients at a practice.

For both types of patients, the utility derived from a treatment of a good doctor is higher than that from that of a bad doctor, $u_G > u_B$. A fraction λ_H of patients is of type H ; the remaining fraction $\lambda_L = 1 - \lambda_H$ is of type L . High- and low-value patients differ in the fees f_p that they pay for a treatment, $f_H > f_L$. As a special case, we can consider the model with a single type of patients and excess capacity (see remark 3 below).

¹¹Note that we use the letter d as index for doctors as well as for the doctors' type; similarly, the letter p is used for patients.

¹²Greek letters are used for probabilities and rates.

¹³Note that it can be shown that the doctor does not have an incentive to sell the practice before he reaches retirement.

$d \in \{G, B\}$	types of doctors
$p \in \{H, L\}$	types of patients
λ_G, λ_B	fraction of good and bad doctors
λ_H, λ_L	fraction of high- and low-value patients
δ_d, δ_p	exit and arrival rates of doctors and patients, respectively
f_H, f_L	fee per treatment
u_G, u_B	utility derived from a treatment
s	switching costs for a change of a practice
χ_G, χ_B	fractions of high-value patients in good and bad practices
T	transfer price of a practice, depends on perceived type χ_{t-1}

Table 1: Notation

In each period, a patient exits with probability $\delta_p > 0$.¹⁴ Assuming stochastic independence, this results in a share of δ_p patients exiting in each period. In each period, a continuum of mass δ_p of new patients arrives; hence at each point in time there is a continuum of doctors of mass 1 of which $1 - \delta_p$ are experienced and δ_p are new.

Patients who do not exit may decide to switch practices. If a patient switches practices he incurs an exogenous switching cost $s > 0$. This switching cost is independent of the patient's current practice and independent of any loss of information. For instance, when accepted by a new practice, the newly arrived patient must fill out forms, and medical records must be transferred from the practice where the patient used to be. Patients maximize expected utility; there is no discounting.

A share of patients χ_d at a particular practice of type d are of high value, and the remaining share $1 - \chi_d$ are of low value. In a stationary world in which the patient base does not change over time and only depends on the doctor's type, we can describe it by χ_G and χ_B , respectively. In table 1, we have summarized the notation of the model. Note that χ_G , χ_B and $T(\chi_{t-1})$ are endogenous. Figure 1 displays the weight of good and bad practices and the composition of their clienteles. White areas add up to λ_L ; gray areas add up to λ_H . The figure can be used to understand the streams of patients between practices.

Time Structure. The sequence of events is the same in each period. First, all doctors observe whether they must retire. They then put their practice up for sale and reveal the composition of their patient base χ to entering young doctors. The latter make take-it-or-leave-it offers to retiring doctors, which can depend on their own type ($d \in \{G, B\}$) and on the composition of the patients at the practice, χ . The bid is denoted by $T = T(d, \chi_{t-1})$, where χ_{t-1} contains the information a bidder can gather about the sold practice.¹⁵ Retiring doctors then accept the highest bid.

¹⁴Note that we model a good treatment as life-improving, but not as life-prolonging.

¹⁵Note that χ_{t-1} need not necessarily be equal to χ_t , because patients may want to switch because of the ownership change.

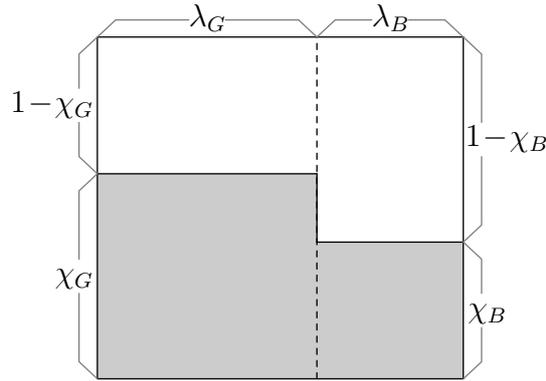


Figure 1: Fractions of Doctors and Patients

In case of a tie between a good and a bad doctor, we use the tie-breaking rule that the doctor who would be willing to pay infinitesimally more wins and pays his bid price.

The ownership change, but not the price paid by the new doctor, is observed by the patients of that practice. Conditional on that information and the doctor's type of the previous period, and provided they do not leave the market, they choose whether to stay or switch practices. We assume that if patients change practices or newly arrive in the market, they apply at new practices with uniform distribution, knowing neither the reputation of the practice nor their chances of being accepted. Hence reputation is only local. If a patient is rejected by a practice he continues to search for a practice until he is accepted. Hence, in each period, every patient is matched to some practice.

Remark 1 *If reputation were publicly observable, then patients could coordinate on this information. This would give rise to the emergence of multiple equilibria. For example, the patients' equilibrium beliefs may be that practices previously run by good doctors are always bought by bad doctors. With local reputation, such counter-intuitive equilibria do not exist.*

Doctors observe the patient's type and can accept or reject the patient. If patients switch practices, they incur a switching cost $s > 0$, which we have introduced above. This switching cost is, for simplicity, assumed to be independent of the number of attempts made to be accepted by another practice. If the switching cost increased in the number of attempts, this would make switching less attractive for low-value patients because, as we will show, they are rejected with positive probability, whereas high-value patients are always accepted.

Finally, once all patients are allocated to practices, each patient receives a treatment, after which he observes the doctor's type. Figure 2 summarizes the sequence of events in one period t .

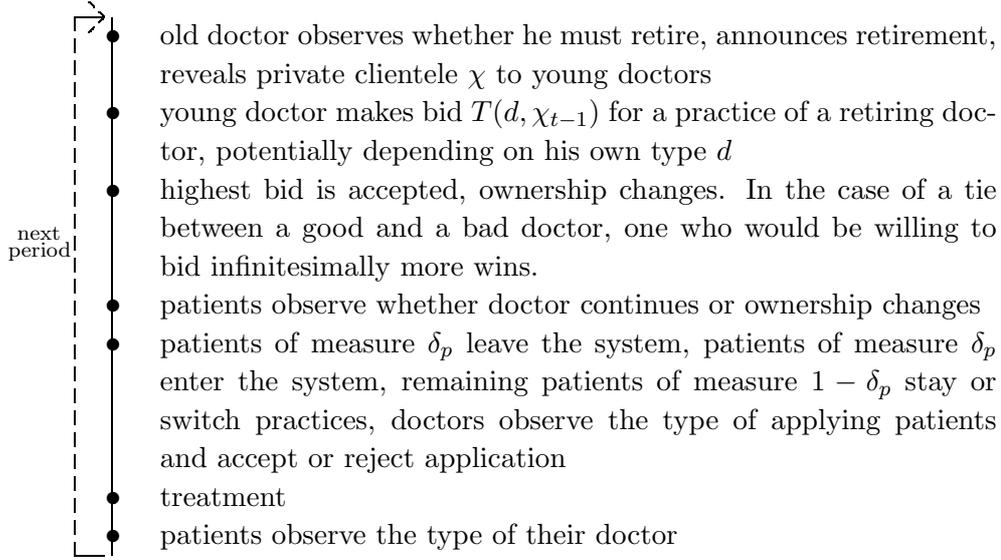


Figure 2: Time Structure of the Game

Strategies. Patients observe d_{t-1} and whether an ownership change has happened in period t , where Y stands for “yes” and N for “no”. Hence, the patients’ information is an element of $\{G, B\} \times \{Y, N\}$. Given the information of a patient of type p , his strategy is to “stay”, to “switch” or to randomize between those. $\sigma_p^t(\cdot, \cdot) \in [0; 1]$ denotes the switching probability. Doctors always accept high-value patients and randomly fill up the remaining capacity with low-value patients.

The strategy of retiring doctors does not need to be formalized, because they simply accept the highest offer from young doctors. The strategy of young doctors is to “make offer T ”. Formally, $\sigma_H^t(\chi) \in \mathbb{R}_+$ and $\sigma_L^t(\chi) \in \mathbb{R}_+$.

Beliefs. Each patient must hold beliefs for three different situations that can occur: he stays and the practice does not change hands; he stays and the practice does change hands; he does not stay. First, his beliefs about the probability of being accepted by a good doctor if he stays with the practice which is not for sale is denoted by $\beta_H^t(d_{t-1}, N) \in [0; 1]$ and $\beta_L^t(d_{t-1}, N) \in [0; 1]$. Because the doctor’s type is perfectly observable after a treatment, we must have

$$\beta_H^t(d_{t-1}, N) = \beta_L^t(d_{t-1}, N) = \begin{cases} 1 & \text{if } d_{t-1} = G \\ 0 & \text{if } d_{t-1} = B \end{cases} .$$

Second, each patient holds beliefs about the probability with which the next doctor will be good if the practice is sold, $\beta_H^t(d_{t-1}, Y) \in [0; 1]$ and $\beta_L^t(d_{t-1}, Y) \in [0; 1]$. If reputation transfer is successful, beliefs depend on the quality of the retiring doctor.

Finally, patients hold beliefs about the probability with which they end up at a good doctor if they switch, denoted by $\beta_p^t(\emptyset, \emptyset)$, where (\emptyset, \emptyset) stands for the lack of information regarding the doctor. Note that beliefs possibly depend on the patient’s

type, i. e. $\beta_H^t(\emptyset, \emptyset) \neq \beta_L^t(\emptyset, \emptyset)$, because the probability of being accepted may differ between patients of different types. We will see that, in equilibrium, high-value patients are always accepted, while low-value patients are sometimes rejected by good doctors.

Retiring doctors do not need to hold any beliefs; they possess all payoff relevant information. Young doctors observe the fraction of high-value patients of the practices they bid for, χ_{t-1} . However, they must form beliefs about whether high-value patients are going to stay with the practice. This in turn depends on the old doctor's quality.¹⁶

Equilibrium. We refer to an equilibrium if agents maximize their utility given their beliefs while anticipating future behavior of other agents, if beliefs are confirmed by the behavior of agents, and if the market for practices clears in each period. Agents update their beliefs using Bayes rule whenever applicable. Hence, in particular, equilibria are perfect Bayesian.

The (Lack of) Information of Patients and New Doctors – Summary.

When a practice is sold, new doctors have to infer the value of a practice. To do so, they have to predict the behavior of the patients previously under treatment at the practice. Similarly, patients have to infer the value of a treatment. To do so, they have to predict the behavior of the new doctors. In the model formulated above, we made a number of assumptions on the information the different agents possess. They are summarized as follows:

- retiring doctors
 - do not know the quality of young (bidding) doctors d
 - do not know the next period's composition of clientele χ_t
- young doctors
 - do not know the quality of retiring doctor
 - know the composition χ_{t-1} of clientele of retiring doctor
 - do not know the next period's composition of clientele χ_t
- patients
 - do not know the quality of new doctor d
 - know the quality of retiring doctor

¹⁶The information problem for the new doctor would not arise if new doctors could either directly observe the old doctor's type or draw unambiguous inferences from the clientele base χ_{t-1} on the old doctor's quality (see the discussion).

4 Symmetric Stationary Equilibria

In this section we characterize stationary equilibria. In doing so, we must specify how large the fractions of good doctors and of high-value patients are. The reason is that the availability of good doctors determines the behavior of consumers. For example, if good doctors were so scarce that their capacity was insufficient to cater to all high-value patients, under perfect information, low-value patients would always be rejected – this case is analyzed in the appendix. In the main text, we will focus on the reverse situation, where high-value patients are always accepted when asking for a treatment at a particular practice. In this case, under perfect information, low-value patients also have a chance to be treated by a good doctor. Because of market clearing ($\lambda_G + \lambda_B = \lambda_H + \lambda_L$), this must also hold under asymmetric information. In particular, a low-value patient who switches practices has a positive probability of being accepted by a good doctor. This implies that, in the absence of switching costs, all patients who learn that their doctor is bad, and continues, will change practices. Such a massive turnover of patients is avoided if moderate switching costs exist. We will describe exactly such a situation regarding switching costs, in which only high-value patients switch after learning that a doctor was bad. To be precise, the main point of this section is to construct an equilibrium in which there is a strictly positive price for a practice with good reputation. In this equilibrium, good new doctors buy practices with good reputation, and bad new doctors buy practices with bad reputation. We show that it is not worthwhile for a good new doctor to buy a cheap practice with bad reputation and build up the reputation himself.

4.1 Informative Equilibrium

In this section we consider a particular class of equilibria, which we call informative. An equilibrium is said to be informative if several properties are met, as spelled out in the definition below. We focus on parameter constellations so that patients do not switch practices unless a high-value patient learns that his doctor is bad.

Definition 1 (Informative Equilibrium) *An equilibrium is called informative if it is characterized by the following observable strategies (along the equilibrium path).*

1. *Old low-value patients stay with their practices. Old high-value patients stay if their doctor was good, otherwise they switch.*
2. *New low- and new high-value patients choose their practice at random.*
3. *New good doctors buy from good old doctors at a high transfer price; new bad doctors buy from bad old doctors at a low transfer price.*

Here, item 1 implies especially that the switching behavior of patients is independent of whether an ownership change takes place. Item 2 is obvious because new patients do not have any information that they can base any decision on. Item 3 really characterizes the informativeness of the equilibrium: The type of the buying doctor is identical to that of the selling doctor, even though buying doctors cannot observe the type of the selling doctor. The market mechanism alone provides for perfect sorting.

Strategies and Beliefs in the Informative Equilibrium. We define strategies and beliefs of doctors and patients. We will show that these strategies and beliefs form an informative equilibrium for certain parameter constellations.

Possible information sets of patients are $\{G, B\} \times \{Y, N\} \cup \{\emptyset, \emptyset\}$. Either patient has experienced a good (G) or a bad (B) doctor, who either keeps his practice (N) or retires (Y); or the patient comes to an unknown practice and knows nothing ($\{\emptyset, \emptyset\}$).

The strategies of high-value and low-value patients are given by

$$\underbrace{\sigma_H^t(G, Y) = 0, \sigma_H^t(G, N) = 0}_{\sigma_H^t(G, \cdot) = 0}, \quad (1)$$

$$\underbrace{\sigma_H^t(B, Y) = 1, \sigma_H^t(B, N) = 1}_{\sigma_H^t(B, \cdot) = 1}, \quad (2)$$

$$\sigma_L^t(\cdot, \cdot) = 0. \quad (3)$$

Here, (1) means that high-value patients stay if they detect good quality, independently of an ownership change. The probability of a change is 0. (2) denotes that high-value patients switch if they detect bad quality, independently of an ownership change. The probability of a change is 1. (3) signifies that low-value patients never switch. Retiring doctors have no strategy; they simply accept the highest offer.

$$\begin{aligned} \sigma_G^t(\chi_G) &= T_{\min}, \quad \sigma_G^t(\chi_B) = 0, \\ \sigma_B^t(\chi_G) &= T_{\min}, \quad \sigma_B^t(\chi_B) = 0, \end{aligned}$$

where $T_{\min} > 0$ will be defined in the following. As a tie breaking rule, if good and bad doctors bid the same price for a practice, the good doctor gets the bid.

The beliefs of patients are given by probability distributions,

$$\beta_H^t(d_{t-1}, N) = \beta_L^t(d_{t-1}, N) = \begin{cases} 1 & \text{if } d_{t-1} = G \\ 0 & \text{if } d_{t-1} = B \end{cases}, \quad (4)$$

$$\beta_H^t(d_{t-1}, Y) = \beta_L^t(d_{t-1}, Y) = \begin{cases} 1 & \text{if } d_{t-1} = G \\ 0 & \text{if } d_{t-1} = B \end{cases}, \quad (5)$$

$$\beta_H^t(\emptyset, \emptyset) = \lambda_G, \quad (6)$$

$$\beta_L^t(\emptyset, \emptyset) = \lambda_G \Pr\{\text{accepted by G|L}\}. \quad (7)$$

Here, (4) means that if high-value and low-value patients stay with their practice and the doctor does not retire, then the doctor's type remains unchanged. (5) means that even if the doctor retires, patients believe that the new doctor will almost surely have the same type. (6) means that a switching high-value patient believes he will find a good doctor with probability λ_G , and a bad doctor otherwise. Finally, (7) means that a switching low-value patient believes he will find a good doctor only with probability $\lambda_G \Pr\{\text{accepted by G|L}\}$ (calculated in the following), and a bad doctor otherwise. Note that the patients' beliefs depend only on the latest performance of their doctor, not on his former quality. In equilibrium, this is irrelevant. The out-of-equilibrium analysis may well depend on these beliefs.¹⁷

Finally, if $\chi_{t-1} \in \{\chi_B, \chi_G\}$, beliefs of young doctors are

$$\beta_G^t(\chi_{t-1}) = \beta_B^t(\chi_{t-1}) = \chi_{t-1} \text{ with prob. } 1.$$

Otherwise, if $\chi_B < \chi_{t-1} < \chi_G$, beliefs are

$$\beta_G^t(\chi_{t-1}) = \beta_B^t(\chi_{t-1}) = \begin{cases} \chi_B & \text{with prob. } \delta_d \\ \chi_B + (1 - \delta_p) \chi_{t-1} & \text{with prob. } 1 - \delta_d \end{cases}.$$

Hence young doctors believe that if they buy a practice with either clientele χ_B or χ_G , the clientele will stay unchanged for the next period. If the clientele is in between, they infer that they are dealing with a practice that used to be bad but has been bought by a good doctor at some point. They also infer that the practice is still possessed by a good doctor with probability $1 - \delta_d$, but is now run by a bad doctor for the recent period with probability δ_d . In this case, the practice is worthless, because all high-value patients will switch. We will now calculate the clientele of practices, χ_G and χ_B , and the equilibrium transfer price T_G .

The Clientele of Practices. The dominant strategy for a high-value patient is to leave his doctor if he is bad, if the probability that he will be accepted by a good doctor is positive (and if switching costs are reasonably low). In each period, the number of high-value patients that apply for places at good practices is

$$\delta_p \lambda_G \lambda_H + \chi_B (1 - \delta_p) (1 - \lambda_G) \lambda_G, \quad (8)$$

because of the newly born patients δ_p , a share λ_H is high-value, and of these a fraction λ_G applies at a good practice. Be χ_B the equilibrium fraction of high-value patients at bad practices. A fraction $(1 - \delta_p)$ of them does not exit; a fraction λ_G of them arrives at a good practice; and $1 - \lambda_G = \lambda_B$ is the weight for the mass of bad practices.

¹⁷However, one may endogenize the short memory by assuming that memorizing earlier information on quality bears an (infinitesimal) cost ϵ . In this case, the patient forgets everything that dates back more than one period, because such information is irrelevant in equilibrium.

Because of the stationarity of equilibrium, this number must be equal to that of exiting high-value patients in good practices, $\delta_p \chi_G \lambda_G$, where χ_G is the fraction of high-value patients at good practices. Hence

$$\begin{aligned}\delta_p \chi_G \lambda_G &= \delta_p \lambda_G \lambda_H + \chi_B (1 - \delta_p) (1 - \lambda_G) \lambda_G, \\ \delta_p \chi_G &= \delta_p \lambda_H + \chi_B (1 - \delta_p) (1 - \lambda_G), \\ \chi_G &= \lambda_H + \chi_B (1 - \lambda_G) \frac{1 - \delta_p}{\delta_p}.\end{aligned}$$

The number of high-value patients at good and bad practices must add up to the number of high-value patients in the economy, thus

$$\chi_G \lambda_G + \chi_B (1 - \lambda_G) = \lambda_H. \quad (9)$$

Substitution yields

$$\begin{aligned}\lambda_H &= \left(\lambda_H + \chi_B (1 - \lambda_G) \frac{1 - \delta_p}{\delta_p} \right) \lambda_G + \chi_B (1 - \lambda_G), \\ \lambda_H &= \lambda_G \lambda_H + \chi_B \left(\lambda_G (1 - \lambda_G) \frac{1 - \delta_p}{\delta_p} + (1 - \lambda_G) \right), \\ \chi_B &= \frac{\lambda_H (1 - \lambda_G)}{(1 - \lambda_G) (1 + \lambda_G \frac{1 - \delta_p}{\delta_p})} = \frac{\delta_p \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)}.\end{aligned} \quad (10)$$

Here, $\delta_p \rightarrow 0$ implies $\chi_B \rightarrow 0$: If patients never exit, all high-value patients land at a good doctor after some time. Furthermore, $\delta_p \rightarrow 1$ implies $\chi_B \rightarrow \lambda_H$: If patients live for one period only, the fraction of high-value patients at a bad doctor's equals the population mix. Substituting (10) into (9) yields

$$\begin{aligned}\chi_G &= \frac{\lambda_H}{\lambda_G} - \chi_B \frac{1 - \lambda_G}{\lambda_G} = \frac{\lambda_H}{\lambda_G} - \frac{\delta_p \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} \frac{1 - \lambda_G}{\lambda_G} \\ &= \frac{\lambda_H}{\lambda_G} \left(1 - \frac{\delta_p (1 - \lambda_G)}{\delta_p + \lambda_G (1 - \delta_p)} \right) = \frac{\lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} = \frac{\chi_B}{\delta_p}.\end{aligned} \quad (11)$$

Here, $\delta_p \rightarrow 0$ implies $\chi_G \rightarrow \lambda_H/\lambda_G$: If patients never exit, high-value patients are eventually shared between good doctors. Furthermore, $\delta_p \rightarrow 1$ implies $\chi_G \rightarrow \lambda_H$: Analogously to bad doctors, good doctors also only get the population mix if patients live for one period only, and no information/reputation can be passed on. One can say that δ_p is a proxy for how much information is handed down from one generation of patients to the next.

From (11), we can derive a condition that guarantees that high-value patients are always accepted by good doctors,

$$\chi_G < 1 \iff \lambda_H < \delta_p + \lambda_G (1 - \delta_p). \quad (12)$$

For the rest of the section, assume that (12) holds.

A Lower Bound on Switching Costs s . If switching costs are very low, then not only high-value patients try to switch to good practices, but also low-value patients. The only difference between the types is that high-value patients are accepted with probability 1 (their only problem is to find a good practice). Low-value patients run the risk of being rejected if they find a good practice. We now look for the minimum switching cost that guarantees no switching for low-value patients, making use of the *single deviation principle*.¹⁸ The critical switching cost is given by

$$s_L = \frac{\lambda_G}{\delta_p} \underbrace{\left(1 - \frac{\lambda_H}{1 - \lambda_H} (1 - \lambda_G) \frac{1 - \delta_p}{\delta_p + \lambda_G (1 - \delta_p)} \right)}_{=: \Pr\{\text{accepted by G|L}\}} (u_G - u_B). \quad (13)$$

For details, see the proof of lemma 1 in the appendix.

Lemma 1 *If $s \geq s_L$ all low-value patients stay with their old practice.*

For lower switching costs, a positive share of low-value patients switches after encountering a bad doctor. Below a critical value \tilde{s}_L all low-value patients switch practices (see figure 3 and remark 2). Note that our main result on reputation transfer does not require any lower bound on switching costs (they only must be positive, see below).

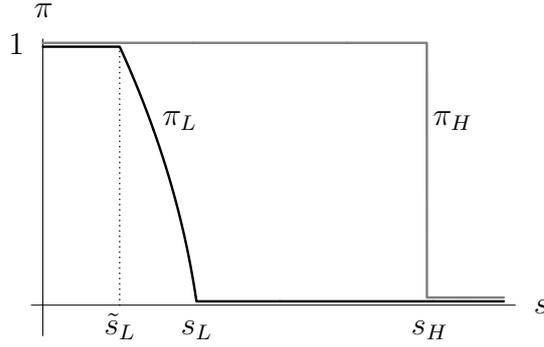
An Upper Bound on Switching Costs s . Clearly, if s is sufficiently high, even high-value patients will not want to switch, despite their relatively high probability of being accepted by a good doctor. In this case, reputation transfer is not possible. Denote the critical switching cost by

$$s_H = \frac{\lambda_G}{\delta_p} (u_G - u_B) > s_L. \quad (14)$$

Lemma 2 *If $s \leq s_H$ all high-value patients switch practices after encountering a bad doctor.*

Because in the main text we want to focus on equilibria in which low-value patients do not switch, we only consider the region $s \in [s_L, s_H]$, as defined by (13) and (14). For lower switching costs, we distinguish between two intervals. In the interval (\tilde{s}_L, s_L) , a positive share of low-value patients would switch practices in each period. If $s = \tilde{s}_L$, low-value patients are indifferent about switching or not, even if all other low-value patients switch. In the range $(0, \tilde{s}_L)$, even all low-value patients would switch. In this case, fact 3 is violated. Note that our qualitative results remain unchanged for $s < s_L$. In particular, a doctor's profits and the decisions of high-value

¹⁸The *single deviation principle* implies that only one-time deviations need to be considered. Furthermore, the notion of equilibrium implies that only unilateral deviations must be taken into account.

Figure 3: Switching Probabilities π_H and π_L of Patients

patients are unaffected by the switching costs s , as long $s < s_H$. The probabilities of switching from a bad practice are illustrated in figure 3.

The Separating Transfer Price T . Bad young doctors buy practices at a price $T_B = 0$. The reason is that they can make the retiring doctors take-it-or-leave-it offers, hence they choose the lowest possible transfer price, $T_B = 0$.¹⁹ If a bad practice is bought by a bad young doctor, expected profits amount to

$$\begin{aligned} \Pi_B(\chi_B) &= \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_B f_H + (1 - \chi_B) f_L) \\ &= \frac{\chi_B f_H + (1 - \chi_B) f_L}{\delta_d}. \end{aligned} \quad (15)$$

How much does a bad young doctor gain if he bids for an good old practice? In this case, his expected profits consist of four components: First, the profits from a good practice, $\chi_G f_H + (1 - \chi_G) f_L$, are earned for one period. Second, with probability δ_d , the bad doctor will retire after one period and will be able to regain the transfer price T_G before his high-value patients switch. Third, if he retires after more than one period, he gets the profits of a bad practice and a low transfer price $T_B = 0$ in the end. Fourth, he must pay the high transfer price T_G right away:

$$\Pi_B(\chi_G) = (\chi_G f_H + (1 - \chi_G) f_L) + \delta_d T_G + (1 - \delta_d) \frac{\chi_B f_H + (1 - \chi_B) f_L}{\delta_d} - T_G.$$

In a separating equilibrium, $\Pi_B(\chi_G) \leq \Pi_B(\chi_B)$ must hold. This can be rewritten as

$$(1 - \delta_d) T_G \geq (\chi_G - \chi_B) (f_H - f_L). \quad (16)$$

¹⁹One may also assume that, as an outside option, young doctors can found their own practices. This has exactly the same value as purchasing an bad old practice: If an bad old practice is bought, the high-value patients are anticipated to leave before the next treatment. Therefore in the next period, the only high-value patients are the ones just arriving. If a new practice is founded, the only high-value patients are, again, the ones just arriving. The proportion is identical.

Therefore,

$$\begin{aligned}
T_G &\geq \frac{1}{1 - \delta_d} (\chi_G - \chi_B) (f_H - f_L) \\
&= \frac{1}{1 - \delta_d} \left(\frac{\lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} - \frac{\delta_p \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} \right) (f_H - f_L) \\
&= \frac{1}{1 - \delta_d} \frac{(1 - \delta_p) \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} (f_H - f_L) =: T_{\min}, \tag{17}
\end{aligned}$$

where T_{\min} is the minimum transfer price that guarantees separating. Note that if $\delta_d \rightarrow 1$, nearly all doctors retire after one period. Therefore, the probability that bad doctors can resell good practices after one period and regain the high transfer price T_G is close to 1. Only if $T_G \rightarrow \infty$, they can be deterred from mimicking. On the other hand, if $\delta_d \rightarrow 0$, the probability that bad doctors can regain T_G is close to 0. The transfer price that suffices to deter mimicking is only slightly higher than the one-time gains for the bad doctor, $T_{\min} \rightarrow (\chi_G - \chi_B) (f_H - f_L)$. If $\delta_p \rightarrow 1$, nearly all patients exit after one period, therefore hardly any information is passed to the next generation, and $T_{\min} \rightarrow 0$, because good and bad practices earn the same money. If $\delta_p \rightarrow 0$, patients never exit, and T_{\min} becomes maximal, $T_{\min} = \frac{1}{1 - \delta_d} \frac{\lambda_H}{\lambda_G} (f_H - f_L)$. This analysis confirms that δ_p contains the information permeability over time.

The Decision Problem of a Young Good Doctor. So far, we have examined how high the transfer price of a good practice T_G must be to deter bad doctors from buying it (i. e., T_{\min}). But at this transfer price, is the purchase profitable for a good doctor? If a good doctor buys a good practice, his expected profits are²⁰

$$\begin{aligned}
\Pi_G(\chi_G) &= \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_G f_H + (1 - \chi_G) f_L) \\
&= \frac{\chi_G f_H + (1 - \chi_G) f_L}{\delta_d}. \tag{18}
\end{aligned}$$

Building up Goodwill. Instead of buying a good practice, a good doctor may buy a bad practice with clientele χ_B , and high-value patients will accumulate at the practice by and by. That is, he builds up reputation over time. As we will show later, the good doctor prefers to buy a good practice at price T_{\min} rather than a bad practice at price 0 if the following two properties hold: $\chi_t \in [\chi_B; \chi_G]$ and $T_t \in [0; T_{\min}]$. We now will show that there is a perfect Bayesian equilibrium off the equilibrium path with these properties; we show this by fully characterizing it. In period $t = 0$, the fraction of high-value patients is χ_B . In $t = 1$, all surviving high-value patients stay, and additionally a fraction χ_B of high-value patients arrive. Because every high-value patient leaves a bad practice immediately, the fraction of high-value patients at a bad practice is identical to those arriving at a practice in

²⁰Note that T_{\min} does not appear in the equation, because it is spent in at the date of purchase and regained at the date of retirement with probability 1.

one period. The fraction goes up to $\chi_B(1 - \delta_p) + \chi_B$. In $t = 2$, the fraction is $\chi_B(1 - \delta_p)^2 + \chi_B(1 - \delta_p) + \chi_B$. In period τ , it amounts to

$$\chi_\tau = \sum_{t=0}^{\tau} \chi_B (1 - \delta_p)^t = \chi_B \frac{1 - (1 - \delta_p)^{\tau+1}}{\delta_p}. \quad (19)$$

As $\tau \rightarrow \infty$, this fraction approaches $\chi_\infty = \chi_G = \chi_B/\delta_p$, and the good doctor has fully built up his reputation.

We now construct a perfect Bayesian equilibrium, regarding the off-equilibrium action by a good doctor buying a bad practice. The main difficulty consists in constructing a sequence T_τ which supports a perfect Bayesian equilibrium, i. e., the price a doctor gets if he sells this practice (with an accrued clientele $\chi_t < \chi_G$).

The buying doctor must consider the probability with which he will buy from a good doctor. If he buys from a bad doctor, patients will leave the practice before the new doctor can convince them that he is good. The practice is hence worthless. If χ_t is observed by the new doctor, the relative probability that the old doctor is good and has built up this clientele on his own is

$$\delta_d (1 - \delta_d)^{t-1},$$

because the doctor must have “survived” for $t - 1$ periods, and retired in period t . The relative probability that the doctor is already bad and that the practice is hence worthless is

$$\delta_d^2 (1 - \delta_d)^{t-2}.$$

Hence no matter which χ_t a doctor observes, the probability that the current doctor is still good (thus that he can reap some of the gains from this clientele) is

$$\frac{\delta_d (1 - \delta_d)^{t-1}}{\delta_d (1 - \delta_d)^{t-1} + \delta_d^2 (1 - \delta_d)^{t-2}} = \frac{1 - \delta_d}{(1 - \delta_d) + \delta_d} = 1 - \delta_d.$$

Now the crucial question is at which transfer price $T(\chi_t)$ a practice with a fraction χ_t of high-value patients is traded. Be T_τ the transfer price that is paid for a practice with a clientele that has been built up for $\tau \geq 1$ periods, hence $T_\tau = T(\chi_\tau)$. For a bad doctor considering the purchase of a practice with some clientele χ_t , we have (at the indifference point)

$$T_\tau = (1 - \delta_d) ((\chi_\tau - \chi_B) (f_H - f_L) + \delta_d T_{\tau+1}). \quad (20)$$

On the one hand, the bad doctor pays the transfer price T_τ ; on the other hand, he has a chance $1 - \delta_d$ to get a practice from a good doctor, in which case he receives higher fees for one period and can resale the practice for a high transfer price with probability δ_d .

Lemma 3 (Transfer Price for a Practice with Medium Clientele) *The transfer price that a bad doctor is willing to pay for a practice with medium clientele ($\chi_\tau \in (\chi_G, \chi_B)$) is given by*

$$T_\tau = \chi_B (f_H - f_L) (1 - \delta_d) \frac{1 - \delta_p}{\delta_p} \left(\frac{1}{1 - \delta_d (1 - \delta_d)} - \frac{(1 - \delta_p)^\tau}{1 - \delta_d (1 - \delta_d) (1 - \delta_p)} \right) \quad (21)$$

if $\tau \geq 1$. However, $T_1 = 0$.

Note that T_τ increases with τ , and $\lim_{\tau \rightarrow \infty} T_\tau \rightarrow T_\infty$ as derived in (4a). The reason that $T_1 = 0$ is that, if a good doctor sells directly after the first period, his clientele is still $\chi_0 = \chi_B$. Therefore, buying doctors infer that the selling doctor is bad with probability 1. In later periods, $\chi_t > \chi_B$, and buying doctors believe that they will be able to buy from a good doctor with positive probability. Because χ_τ is just a monotone function of τ , we can write τ as a function of the clientele χ , and hence derive $T(\chi)$ from T_τ . From (19), we get

$$\tau(\chi) = \frac{\log(1 - \delta_p \chi / \chi_B)}{\log(1 - \delta_p)} - 1.$$

Substitution into (21) yields

$$\begin{aligned} T(\chi) &= \chi_B (f_H - f_L) (1 - \delta_d) \frac{1 - \delta_p}{\delta_p} \left(\frac{1}{1 - \delta_d (1 - \delta_d)} - \frac{(1 - \delta_p \chi / \chi_B) / (1 - \delta_p)}{1 - \delta_d (1 - \delta_d) (1 - \delta_p)} \right) \\ &= (f_H - f_L) \frac{1 - \delta_d}{\delta_p} \left(\frac{\chi_B (1 - \delta_p)}{1 - \delta_d (1 - \delta_d)} + \frac{\delta_p \chi - \chi_B}{1 - \delta_d (1 - \delta_d) (1 - \delta_p)} \right). \end{aligned} \quad (22)$$

Hence the transfer price $T(\chi)$ is an affine linear function of the clientele χ of a practice (given that $\chi \notin \{\chi_B, \chi_G\}$).

Two properties of the sequence T_τ remain to be shown. First, in order to prove that T_τ really renders the out-of-equilibrium transfer prices for a practice that has accumulated a clientele χ_τ , we must prove that it really is the bad doctors that buy a practice with this clientele, i. e. that good doctors prefer to buy good practices. Second, we must show that good doctors never buy bad practices, given the sequence T_τ of expected future transfer prices. Both question can be answered at once. We prove that a good doctor would not buy a practice with clientele χ_τ for any $\tau \geq 0$. Because $T_0 = T_B = 0$, we know in particular that he would not buy a bad practice for $T_B = 0$. We have

$$T_\tau - T_{\min} \leq \sum_{t=0}^{\infty} (1 + \delta_d)^t (\chi_{t+\tau} - \chi_G) (f_H - f_L) + (1 - \delta_d)^t \delta_d (T_{t+\tau} - T_{\min}),$$

which is implied by

$$\begin{aligned} 0 &\leq 1 - (1 - \delta_d) (\delta_d + \delta_p - 2 \delta_d \delta_p) \quad \text{and} \\ 0 &\leq 1 - \delta_d (1 - \delta_d) (1 - \delta_p). \end{aligned}$$

Both are true for $\delta_d, \delta_p \in [0; 1]$.

Sorting of Doctors' Types. Recall that at the transfer price T_{\min} , bad doctors are indifferent about buying a good or a bad practice. To complete the analysis, we must show that at this price, good doctors strictly prefer to buy good over bad practices, i. e. $\Pi_G(\chi_G) > \Pi_G(\chi_B)$. In their decision, they take into account that after τ periods, they could sell at a transfer price T_τ if they bought a bad practice. Keeping in mind that $T_1 = T_B = 0$, this is equivalent to showing

$$\begin{aligned} \Pi_G(\chi_G) - \Pi_G(\chi_B) &> \Pi_B(\chi_G) - \Pi_B(\chi_B), \\ (f_H - f_L) \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_G - \chi_{t+1}) + \sum_{t=0}^{\infty} (1 - \delta_d)^t \delta_d (T_{\min} - T_t) \\ &> (f_H - f_L) (\chi_G - \chi_B) + \delta_d T_{\min}, \\ (f_H - f_L) \sum_{t=1}^{\infty} (1 - \delta_d)^t (\chi_G - \chi_t) + \sum_{t=1}^{\infty} (1 - \delta_d)^t \delta_d (T_{\min} - T_{t+1}) &> 0. \end{aligned} \tag{23}$$

This is true because $T_{\min} > T_t$ and $\chi_G > \chi_t$ for all $t \geq 1$. Both χ_t and T_t rise monotonously; χ_t converges against χ_G ; and T_t is even bounded away from T_{\min} , cf. (5a). The intuition is that in the first period, both good and bad doctor benefit equally from buying a good practice. Henceforth, the bad doctor no longer benefits, whereas benefits keep accruing for the good doctor.

The Informative Equilibrium – Main Results. Summing up our previous analysis, we have the following results.

Proposition 1 (Informative Equilibrium) *Suppose $\lambda_H < \lambda_G + \delta_P(1 - \lambda_G)$. If $s \in [s_L; s_H]$, then there is an informative equilibrium.*

Good new doctors pay a transfer price $T(\chi_G) = T_{\min}$; bad new doctors pay $T(\chi_B) = 0$.

Remark 2 (Low Switching Costs) *If switching costs s are low ($s < s_L$), a different sort of informative equilibrium exists (see appendix). The only difference from the equilibrium of proposition 1 is that low-value patients also switch with a certain positive probability $\sigma_L^t(B, \cdot) > 0$. Their strategies must be modified accordingly. If $s \leq \tilde{s}_L$, this probability is 1, with*

$$\tilde{s}_L = (u_G - u_B) \frac{\lambda_H}{\delta_p + \delta_d(1 - \delta_p)}.$$

A proof of this result is given in appendix A.1. What happens if $\lambda_H < \lambda_G + \delta_P(1 - \lambda_G)$? Then, in equilibrium, low-value patients are always rejected by a good doctor and a high-value patient runs the risk of being rejected by a good doctor. This means that the calculations have to be modified substantially. The new feature under this parameter constellation is that a good doctor can build up a reputation in finite

time. Still, there is an informative equilibrium so that building up a reputation is not an equilibrium phenomenon. The formal analysis is relegated to section A.2 of the appendix.

To summarize, if switching costs are not excessive reputation transfer is supported as an equilibrium phenomenon provided that the share of high-value patients is sufficiently small. A special case of our model is one with a single patient type and aggregate excess capacity of practices.

Remark 3 (Single Patient Type) *Suppose that only a single type of patient exists, but that there is excess capacity in the market. This means that doctors are interested in a large patient base. A new doctor then makes his bid for a practice, dependent upon the number of treatments in the previous period. Such a situation is a special case of our model, where low-value patients are effectively ignored (by setting $f_L = 0$). In equilibrium, good and bad doctors have excess capacity but good doctors provide more treatments. Bad doctors pick up patients who, after the treatment, switch immediately. If there are more patients such that the parameter condition of appendix A.2 holds, good doctors operate under full capacity and only bad doctors have excess capacity.*

4.2 Non-Existence of Uninformative Equilibrium

In this section we show that equilibria in which types do not sort do not exist.

Definition 2 (Uninformative Equilibrium) *An equilibrium is called uninformative if it is characterized by the following observable strategies (along the equilibrium path).*

1. *Old low-value patients stay with their practices. Old high-value patients stay if their doctor was good or retires; otherwise they switch.*
2. *New low- and high-value patients choose their practices at random.*
3. *New doctors buy from old doctors at random. The type of the old doctor contains no information on the new doctor.*

In comparison to the first item of definition 1, item 1 here says that high-value patients switch away from bad doctors unless ownership changes. Item 2 is the same as in definition 1. Item 3 justifies the name “uninformative equilibrium”. There is no relation between the type of the selling and that of the buying doctor.

Proposition 2 (Uninformative Equilibria) *Assume that*

$$s < (u_G - u_B) \frac{\lambda_H}{\delta_p + \delta_d(1 - \delta_p)}.$$

Then there is no uninformative stationary equilibrium.

Intuitively, the proof can be made by contradiction: Assume there is an uninformative equilibrium. Then patients consistently must believe that the expected quality of a new doctor is average, independently of whether the practice used to be good or bad before. The retiring doctor's type contains no information about the type of the new doctor. Now patients always stay with good doctors, whereas high-value patients switch away from bad doctors. This implies that, during their professional life, good doctors accumulate a better clientele than bad doctors. Because patients expect that their probability of getting a good doctor is independent of whether they stay or switch after the retirement of their doctor; they stay because of the (possibly infinitesimal) switching costs s . This implies that doctors must believe that they can inherit a good clientele and turn it into profit. As in section 4.1, a high-value clientele is relatively more valuable for good doctors. As a result, good doctors bid higher transfer prices; hence patients of good practices tend to remain in the hands of good doctors. This contradicts the uninformative nature of stationary equilibrium. A formal proof follows.

Proof of proposition 2: Assume that at a point in time, practices with differing clienteles χ_B and χ_τ are for sale. We show that a good doctor benefits more from buying the χ_τ -practice. Expected profits of a good doctor, should he buy the χ_B -practice, are

$$\Pi_G(\chi_B) = \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_t f_H + (1 - \chi_t) f_L) + \sum_{t=0}^{\infty} (1 - \delta_d)^t \delta_d T_{t+1},$$

where $T_1 = 0$, and T_τ increases strictly with τ .

For a bad doctor, projected profits from buying the χ_B -practice are

$$\Pi_B(\chi_B) = \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_B f_H + (1 - \chi_B) f_L),$$

whereas profits from buying the χ_τ -practice are

$$\Pi_B(\chi_\tau) = -T_\tau + \delta_d T_{\tau+1} + (\chi_\tau f_H + (1 - \chi_\tau) f_L) + \sum_{t=1}^{\infty} (1 - \delta_d)^t (\chi_B f_H + (1 - \chi_B) f_L).$$

Comparing profits, both good and bad doctors pay the same transfer price T_τ for the χ_τ -practice; their additional revenue in the first period is the same; and if they sell immediately after one period, the transfer $T_{\tau+1}$ they get is the same. Yet for the remaining time, the good doctor benefits more. Comparably to (23),

$$\begin{aligned} & (\Pi_G(\chi_\tau) - \Pi_G(\chi_B)) - (\Pi_B(\chi_\tau) - \Pi_B(\chi_B)) \\ &= \left(\sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_{t+\tau} - \chi_t) (f_H - f_L) + \sum_{t=0}^{\infty} (1 - \delta_d)^t \delta_d (T_{t+\tau+1} - T_{t+1}) \right) \\ & - \left((\chi_\tau - \chi_B) (f_H - f_L) + \delta_d T_{\tau+1} \right) \\ &= \sum_{t=1}^{\infty} (1 - \delta_d)^t (\chi_{t+\tau} - \chi_t) (f_H - f_L) + \sum_{t=1}^{\infty} (1 - \delta_d)^t \delta_d \underbrace{(T_{t+\tau+1} - T_{t+1})}_{>0}, \end{aligned} \quad (24)$$

which is strictly positive if χ_t is strictly increasing in t , hence if high-value patients switch from bad doctors.²¹ \square

Note that the proof is even *more general* than the claim of proposition 2. It shows that not only perfectly uninformative equilibria are impossible. It also demonstrates that the informative equilibrium from proposition 1 is unique in the following sense. In a stationary equilibrium, good doctors always buy good practices, and bad doctors always buy bad practices. Otherwise, both good and bad doctors would have to adopt a mixed strategy. This is impeded by (24).

In addition, the proof indicates that the informative equilibrium from proposition 1 is even unique in a non-stationary framework. Starting from any distribution of clienteles, good doctors always buy the practices with high-value clienteles. At retirement, they leave behind a practice with an even better clientele. Bad doctors buy practices with low-value clientele, and with a large probability, this clientele even breaks down to χ_B . Hence, from any initial distribution, we have convergence towards the informative equilibrium.

In particular, there is no equilibrium in which $T_G = T_B = 0$, in which the price of an old practice is independent of its clientele. However, if switching costs s are considerably large ($(u_G - u_B) \frac{\lambda_H}{\delta_p + \delta_d(1 - \delta_p)} < s < (u_G - u_B) \frac{\lambda_H}{\delta_p}$), informative and uninformative equilibrium may coexist.

As already stated in remark 1, our result is more than a coordination result. In particular, a situation in which new good owners buy bad practices cannot be supported in equilibrium because customers have only information about the quality of the doctor they visited in the previous period, that is, information is local.

5 Discussion and Conclusion

In this paper, we have presented a model of reputation as a tradeable intangible asset. We have placed the model in the context of the sale of a doctor's practice. As

²¹The maximal switching costs s_H under which high-value patients switch from bad doctors are lower in the uninformative equilibrium. The reason is that if they find a good doctor, they can only be sure that the practice will remain good until the old doctor retires. Hence

$$\begin{aligned} s_H &= \lambda_G (u_G - u_B) \sum_{t=0}^{\infty} (1 - \delta_p)^t (1 - \delta_d)^t \\ &= (u_G - u_B) \frac{\lambda_G}{1 - (1 - \delta_p)^t (1 - \delta_d)^t} = (u_G - u_B) \frac{\lambda_G}{\delta_p + \delta_d(1 - \delta_p)}. \end{aligned}$$

In any case, profits of a good doctor from buying the χ_τ -practice are

$$\Pi_G(\chi_\tau) = -T_\tau + \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_{t+\tau} f_H + (1 - \chi_{t+\tau}) f_L) + \sum_{t=0}^{\infty} (1 - \delta_d)^t \delta_d T_{t+\tau+1}.$$

mentioned in the introduction, the same formal argument can be directly applied to other markets such as dentists' practices, pharmacies, and lawyers', notaries', and tax advisors' offices.

The Sale of a Doctor's Practice: Summary. We have characterized stationary equilibria with a price difference between practices sold by good doctors and practices sold by bad doctors exist. In equilibrium, the two types of doctors are separated, that is:

1. Bad new doctors do not buy practices from good old doctors.
2. Good new doctors do not buy practices from bad old doctors.

With respect to the first item, note that there is a free-riding problem because the quality of a doctor is an experience good so that patients can leave only after they see that the current doctor is bad. However, if the price for a practice of a good old doctor is sufficiently high, bad new doctors do not have an incentive to free-ride on the reputation of a good old doctor. With respect to the second property, note that new doctors can save the money needed to buy a practice with a good reputation and build up their reputation on their own, because over time, more and more high-value patients visit their practice and stay. Here, the share of high value patients can be seen as the intangible asset: since high-value patients stay unless the treatment is bad, reputation can be traded. Indeed, if the price is sufficiently low, buying reputation is more attractive than building up reputation over time. The equilibrium price satisfies both properties.

To transfer reputation it is crucial that new doctors can distinguish between practices with a good and with a bad reputation. Although new doctors do not observe the old doctors' types, they can make informed decisions because they observe the composition of patients within a practice – a large share of high-value patients is an indicator that the old doctor is good. Otherwise, new doctors would not be able to distinguish between the different practices put up for sale.²²

Patients have only local information about a practicing doctor, so that reputation can only be local: Only patients frequenting a practice of a doctor in a particular period learn the quality of that doctor in that period. Otherwise, patients do not have any idea about the quality of other doctors. This endogenously creates switching costs (in addition to exogenous switching costs) because of quality uncertainty. In equilibrium, these switching costs are present even when patients observe that the old doctor is replaced by some new doctor. In other words, patients believe

²²Other model versions could be constructed, in which the old doctor's type is directly observable to new doctors or in which some indicators for the attractiveness of a practice are available before the sale of the practice is completed (see below).

that good doctors are followed by good doctors and bad doctors are followed by bad doctors.

For sufficiently low switching costs, an uninformative equilibrium does not exist. For higher switching costs, informative and uninformative equilibria coexist on a certain range. From a normative point of view, an uninformative equilibrium has the attractive feature that switching costs are avoided. However, switching may be socially desirable if there are efficiency gains from matching high-value patients to good doctors. This can be exemplified by the one-type model (see remark 3): The informative equilibrium corresponds to a situation in which patients are likely to visit a good doctor, $\chi_G > \chi_B$. In contrast, fewer patients visit good doctors in an uninformative equilibrium.

The Sale of a Doctor’s Practice: Discussion. Certain aspects of the ambulatory health care sector have not been included in our model. In particular, we have postulated that treatments have the nature of experience goods. However, they may be viewed as, in part, having attributes of credence goods. This would mean that patients often do not observe the quality directly after a treatment (or series of treatments) – see the general discussion on the information of customers below. Here, we discuss aspects which are unrelated to the information patients or doctors receive; the corresponding model modifications are also partly of interest for other examples.

In our model we have focused on the quality of a treatment, abstracting from the different amount of time and number of visits it takes for a doctor, for example, to make the correct diagnosis. To fix ideas, suppose a treatment consists of a number of visits and tests in a given period. A good doctor can then be defined as a doctor who makes the correct diagnosis faster, that is, fewer visits and tests are required. This increases the utility of the patient. However, since treatment fees are not based on the “output” (that is, the time it takes to make the correct diagnosis) but on “input” (that is, the number of visits and the number and type of tests), a bad doctor may obtain higher profits in the short term.²³ The increase in visits and tests per patient by bad doctors is reminiscent to supply-induced demand, an important topic in health economics. Our model can accommodate this feature: bad doctors then have a stronger incentive to bid for practices with a large share of high-value patients, and reputation transfer becomes more difficult to support.

In our model we have focused on the quality of doctors. Yet certainly, not only qualitative attributes of a doctor play a role in a patient’s decision on whether or not to switch practice. A new doctor certainly has horizontal characteristics different from the retiring doctor, an obvious one is the different age and all the implications this has. Therefore, incorporating these horizontal taste idiosyncracies

²³There are two reasons for this. First, he may need more intensive or expensive tests. Second, provided that he generates more visits by a patient per treatment, he in particular has a larger number of visits by privately insured patients than a good doctor.

into the model, we would obtain that a larger share of patients switch practices after an ownership change (immediately, if this characteristic is *ex ante* observable or with a lag if this requires experience). Our results appear robust provided that quality aspects are an important determinant of customer behavior.

In our model we have focused on the intangible asset of a practice. Clearly, the sale of a practice often also involves tangible assets (long-term rental contracts, apparatuses etc.). If these tangible assets have the same value for good and for bad doctors, our analysis can simply be interpreted as conditional on the value of the tangible asset. However, complementarities between tangible and intangible assets may exist. In particular, certain apparatuses may be more useful for good doctors than for bad doctors. Then the presence of such tangible assets reinforces our argument for the self-selection of doctor types.

In our model we have focused on the composition of patients at a practice. In reality, practices do not necessarily operate under full capacity. As we have pointed out in remark 3, a model in which there is an overcapacity for treatments and a single type of patient is a special case of our model. More generally, our result of reputation transfer can also be shown in a model with two types of patients and an overcapacity for treatments.

In the remainder we no longer refer to our example, the sale of a doctor's practice. We first discuss changes in assumptions with respect to the information new owners and patients possess. We then discuss ways to endogenize the population shares in our model. We finally discuss the possibility that the price for a good or service is determined on the market.

The Information of New Owners. In our model, we have assumed that new owners do not observe the type of the old owner. The only information they receive is concerned with the composition of the customer base. Here, we discuss a number of modifications of this assumption.

A simpler setting would consist in postulating that the type of old owners can be directly observed by new owners. In this case, new owners no longer have any inference problem from the observed composition of the customer base to next period's composition; that is, they perfectly learn the type of the old owner of a particular practice also off-equilibrium and, given customers' beliefs, can always perfectly predict the composition of the customer base. Our results remain valid under this alternative assumption.

Along the same lines, new owners may also observe how long the retiring owner was in charge of the practice. This means that although new owners do not directly observe the old owner's type, they know that if an owner has attracted a high share of high-value customers and if that owner was active for more than one period, he must be a good owner. Again, the off-equilibrium analysis would need to be modified, but our results remain valid.

Worsening the precision of information new owners receive about a practice for sale, we may introduce a probability that the books of old owners are cooked, so that there is a positive probability that they are uninformative. On the whole, our mechanism of reputation transfer is robust to such an extension, provided that the books are sufficiently informative. Endogenizing the precision of information, it must be sufficiently costly to cook the books.

The Information of Customers. In our model, customers learn the quality of the product or service after consumption. Furthermore, customers have a memory of one period only. Note that a bounded memory could be made endogenous because more memory would not lead to additional utility along the equilibrium path.

In our model, customers do not receive informative signals about the quality of the products or services of firms they did not visit before. In reality, word-of-mouth communication is likely to be relevant as a means to acquire information on owners who are not personally known. This information typically will be noisy. Suppose that customers can get noisy signals about owners of other firms; then the informative equilibrium survives. However, for low switching costs, also an uninformative equilibrium may coexist. In this equilibrium, high-value customers switch after ownership change to a recommended owner, whose conditional expected quality is above average.

Word-of-mouth communication may be predominant as a means for new customers to obtain information from experienced customers. If a share of new customers has received information about the owner's type from old customers and if this information is reliable, we can simply relabel the groups of old and new customers by including new customers who receive information in the group of experienced customers.

Customers learn the owner's type after one period. We can extend the model and allow for noisy signals for customers about the quality of the owner. Suppose the quality is observed with a probability of less than 1. This means that the product or service is to some extent a credence good. In our model this leads to a smooth deterioration of the clientele, should a bad type acquire a firm previously owned by a good one. If the detection probability is sufficiently high, our results remain valid.

The Distribution of Owner and Customer Types. Population shares are parameters in our model. In an extended model, these can be endogenized.

We postulated that there is a fixed share of good and bad types of owners. The model can be extended to allow for investments in education, so that those who *have* invested are good doctors, whereas those who have *not* are bad. The investment occurs before acquiring a firm, and it must be unobservable to customers. With ex ante homogeneous owners the cost of investment must equal the expected difference

in lifetime profits for good compared to bad owners. In a model with heterogeneous owners, this must hold for the marginal owner.²⁴

The analysis can also be extended to combine adverse selection and moral hazard. A share λ_G of owners, the potentially good owners, decides whether to invest in education leading to good quality, whereas a share $\lambda_B = 1 - \lambda_G$ is not able to improve quality by investing in education. Then an informative equilibrium corresponds to a situation in which potentially good owners *do* invest in education. A reputation transfer alleviates the moral hazard problem. In an uninformative equilibrium, owners have a weaker incentive to invest in education, which is an additional source for a welfare loss.

We also postulated that there is a fixed share of high-value and low-value customers. The model could be extended to allow for customers to select their type. In the example this would be the insurance contract (e. g. full insurance/partial insurance). With ex ante homogeneous customers, the extra cost of full insurance must equal the expected lifetime gain from the higher probability of being served by good owners. In a model with heterogeneous customers, this must hold for the marginal customer.

The Price of a Product or Service. In our model the price of a product or service was exogenously given and larger for high-value than for low-value customers. This price can be endogenized by considering bargaining between an owner and a customer and postulating that any expected surplus is equally split. Here a low-value customer refers to a customer with a low willingness-to-pay; correspondingly for a high-value customer. Suppose the owner's quality is known. Then Nash bargaining between owners and customers leads to a higher price for high-value customers. In addition, the net surplus for customers is larger from bargaining with a good owner than with a bad one. In case of ownership change or in case the customer has switched firm, bargaining takes place given the beliefs of the customer.²⁵ To obtain our results, we only need that the customer gets a higher discounted expected net surplus when facing an owner of uncertain type than when facing a bad owner. Note that this is compatible with a situation in which the uninformed customer receives less in the period after switching than if she stayed with a bad owner.

Perhaps more relevant is the issue of price setting by the owner. To show that our argument also applies to this case, we consider a simple market: suppose that there is a single type of customer of mass λ_p with linear demand $a_d - P$, $d \in \{G, B\}$, where $a_G > a_B$, and that firms have constant marginal cost of production c . The firm sets a price P to all customers. Suppose furthermore, that customers observe

²⁴As is well understood in the literature on network effects, there may exist situations with multiple equilibria: If all other owners are good, a bad owner has a very unattractive composition of patients, whereas if all others are bad, he will get the population average. This makes new owners more inclined to invest in education if others do.

²⁵Note that this is a situation with one-sided asymmetric information and interdependent values. Unfortunately, we are not aware of results in the literature on non-cooperative bargaining that we could directly apply (see Ausubel et al. (2002)).

the price only after selecting the firm, that is, there is no price transparency (to avoid multiplicity of equilibria we implicitly assume that each additional search involves an infinitesimal cost ε). However, upon observing the price, they may search again. For everything else, the assumptions of section 3 are assumed to hold.

We then can show that under some parameter restrictions, our informative equilibrium takes a similar form to that with fixed prices. In equilibrium, good owners maximize expected profits, taking into account the equilibrium mix of experienced and unexperienced customers. This price P^* is between $P_G = (a_G + c)/2$ and $P_B = (a_B + c)/2$. Bad owners mimic this price P^* in order not to be perceived to be bad. Hence, in equilibrium, customers switch whenever they observe bad quality; they stay if the quality was good. Customers effectively ignore ownership change and the price of a market transaction P does not signal quality.²⁶

Comparing this simple model to our one-type model with fixed prices, we note a small but interesting difference: the net surplus derived from an experienced customer is greater than the net surplus derived from an unexperienced customer. In contrast, in our fixed price model the net surplus is independent of experience. This difference arises because in the model with price setting a customer has variable demand and a customer who has newly arrived or switched buys less than one who is experienced. In other words, in this model, experienced customers are more valuable than unexperienced customers for a good owner.

Conclusion While we framed the possibility of reputation transfer in the context of the sale of a doctor's practice, our argument can be applied to a wide variety of industries. In many industries in which businesses are run by the owner, ownership changes frequently occur (and are exogenous events). Furthermore, the change in ownership of a firm is often observable to customers. Nevertheless, because the customer base is an intangible asset, reputation can be transferred at a positive price from one generation of owners to the next.

A Technical Appendix

A.1 Proofs

Proof of lemma 1: Given that low-value patients at bad doctors' practices switch with probability $\pi_L = 0$, the number of low-value patients applying at good practices is equal to the rate of newborn patients, weighted with the fractions of good practices and low-value patients,

$$\delta_p \lambda_G \lambda_L = \delta_p \lambda_G (1 - \lambda_H).$$

²⁶We would like to point out that, in the underlying monopoly pricing problem, price cannot be a signal of product quality.

The number of applying high-value patients can be taken from (8); it is

$$\begin{aligned} \delta_p \lambda_G \lambda_H + \chi_B (1 - \delta_p) (1 - \lambda_G) \lambda_G &= \delta_p \lambda_G \lambda_H + \frac{\delta_p \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} (1 - \delta_p) (1 - \lambda_G) \lambda_G \\ &= \frac{\delta_p \lambda_G \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)}. \end{aligned}$$

After all these high-value patients have been accepted, the spare capacity κ_G of a good doctor is

$$\begin{aligned} \kappa_G &= \delta_p \lambda_G - (\delta_p \lambda_G \lambda_H + \chi_B (1 - \delta_p) (1 - \lambda_G) \lambda_G) \\ &= \delta_p \lambda_G (1 - \lambda_H) - \frac{\delta_p \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} (1 - \delta_p) (1 - \lambda_G) \lambda_G. \end{aligned} \quad (1a)$$

The probability of a low-value patient being accepted after having applied at a good practice is

$$\begin{aligned} \Pr\{\text{accepted by G|L}\} &= \frac{\kappa_G}{\delta_p \lambda_G (1 - \lambda_H)} \\ &= 1 - \frac{\lambda_H}{1 - \lambda_H} (1 - \lambda_G) \frac{1 - \delta_p}{\delta_p + \lambda_G (1 - \delta_p)}. \end{aligned} \quad (2a)$$

For low-value patients, there is no incentive to try and switch from a bad practice if the costs of a trial s are sufficiently large,

$$\begin{aligned} s \geq s_L &:= \lambda_G \Pr\{\text{accepted by G|L}\} (u_G - u_B) \sum_{t=0}^{\infty} (1 - \delta_p)^t \\ &= \frac{\lambda_G}{\delta_p} \Pr\{\text{accepted by G|L}\} (u_G - u_B), \end{aligned} \quad (3a)$$

where $\Pr\{\text{accepted by G|L}\}$ is taken from (2a). \square

Proof of lemma 2: (14) is structurally similar to (13). The only difference is that for a high-value patient, the probability of being accepted is 1, hence we have to set $\Pr\{\text{accepted by G|L}\} = 1$ in (13). \square

Proof of lemma 3: We are looking for a solution to (20), which gives us a recursive formula: We can derive $T_{\tau+1}$ from T_{τ} . However, we would need some beginning for the recursion. Although $T(\chi_0) = T(\chi_B) = 0$, this does not serve our purposes, because (20) does not hold for the step from T_0 to T_1 . The reason is that, in equilibrium, a bad doctor buying a practice at $T_0 = 0$ cannot expect with any probability to sell the practice at a higher price after one period. We can analyze the property of T_{τ} as $\tau \rightarrow \infty$, though. Because $\chi_{\tau} \rightarrow \chi_G$, we get

$$\begin{aligned} T_{\infty} &= (1 - \delta_d) ((\chi_G - \chi_B) (f_H - f_L) + \delta_d T_{\infty}) \\ T_{\infty} &= \frac{1 - \delta_d}{1 - \delta_d (1 - \delta_d)} (\chi_G - \chi_B) (f_H - f_L). \end{aligned} \quad (4a)$$

Interestingly, $T_\infty < T_{\min}$, although the clientele after many periods is the same. This is because a doctor who buys a practice that has only infinitesimally lower clientele than χ_G runs the risk of getting a valueless practice with probability δ_d . We have

$$\begin{aligned}
T_\infty < T_{\min} &\iff & (5a) \\
\frac{1 - \delta_d}{1 - \delta_d(1 - \delta_d)} (\chi_G - \chi_B) (f_H - f_L) &< \frac{1}{1 - \delta_d} \frac{(1 - \delta_p) \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} (f_H - f_L) \\
1 - \delta_d(1 - \delta_d) &< (1 - \delta_d)^2 \\
1 - \delta_d + \delta_d^2 &< 1 - 2\delta_d + \delta_d^2.
\end{aligned}$$

We now find a solution for T_τ by an ‘‘educated’’ guess, derived from the following intuition. Imagine the doctors determine their T_τ ‘‘dynastywise’’, i.e. such that a doctor takes into account the gains a buying doctor expects after one period (which is known) instead of a transfer price that he can get after one period (which is unknown). In (6a), $1 - \delta_d$ is the probability with which a practice is valuable at all, and δ_d the probability that it will be sold after one period.

$$\begin{aligned}
T_\tau &= \sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t (\chi_{t+\tau} - \chi_B) (f_H - f_L) & (6a) \\
&= (f_H - f_L) \sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t \chi_B \left(\frac{1 - (1 - \delta_p)^{t+\tau+1}}{\delta_p} - 1 \right) \\
&= \chi_B (f_H - f_L) \sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t \frac{(1 - \delta_p) - (1 - \delta_p)^{t+\tau+1}}{\delta_p} \\
&= \chi_B (f_H - f_L) (1 - \delta_d) \frac{1 - \delta_p}{\delta_p} \sum_{t=0}^{\infty} (1 - \delta_d)^t \delta_d^t (1 - (1 - \delta_p)^{t+\tau}) \\
&= \chi_B (f_H - f_L) (1 - \delta_d) \frac{1 - \delta_p}{\delta_p} \left(\frac{1}{1 - \delta_d(1 - \delta_d)} - \frac{(1 - \delta_p)^\tau}{1 - \delta_d(1 - \delta_d)(1 - \delta_p)} \right).
\end{aligned}$$

From (6a) one can see directly that the recursion equation (20) is satisfied. Dividing (20) by $(f_H - f_L)$ yields

$$\begin{aligned}
&\sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t (\chi_{t+\tau} - \chi_B) \\
&= (\chi_{t+\tau} - \chi_B) (1 - \delta_d) + (1 - \delta_d) \delta_d \sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t (\chi_{t+\tau+1} - \chi_B) \\
&= (\chi_{t+\tau} - \chi_B) (1 - \delta_d) + (1 - \delta_d) \delta_d \sum_{t=1}^{\infty} (1 - \delta_d)^t \delta_d^{t-1} (\chi_{t+\tau} - \chi_B) \\
&= (\chi_{t+\tau} - \chi_B) (1 - \delta_d) + \sum_{t=1}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t (\chi_{t+\tau} - \chi_B) \iff \\
(1 - \delta_d) (\chi_\tau - \chi_B) &= (\chi_{t+\tau} - \chi_B) (1 - \delta_d). \quad \square
\end{aligned}$$

Proof of remark 2: We first calculate the probability π_L with which a low-value patient switches away from a bad practice in equilibrium. In analogy to (8), the number of low-value patients arriving at good practices is

$$\delta_p \lambda_G (1 - \lambda_H) + (1 - \chi_B) (1 - \delta_p) (1 - \lambda_G) \lambda_G \pi_L.$$

The spare capacity of good doctors after all arriving high-value patients have been accepted is given in (1a); it is independent of how many low-value patients arrive. Hence for a low-value patient, the probability of being accepted after having applied at a good practice is

$$\Pr\{\text{accepted by G|L}\} = \frac{\kappa_G}{\delta_p \lambda_G (1 - \lambda_H) + (1 - \chi_B) (1 - \delta_p) (1 - \lambda_G) \lambda_G \pi_L}.$$

In equilibrium, the low-value patient must be indifferent about switching or not, hence necessarily

$$s = \frac{\lambda_G}{\delta_p} \Pr\{\text{accepted by G|L}\} (u_G - u_B).$$

When setting $\pi_L = 0$, we receive s_L , which obviously is equal to that calculated in (3a). When setting $\pi_L = 1$, we get \tilde{s}_L . Because $\Pr\{\text{accepted by G|L}\}$ rises with π_L , it is clear that $s_L > \tilde{s}_L$. Furthermore, one can easily prove that $\tilde{s}_L > 0$ iff $\lambda_H < \delta_p + \lambda_G (1 - \delta_p)$, which is true because of (12).

We now argue for why we obtain an informative equilibrium. For doctors and high-value patients, expected profits are independent of how many low-value patients try to switch practices. Therefore, also the clienteles of practices remain the same. Consequently, all three properties of an informative equilibrium (definition 1) remain unchanged. \square

A.2 More High-Value Patients than Good Doctors

Because there are more high-value patients than good doctors, good practices may possibly be filled with high-value patients. In this case, it is no longer always optimal for high-value patients at bad doctors to switch practices – if their probability of being accepted by a good practice is close to zero, they should rather stay. In the following, we analyze a mixed equilibrium in which high-value patients at bad practices switch with probability π_H .

The Clientele of Practices. First, note that in a mixed stationary equilibrium, good doctors treat only high-value patients. If they also treated low-value patients, they would have spare capacity for more high-value patients, thus a high-value patient would be accepted by a good practice with certainty. Consequently, $\chi_G = 1$, and from the market clearing condition (analogously to (9)), we receive

$$\lambda_H = \lambda_G + \chi_B (1 - \lambda_G) \iff \chi_B = \frac{\lambda_H - \lambda_G}{1 - \lambda_G}. \quad (7a)$$

Boundaries on Switching Costs s . Like in section 4.1, we have qualitatively different switching behavior for different values of s . For very large s (if $s \geq \tilde{s}_H$, where \tilde{s}_H is to be specified), neither type of patients switch. For smaller s (if $s \in (s_H, \tilde{s}_H)$), low-value patients never switch, and high-value patients adopt a mixed strategy, switching away from bad doctors with probability π_H . For $s < s_H$, high-value patients always switch away from bad doctors. Even for infinitesimal s , low-value patients do not attempt to switch from bad doctors, because their probability of being accepted by a good doctor is zero.

The number of good patients that switch practices is $(1 - \delta_p) \lambda_B \chi_B \pi_H$, as $1 - \delta_p$ is the fraction of surviving patients, λ_B is the fraction of bad practices, χ_B the fraction of high-value patients. Additionally, there are $\delta_p \lambda_H$ newborn patients looking for practices. The total number of high-value patients searching new practices is thus

$$\begin{aligned} & \delta_p \lambda_H + (1 - \delta_p) \lambda_B \chi_B \pi_H \\ &= \delta_p \lambda_H + (1 - \delta_p) (1 - \lambda_G) \frac{\lambda_H - \lambda_G}{1 - \lambda_G} \pi_H \\ &= \delta_p \lambda_H + (1 - \delta_p) (\lambda_H - \lambda_G) \pi_H. \end{aligned} \quad (8a)$$

Each patient only has the probability λ_G of finding a good practice, and this good practice has free capacity δ_p , the number of patients that have exited recently. Hence the probability of being accepted after having applied by a good practice is

$$\Pr\{\text{accepted by G|H}\} = \frac{\delta_p}{\delta_p \lambda_H + (1 - \delta_p) (\lambda_H - \lambda_G) \pi_H}. \quad (9a)$$

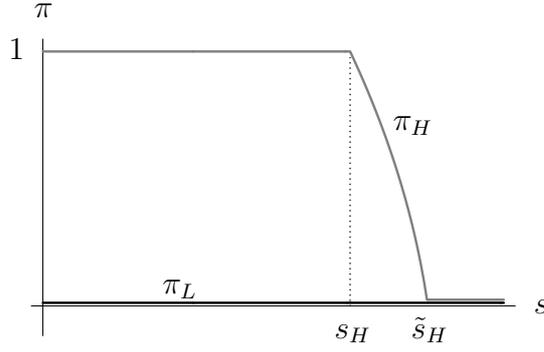
In equilibrium, expected utilities must coincide, hence

$$\begin{aligned} s &= (u_G - u_B) \Pr\{\text{accepted by G|H}\} \sum_{t=0}^{\infty} (1 - \delta_p)^t \\ &= (u_G - u_B) \frac{\delta_p}{\delta_p \lambda_H + (1 - \delta_p) (\lambda_H - \lambda_G) \pi_H} \frac{1}{\delta_p} \\ &= \frac{u_G - u_B}{\delta_p \lambda_H + (1 - \delta_p) (\lambda_H - \lambda_G) \pi_H} \implies \\ \pi_H^* &= \frac{(u_G - u_B)/s - \delta_p \lambda_H}{(1 - \delta_p) (\lambda_H - \lambda_G)}. \end{aligned} \quad (10a)$$

We can now derive the critical s_H (for which $\pi_H^* = 1$) and \tilde{s}_H (for which $\pi_H^* = 0$),

$$\begin{aligned} \pi_H^* = 1 &\implies s_H = \frac{u_G - u_B}{\lambda_H - (1 - \delta_p) \lambda_G}, \\ \pi_H^* = 0 &\implies \tilde{s}_H = \frac{u_G - u_B}{\delta_p \lambda_H}. \end{aligned}$$

Again, we concentrate on pure equilibria, hence we assume that $s < s_H$; hence high-value patients always switch away from bad practices.

Figure 4: Switching Probabilities π_H and π_L of Patients

The Separating Transfer Price T . In complete analogy to (16), a transfer price T_G that guarantees a separating equilibrium must satisfy

$$\begin{aligned}
 T_G &\geq \frac{1}{1 - \delta_d} (\chi_G - \chi_B) (f_H - f_L) \\
 &= \frac{1}{1 - \delta_d} \left(1 - \frac{\lambda_H - \lambda_G}{1 - \lambda_G}\right) (f_H - f_L) \\
 &= \frac{1}{1 - \delta_d} \frac{1 - \lambda_H}{1 - \lambda_G} (f_H - f_L) =: T_{\min}. \tag{11a}
 \end{aligned}$$

Building up Goodwill. We now consider the case in which a good doctor buys a bad practice. The issue now becomes even more involved than in section 4, because a good doctor will reach a clientele of 100% high-value patients already after finite time. Integer problems arise.

In $t = 0$, the fraction of high-value patients is χ_B . In $t = 1$, it goes up to $\chi_B (1 - \delta_p) + \chi_B$. In $t = 2$, it is $\chi_B (1 - \delta_p)^2 + \chi_B (1 - \delta_p) + \chi_B$. In period τ , it amounts to

$$\begin{aligned}
 \chi_\tau &= \min \left\{ 1, \chi_B \sum_{t=0}^{\tau} (1 - \delta_p)^t \right\} \\
 &= \min \left\{ 1, \frac{\lambda_H - \lambda_G}{1 - \lambda_G} \frac{1 - (1 - \delta_p)^{\tau+1}}{\delta_p} \right\}.
 \end{aligned}$$

One can see immediately that χ_τ reaches 1 in finite time iff

$$\begin{aligned}
 \frac{\lambda_H - \lambda_G}{1 - \lambda_G} \frac{1}{\delta_p} &> 1, \\
 \lambda_H &> \delta_p + \lambda_G (1 - \delta_p).
 \end{aligned}$$

We again found (12), using the opposite approach. In this case, a good doctor has

regained 100% high-value patients after

$$\begin{aligned}\tau_1 &= \left\lceil \frac{\log(1 - \delta_p/\chi_B)}{\log(1 - \delta_p)} - 1 \right\rceil \\ &= \left\lceil \frac{\log\left(1 - \delta_p \frac{1-\lambda_G}{\lambda_H-\lambda_G}\right)}{\log(1 - \delta_p)} \right\rceil - 1,\end{aligned}$$

where $\lceil \cdot \rceil$ stands for rounded up numbers.

Because of the integer problem, further calculations with χ_τ become rather tedious. All we have to show is that, for a good doctor, buying a bad practice and building up reputation is inferior to buying a good practice right away. We can follow the same approach as used in proposition 2. If χ_t and T_t increase with time, the benefit that good young doctors have from buying a good practice is higher than that of bad doctors (analogously to (24), bearing in mind that sums end at time τ_1). However, there is one additional prerequisite: If there are an abundance of high-value patients, practices of good doctors reach the maximal clientele $\chi = 1$ already after one period ($\tau_1 = 1$). In this case, benefits from buying a good practice are the same for both types of young doctors. We have

$$\begin{aligned}\tau_1 > 1 &\iff \\ \left\lceil \frac{\log\left(1 - \delta_p \frac{1-\lambda_G}{\lambda_H-\lambda_G}\right)}{\log(1 - \delta_p)} \right\rceil &> 2 \\ \log\left(1 - \delta_p \frac{1-\lambda_G}{\lambda_H-\lambda_G}\right) &> 2 \log(1 - \delta_p) \\ 1 - \delta_p \frac{1-\lambda_G}{\lambda_H-\lambda_G} &> (1 - \delta_p)^2 = 1 - 2\delta_p + \delta_p^2 \\ \lambda_H &< \frac{1 + (1 - \delta_p)\lambda_G}{2 - \delta_p}.\end{aligned}$$

Resuming, we have the following remark.

Remark 4 (Informative Equilibrium) *Assume that*

$$\lambda_H < \frac{1 + (1 - \delta_p)\lambda_G}{2 - \delta_p}.$$

Then an informative equilibrium as described in proposition 1 exists.

Again, for relatively low switching costs, there is no uninformative equilibrium.

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