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Bidding with Outside Options

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Abstract

We introduce and experimentally test an auction model that allows for outside options of bidders as substitutes for the auctioned object under the private values assumption. Theoretically and in the experiments, bidders respond to their individual outside options and to variations of common knowledge about competitors' outside options. Interestingly, private outside options induce concave equilibrium bidding functions with uniformly distributed valuations. The bidding data does support this property. As theoretically predicted, lower-valued outside options lead individuals to bid more aggressively in the experiments.

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1 Introduction

During the last decade, auctions have increasingly attracted attention from academia and the wider public. A major part of this increased interest is due to growing popularity of using auctions as market institutions for C2C and B2C transactions, allocating public resources and procurement contracts. Cases in point are worldwide spectrum auctions, online auction platforms such as eBay and Ricardo and virtual B2B market places, e.g. Covisint for the automotive industry or Consip's AiR for Italian public procurement offers.

Often outside options are available to bidders in addition to the object offered in the particular auction. For the purpose of illustration, suppose that individual A has the opportunity to buy a used watch either from a friend at some price or to participate in an online auction where a similar watch is offered. A's decision on the amount to bid for the watch in the auction might depend on the value she derives from seizing her outside option, i.e. from transacting with her friend.

To our knowledge, there is no literature on the effects of auction-exogenous outside options on behavior of bidders with heterogenous valuations. With identical bidder valuations and private, independent outside options, the model reduces to the standard symmetric independent private values model (SIPV), see Holt (1980). In Weber (1983), Gale and Hausch (1994), and Reiß and Schöndube (2002), outside options are endogenized by another, subsequently conducted auction as a mutually exclusive transaction alternative since bidders have single-object demand. The laboratory study of the sequential auction model in the latter paper suggests that alternative transaction opportunities are an important determinant of bidding behavior (see Brosig and Reiß, 2003). In order to isolate outside option effects from other strategic considerations arising with transaction alternatives such as entry decisions or Bayesian updating, we augment the SIPV model to allow for auction-exogenous public and private outside options and implement it in the laboratory to assess its predictive power. According to the data generated in our extensive laboratory experiment, the comparative static predictions of the augmented theory match observed behavior and clearly provide evidence for the bidding decision's dependence on available outside options.

The plan of the paper is as follows: in section 2 we introduce outside options into the symmetric independent private values auction model and derive equilibrium bidding strategies for the first-price and second-price auctions, section 3 describes our experimental design and section 4 discusses the experimental data.

2 The Symmetric Independent Private Values auction model with outside options

There are n risk-neutral individuals who maximize expected utility. Each individual has a valuation for an object that is for sale in an auction. Individual i's valuation is denoted by v_i. Individuals have unit-demand for that object. In addition, each individual has access to a transaction alternative that can be substituted for the object in the auction. The value that an individual derives from executing her outside option instead of receiving the object auctioned off is denoted by w.[1] We distinguish between public and private outside options. With public outside options, each individual derives the same benefit from the outside option. This is common knowledge. In contrast, private outside options are individual-specific and private information. In subsection 2.2 we introduce public outside options into the SIPV model and solve for bidding strategies in the first-price and second-price auction, in subsection 2.3 we extend the SIPV model to allow for private outside option, derive equilibrium bidding behavior in the first-price and second-price auction, and describe the model's efficiency properties.

2.1 Bidding without outside options

Suppose that individual valuations v_i of the object that is offered in an auction are private information and independently and identically distributed according to cumulative density function $F(v_i)$ where $v_i \in [\underline{v},\bar{v}]$. We assume throughout auctions without reserve price to concentrate on bidder behavior. Without outside options, the Bayes-Nash equilibrium bidding functions for the first- and second-price auctions are well-known (e.g. Riley and Samuelson, 1981, and Vickrey, 1961): for the first-price auction we have $b^{\text{fp}}(v) = v - \int_{\underline{v}}^{v} F^{n-1}(x)dx/F^{n-1}(v)$ and for the second-price auction $b^{\text{sp}}(v) = v$.

2.2 Public outside options

This is the easy case. In addition to valuation v_i that individual i places on the auctioned object, the value that individual i derives from executing her public outside option instead of receiving the object is the same for all individuals and equals $w_i = w \ \forall i = 1,..,n$. In particular, we assume $w \leq \underline{v}$. This ensures that every individual voluntarily participates in the auction. For any auction design for which payoff equivalence applies, it is straightforward to derive equilibrium bidding functions by application of the payoff equivalence theorem. We illustrate this for the case of the first-price sealed-bid auction.

From payment equivalence with the boundary condition that the lowest valuation type $\underline{v}$ which never wins the auction in equilibrium always receives his outside option w, we obtain (cf. Riley and Samuelson, 1981, eq. 8b) the expected equilibrium payment for a representative bidder with valuation v:

$$P(v) = vF^{n-1}(v) - w - \int_{\underline{v}}^{v} F^{n-1}(x)dx. \tag{1}$$

For the first-price design with the modification that any bidder receives w if he is unsuccessful in the auction, the expected equilibrium payment is given by

$$P(v) = F^{n-1}(v)\, b^{\text{fp-pb}}(v) - \left[1 - F^{n-1}(v)\right] \ w. \tag{2}$$

[1] Valuations of transaction alternatives are net of transaction costs. If, for instance, an alternative object is offered at a posted price, then w represents the value of the outside option net of its price. If there are many alternatives, then w corresponds to the best alternative net of prices.

Setting both expressions for the expected equilibrium payment, (1) and (2), equal to each other and solving for $b^{\text{fp-pv}}(v)$ leads to the intuitive result that the equilibrium bid under the first-price design with public outside option precisely matches that without public outside options reduced by the outside option's value:

$$b^{\text{fp-pb}}(v, w) = b^{\text{fp}}(v) - w$$

where $b^{\text{fp}}(v)$ is the equilibrium bidding function in the first-price auction without outside options defined in the preceding subsection.

In the second-price auction the weakly dominant bidding strategy is given by:[2]

$$b^{\text{sp-pb}}(v, w) = v - w.$$

Since without outside options the above bidding functions always imply efficient allocations in equilibrium, it is obvious that public outside options do not destroy this property.

2.3 Private outside options

Again, individual i's valuation is v_i and the value that she derives from executing her outside option is w_i. Valuation pairs $(v, w) \in [\underline{v}, \bar{v}] \times [\underline{w}, \bar{w}]$ are independently distributed across individuals according to the probability density function $f(v, w)$ and are private information. The assumption of jointly distributed valuations for a given individual allows for correlation between v and w: it might be sensible to assume that a higher v implies a higher w and lower v tend to occur together with lower w. Again we assume that the lowest valuation is not lower than the outside option, i.e. $\underline{v} \geq \bar{w}$ ensuring that each individual submits a bid in the auction, otherwise there are some types who prefer the outside option to the object auctioned off even if the auction price equals zero. Figure 1 illustrates the type space.

Next we derive and discuss equilibrium bidding strategies for the first-price auction. In subsection 2.3.3 we show that private outside options in the first-price bidding model with uniformly distributed valuations yield concave bidding functions. In subsection 2.3.4 we derive equilibrium bidding functions for the second-price auction and in 2.3.5 we describe the efficiency properties of the augmented model.

2.3.1 Equilibrium bidding: first-price auction

In order to derive the equilibrium bidding strategy in the first-price auction, we represent the bidding model in a way that allows its solution with standard procedures. For this, consider the utility maximization problem of the representative risk-neutral individual i that submits bid b_i in the auction and faces the outside option w_i:

$$\max_{b_i} \quad \Pr(b_i \text{ wins}) \cdot (v_i - b_i) + [1 - \Pr(b_i \text{ wins})] \cdot w_i$$

[2] The proof is identical to that for the equilibrium bidding strategy under the second-price auction with private outside options, see below.

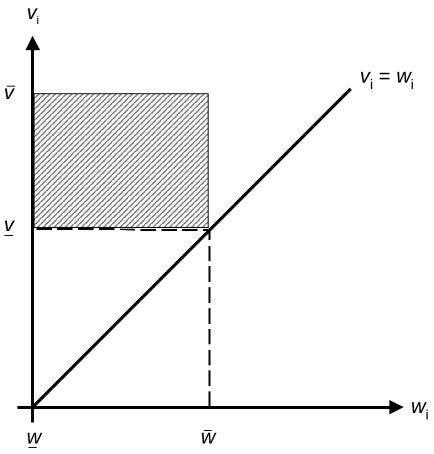


Figure 1: Type space (v, w)

This program can be rearranged to

$$\max_{b_i} \quad \Pr(b_i \text{ wins}) \cdot (v_i - w_i - b_i) + w_i$$

Since the outside option w_i is known to the individual and a constant, the argmax of that problem is the same as the one of the following problem where the new variable $x_i := v_i - w_i$ is introduced

$$\max_{b_i} \quad \Pr(b_i \text{ wins}) \cdot (x_i - b_i) \tag{3}$$

We interpret the $x \in [\underline{v} - \bar{w}, \, \bar{v} - \underline{w}]$ as an individual's net valuation of the object that is offered in the auction that takes into account the opportunity cost of winning the auction and foregoing the outside option. The logic of the transformation of the original maximization problem into (3) is to suppose that the representative individual executes her outside option w_i before bidding in the auction, then bids in the auction, and, since she has unit-demand, in case she wins the auction "repays" her outside option in addition to the price of the auctioned object.

By this simple transformation we have a standard bidding problem in net valuations x. All there remains to do is to identify the probability density function that governs the distribution process of net valuations. Note that a number of (actually infinitely many) valuation pairs (v, w) leads to the identical net valuation $\bar{x}$. Figure 2 depicts some iso-net-valuation-curves in two different type spaces to illustrate this. By definition, iso-net-valuation-curves are given in w-v-space since $v = \bar{x} + w$. Obviously "higher" iso-net-valuation-curves are associated with higher net valuations. The probability density function of a given net valuation x is obtained by "summing up" all densities over the corresponding iso-net-valuation curve as follows

$$f_X(x) = \int\limits_{\max\{\underline{w}, \underline{v} - x\}}^{\min\{\bar{w}, \bar{v} - x\}} f(x + w, w) \, dw.$$

Given a well-defined $f_X(x)$ with support $[\underline{v}-\bar{w}, \bar{v}-\underline{w}]$ and cumulative density function $F_X(x)$, we can invoke standard results to derive the equilibrium bidding function (e.g. Riley and Samuelson, 1981) assuming that there is no reserve price in the auction:

$$b^{\text{fp-pr}}(x) = x - \frac{\int_{\underline{v}-\bar{w}}^{x} F_X^{n-1}(y)\,dy}{F_X^{n-1}(x)}.$$

By resubstitution, the equilibrium bidding function for our model is:

$$b^{\text{fp-pr}}(v,w) = v - w - \frac{\int_{\underline{v}-\bar{w}}^{v-w} F_X^{n-1}(y)\,dy}{F_X^{n-1}(v-w)}$$

which is strictly increasing in v and strictly decreasing in w since $\partial b/\partial x > 0$ and $\partial x/\partial v = -\partial x/\partial w = 1$.

2.3.2 Example: $v \in U[50, 100]$, $w \in U[0, 50]$, $n = 2$

Suppose $(v, w) \in [50, 100] \times [0, 50]$ with $f(v, w) = 1/2,500$ and $n = 2$. It follows for net valuations that $x \in [0, 100]$. The cumulative density function of $X \equiv V - W$ is given by

$$F_X(x) = \begin{cases} \frac{x^2}{5{,}000} & \text{if } x \in [0, 50] \\ \frac{200x - x^2 - 5{,}000}{5{,}000} & \text{if } x \in [50, 100]. \end{cases}$$

As a result, the symmetric Bayes-Nash equilibrium bidding function is given by

$$b^{\text{fp-pr}}(x) = \begin{cases} \frac{2}{3}x & \text{if } x \in [0, 50] \\ \frac{300x^2 - 2x^3 - 250{,}000}{600x - 3x^2 - 15{,}000} & \text{if } x \in [50, 100]. \end{cases}$$

Please note that bids and slopes at the potential discontinuity at $x = 50$ coincide at "both sides". The bidding function depending on (v, w) is simply obtained by resubstitution:

$$b^{\text{fp-pr}}(v,w) = \begin{cases} \frac{2}{3}(v - w) & \text{if } (v - w) \in [0, 50] \\ \frac{300(v-w)^2 - 2(v-w)^3 - 250{,}000}{600(v-w) - 3(v-w)^2 - 15{,}000} & \text{if } (v - w) \in [50, 100]. \end{cases}$$

Figure 3 displays a corridor of equilibrium bidding functions $b^{\text{fp-pv}}(v, w)$ and the equilibrium bidding function that ignores outside options $b^{\text{fp}}(v) = 25 + v/2$.

2.3.3 Concavity of equilibrium bidding functions

The parameterization of the preceding example with uniformly valuations and outside options leads to concave equilibrium bidding functions. The equilibrium bidding function in net valuations x is shown in figure 4. It is linear over interval $[0, 50]$ and strictly concave over interval $(50, 100]$. Every bidding function $b^{\text{fp-pr}}(v, w)$ is strictly concave in valuation v over some range if $w < 50$. The function $b^{\text{fp-pr}}(v, w = 50)$ is linear in v. Indeed, any bidding function $b^{\text{fp-pr}}(v, w)$ that is in the corridor of the

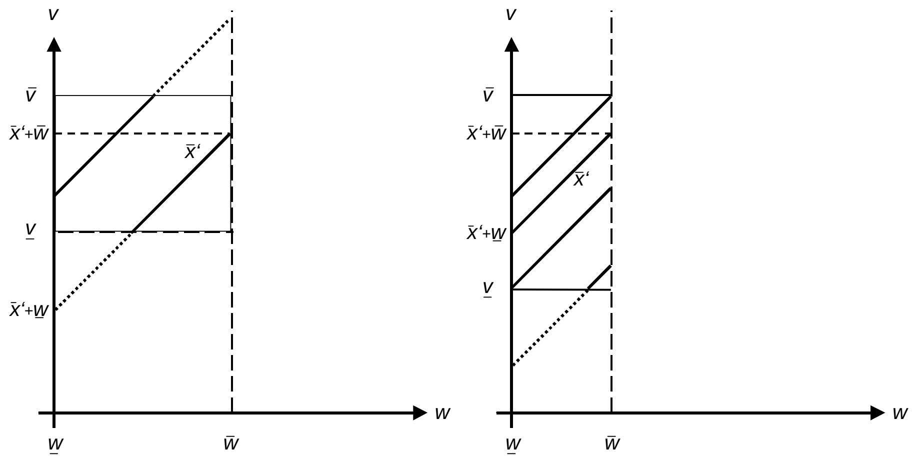


Figure 2: Iso-net-valuation-curves $v = \bar{x} + w$ in two type spaces.

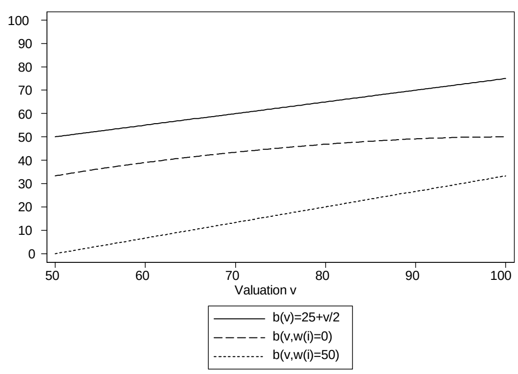


Figure 3: Equilibrium bidding functions

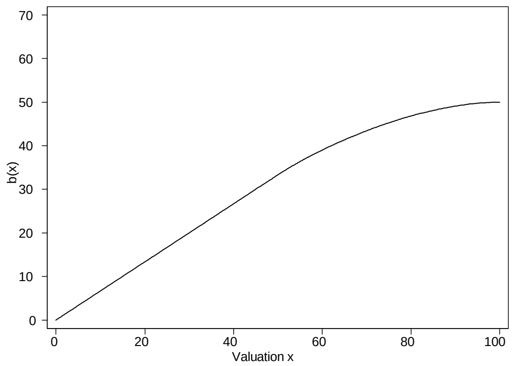


Figure 4: Equilibrium bidding function $b^{\text{fp-pr}}(x)$

preceding figure 3 for a given outside option value w and valuations $v \in [0, 50]$, is nested in figure 4 where its image corresponds to the image of net valuations over the subdomain $[50 - w, 100 - w]$. It can be seen from the figure that equilibrium bidding functions for given w become "more concave" in v as the value of the outside option decreases. For the largest outside option value $w = 50$, it is linear. As the outside option's value decreases, the bidding function becomes strictly concave over a larger portion of its domain. With the lowest outside option value $w = 0$, the bidding function is strictly concave over its full domain.

2.3.4 Equilibrium bidding: second-price auction

Any individual i prefers to win the auction if the value net of payment that she derives from receiving the object in the auction is not lower than the value she derives from executing her outside option. Thus, individual i prefers winning the second-price auction if

$$v_i - \hat{b}_{-i} \geq w_i$$

where the left-hand side is the return that she receives if her bid exceeds the largest competing bid $\hat{b}_{-i}$. Using the definition of net valuations $x := v - w$, this can be rewritten to

$$x_i \geq \hat{b}_{-i}.$$

Since winner i's payment for the auctioned object is always determined by the fiercest competitors' bidding behavior (denoted by $\hat{b}_{-i}$), utility maximization reduces to the selection of a bid that always wins the auction if the above inequality holds and never wins it if the inequality is violated. Clearly, bidding exactly the net valuation x_i is a weakly dominant strategy, the reasons are standard and detailed below.

There are two alternatives to bidding x_i: either bidding a larger or a smaller amount (overbidding or undercutting, respectively). Consider both bidding alternatives:

- Overbidding: if bidder i overbids, say he submits $x_i + \delta$, then there are three possibilities: (a) $\hat{b}_{-i} > x_i + \delta > x_i$: both, overbidding and bidding x_i, doesn't win the auction leading to the same outcome; (b) $x_i + \delta > x_i \geq \hat{b}_{-i}$: again, overbidding and bidding x_i lead to the same return, this time both strategies win the auction at the same price below x_i (or both generate a return of zero if $x_i = \hat{b}_{-i}$); (c) now consider $x_i + \delta \geq \hat{b}_{-i} > x_i$: here overbidding wins the object resulting in a loss ($\hat{b}_{-i} - x_i$) while bidding x_i prevents this. It follows that overbidding either leads to the same payoff (a and b) or to a lower payoff (c) than bidding x_i.

- Undercutting: if bidder i submits a lower bid than his net valuation, say $x_i - \delta$, then, again it can be made the argument that payoffs are either same under both strategies or undercutting performs worse. Undercutting precisely leads to lower payoffs if $x_i > \hat{b}_{-i} > x_i - \delta$: then the bidder doesn't win the auction though bidding x_i would have allowed him winning it with a profit at $\hat{b}_{-i}$.

Since overbidding and undercutting either result in the same or lower payoffs than bidding the true net valuation x_i, the latter must be weakly dominant.

2.3.5 Efficiency with private outside options

We know that in the case without outside options and also in the case with public outside options, always efficient allocations are implemented in equilibrium. Here we show that efficiency is also obtained with private outside options. If there is the possibility of an inefficient allocation then there must be any unsuccessful bidder who can engage in a trade with the successful bidder such that the successful bidder is compensated for giving up the object he won in the auction for her outside option and the unsuccessful bidder is better off with the object he didn't win in the auction. Suppose that such a trade is feasible, then the lowest amount the winner of the auction must receive in order to be compensated for resorting to his outside option is $p := v_s - w_s$. The amount that the unsuccessful bidder gets if he receives the auctioned object from the auction winner instead of executing his outside option is $q := v_u - w_u$. In an inefficient allocation, the unsuccessful bidder's gain must exceed his payment to the auction winner, i.e. $q > p$. By the definition of net valuations this is equivalent to $x_u > x_s$ which implies via $b'(x) > 0$ that $b_u > b_s$. This, of course, contradicts the assumption that the unsuccessful trader lost the auction. Therefore, first-price and second-price auctions where outside options are taken care of appropriately result in Pareto-efficient allocations since these award the auctioned object to the bidder with the largest net valuation.

3 Experimental design and procedures

To test the theoretical implications of public and private outside options in the SIPV model, we devised three treatments that were implemented in a between-subjects design. In the baseline treatment A we ran standard auction games without outside options under the independent private value assumption with two bidders where the highest bid wins. Bids and valuations were denominated in experimental currency units (ECU). Each experiment session had twelve auction rounds in the strangers-matching design. In each auction round, we used the strategy method to elicit a subject's continuous bidding function by asking for bids that correspond to the six hypothetical valuations 50,60,70,80,90,100. Bids for intermediate valuations were determined by linear interpolation between the discrete bids. The particular bidding function that a subject specified by entering a set of bids was graphically displayed at all times.[3] Subjects were free to adjust their set of entered bids and thereby their specified bid functions as often as they wished. Bids had to be nonnegative and were entered via keyboard. Recently, the collection of entire bidding schedules in each round of auction experiments gained popularity but the particular implementation varies between studies. Selten and Buchta (1999) introduced the strategy method for auction experiments. In their experiments, subjects could specify a piecewise linear bidding functions with up to 10,000 segments either using a graphical input mode or via keyboard. Their implementation suffered from the problem that subjects might have been drawing "interesting landscapes" since 46% of observed bidding functions were nonmonotonic (p. 81). Pezanis-Christou and Sadrieh (2004) used a simplified version of the Selten and Buchta (1999) implementation where subjects could specify two piecewise segments to allow for a single kink in the bidding schedule. In their study, approximately 15% of bidding functions are nonmonotonic in their asymmetric auction treatments and approximately 5% in their symmetric auction treatments.[4] In the experimental studies of Güth et al. (2002, 2003) the set of possible valuations was restricted to eleven values. For each of the eleven valuations, subjects had to enter a corresponding bid. Unfortunately, these two studies do not report the share of observed bidding schedules that are nonmonotonic. A characteristic common to all implementations including ours is that subjects were required to submit their bidding schedules before their valuations were drawn unlike in earlier studies.

A distinctive feature of our design is that pairs of matched subjects participated in five unrelated auctions after specification of their bidding schedules instead of one single auction. For each of these five auctions and for each of the subjects, a valuation was independently drawn from a uniform distribution with support [50,100] and rounded to two decimal places. In each of the five auctions, subjects' bids were determined according to their specified bidding functions and their valuations. In each auction, the high bidder received the difference between valuation and bid. In our baseline treatment A, the other bidder received nothing. Subjects were informed about their five valuations, their submitted bids, the high bidder, the high-bid, and their income in all five auctions; they were also informed about their income in this round which was the sum of the five auction incomes. No information about competitor's valuations, incomes, and losing bids was revealed. A list of experiments is given in table 1.

[3] Screenshots are provided in the appendix.

[4] The percentages are inferred from clearly visible bar charts, figures 3 and 5.

Table 1: List of experiment sessions

Treatment	Session	Number of Subjects	Type of outside option	Number of matching groups	ECU/Euro
A	1	14	none	1	25
A	2	14	none	1	25
A	3	18	none	2	35
B	4	18	public	2	120
B	5	16	public	2	120
B	6	18	public	2	120
C	7	16	private	2	150
C	8	18	private	2	150
C	9	20	private	2	150
C	10	18	private	2	150

In treatment, B and C, the procedures of treatment A were modified to allow for an outside option that was implemented as a fixed income for the low bidder. The values of the outside options were drawn from a uniform distribution with support [0,50] for four auction rounds, rounded to two decimal places and announced to each individual bidder before subjects specified their bidding functions. Treatments B and C differed in the amount of information that subjects had about their competitors' outside options. In treatment B, the outside option was public in the sense that both bidders would have received the same fixed income if they had happened to be the low bidder. In treatment C, outside options were private information and it was common knowledge that outside options were independently and uniformly distributed over the interval [0,50].

At the beginning of each experiment session, subjects took a brief treatment-specific computerized quiz to ensure their familiarity with the instructions and the experiment they were about to participate in. At the end of the last auction round, subjects went through a brief computerized questionnaire. Upon completion, they received their earned income converted into Euros in cash. Subjects that participated in treatment A received in addition a show-up fee of 3 Euro at the end of the experiment. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 1999).

Equilibrium predictions Table 2 presents the equilibrium bidding strategies derived from our augmented auction model for our specific treatments (obviously treatment A is a special case of treatment B without outside option).

In figure 5, the equilibrium bidding function for the baseline treatment, $b^{\mathrm{A}}(v)$ is drawn as well as a corridor of bidding functions for treatment C that is characterized by its lower bound $b^{\mathrm{C}}(x = v - 50)$ and its upper bound $b^{\mathrm{C}}(x = v - 0)$; any bidding function for treatment B, $b^{\mathrm{B}}(v, w)$ is a parallel line to

Table 2: Equilibrium bidding predictions

Treatment	RNNE equilibrium prediction	Bid range
A	$b^{\text{A}}(v) = 25 + \frac{v}{2}$	$b^{\text{A}} \in [50, 75]$
B	$b^{\text{B}}(v, w) = 25 + \frac{v}{2} - w$	$b^{\text{B}} \in [0, 75]$
C	$b^{\text{C}}(x) = \begin{cases} \frac{2}{3}x & \text{if } x \in [0, 50] \\ \frac{100x^2 - 2/3x^3 - 250{,}000/3}{200x - x^2 - 5{,}000} & \text{if } x \in [50, 100] \end{cases}$ where $x \equiv v - w$	$b^{\text{C}} \in [0, 50]$

$b^{\text{A}}(v)$ where the "vertical distance" is equal to the outside option's value w.

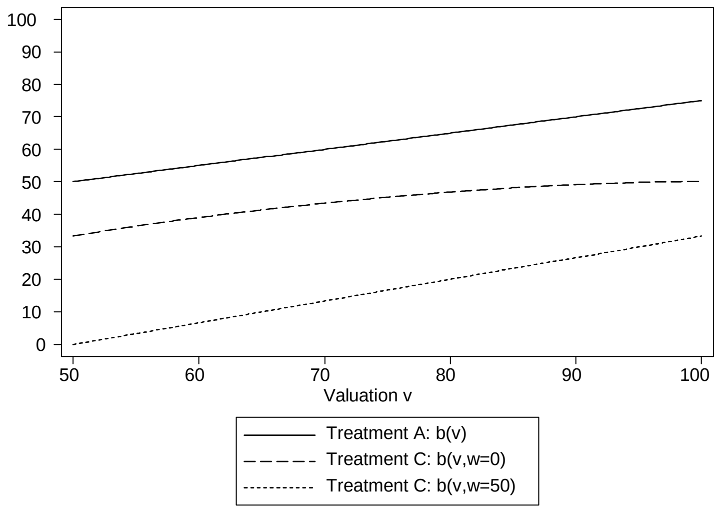


Figure 5: Bidding predictions for Treatments A, B, and C

4 Experimental results

The plan of this section is to provide a brief description of the data that we obtained, to discuss the monotonicity of observed bidding functions and then to discuss the effects of outside options and efficiency. In the remainder of this section, we investigate if observed bidding functions are concave in treatment C as predicted.

4.1 Monotonicity of collected bidding functions

A key feature of our experimental design is that subjects specified an entire bidding function in each auction round. Across all treatments and rounds, 2,040 bidding functions were specified by 170 subjects. From all of these bidding functions, 1,804 are monotonic increasing in valuation v (88.4%), 44 are constant (2.2%), five are decreasing (0.3%), and 186 are nonmonotonic with either peaks or troughs (9.1%). Table 3 provides a breakdown by treatment.

Table 3: Monotonicity of observed bidding functions

Monotonicity of bidding functions	treatment A (no outs. opt.)	treatment B (public outs. opt.)	treatment C (private outs. opt.)	sum
strictly decreasing			1 (0.1%)	1 (0.05%)
weakly decreasing		1 (0.1%)	4 (0.5%)	5 (0.25%)
constant	2 (0.4%)	19 (3.1%)	23 (2.7%)	44 (2.16%)
weakly increasing	378 (68.5%)	410 (65.7%)	543 (62.9%)	1,331 (65.25%)
strictly increasing	123 (22.3%)	139 (22.3%)	211 (24.4%)	473 (23.19%)
nonmonotonic	49 (8.9%)	55 (8.8%)	82 (10.0%)	186 (9.12%)
sum	552	624	864	2,040

In contrast to the experimental data reported in Selten and Buchta (1999) where 46% of the collected bidding functions were nonmonotonic, the phenomenon of drawing interesting landscapes with bidding functions seems not to be a problem with our design since only a minority of bidding functions is nonmonotonic (9.1%) and allows for those landscapes.[5]

4.2 Raw bidding data

Before we begin our analysis of the effects of outside options on bidding behavior, it is worthwhile to take a brief look at the bidding data that we obtained. In all treatments, individuals were asked to specify a bidding function given their outside option in every round (in treatment A, the value of the outside options equals zero). In order to represent the bidding functions that participants in our experiments have submitted as stepwise linear functions, we will in the next few paragraphs concentrate on bids. For each bidding function that was submitted we will randomly draw five valuations to represent pairs of bids and valuations. We take the same five randomly drawn valuations that we also used in the experiment to determine the payoff and to give feedback to participants.

4.2.1 Treatment A - no outside options

Figure 6 illustrates the bidding data that we obtained in the baseline treatment A where there was no outside option available to any of the bidders. If bidders have no outside option then valuations of the object that is offered in the auction coincide with net valuations, i.e. $x_i = v_i$ since $w_i = 0$. The lower line in the scatterplot indicates the RNNE-bidding prediction and the upper indicates the naive bidding strategy "bid your valuation". Each dot represents one of the five bids that were calculated according

[5] Some observed bidding functions are provided in the appendix.

to the submitted bidding function and for the five randomly drawn valuations in each round a single bid that was submitted. Since 46 individuals participated in treatment A and 5 valuations were drawn in each of 12 rounds, there are 2,760 data points.[6] The dashed line is a median spline fitted to the bid data. Indeed, we observe bids that are higher than the RNNE prediction, but only for large individual

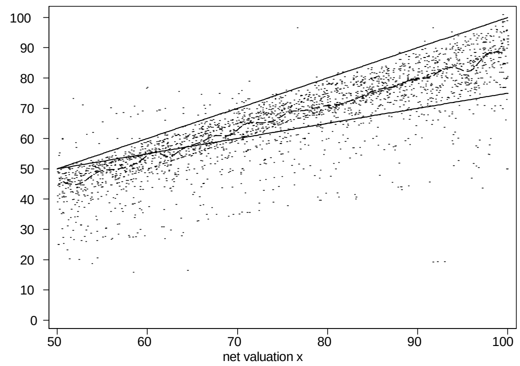


Figure 6: Bidding data for treatment A

valuations. For small individual valuations, bids appear smaller than the RNNE predicts. Why is this so? One difference between the studies by Cox et al. (1982), Cox et al. (1988), Dorsey and Razzolini (2003), Isaac and Walker (1985), Ockenfels and Selten (2002) and ours is that in their studies the lowest valuation is zero or very close to zero. This is not the case in our experiment. We are aware of two other studies where the lowest valuation is significantly larger than zero (Güth et. al. 2003; Chen and Plott 1998). In the first study, the authors report some underbidding with low valuations and overbidding with high valuations, unfortunately statistical tests are not provided. In Chen and Plott (1998), individual-wise estimated intercepts of linear bidding functions are reported to not significantly differ from the RNNE/CRRA prediction for 86% of the subjects (p. 66). Unfortunately, this number includes 50% of subjects that was faced with a lowest possible valuation equal to zero (the other 50% faced a lowest possible valuation equaling 500.) Since negative bids were either excluded (not reported in the paper) or very unlikely to be observed in that experiment (since the provided instructions are silent on this possibility), it is not obvious that there was no underbidding observed. In order to address this issue, we have conducted further experiments that significantly point to the fact that underbidding occurs if it is not precluded by experimental design, see Kirchkamp and Reiß (2004).

[6]The figure depicts only 2,752 data points due to the scale choice for the vertical axis. In the first round, one subject specified a bidding schedule comprising six data points where the lowest boundary equals 110.06 resulting in five invisible data points. Also in the first round, another subject submitted the bids 115.07 and 120.49 for the valuations 93.38 and 96.99, respectively. Another subject once bid 153.04 for valuation 99.72.

4.2.2 Treatment B - public outside options

In treatment B, every individual faced an outside option that coincided with the outside option of his competitor which was commonly known. The next figure illustrates how observed bids relate to the theoretical bid prediction that underlies the corresponding (valuation, outside option)-pair. Since public outside options vary and the RNNE bidding function doesn't depend solely on net valuations, we transform the bidding data by adding $w/2$ to each observed bid such that the data is readily comparable to the transformed RNNE bidding function $b^{\mathrm{B}}(v,w)+w/2\ (=25+x/2)$ that only depends on net valuations. Since there are 52 individuals who participated in this treatment, figure 7 plots 3,120 data points.

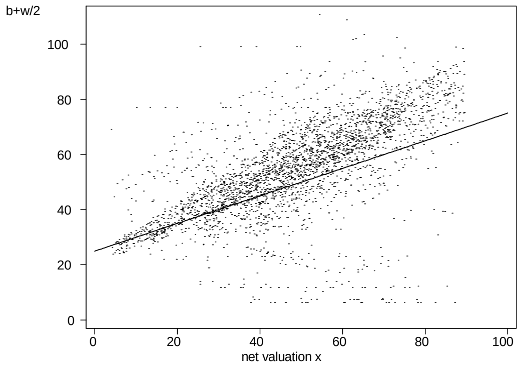


Figure 7: Bidding data for treatment B (all bids)

4.2.3 Treatment C - private outside options

Figure 8 illustrates data for treatment C where every bidder's outside option was private information such that the competitor's outside option were unknown. The lower line in the figure corresponds to the RNNE-equilibrium function, the upper line is 45-degree-line. Please note that the data points that lie beyond the 45-degree line do not necessarily indicate that there was bidding-above valuation since the horizontal axis gives net valuations, i.e. valuations minus the value of the outside option. Again, each dot represents a single bid. Since 72 individuals participated in treatment C over 12 rounds, there are 4,320 data points. Although there is some underbidding, overbidding the RNNE-prediction seems to be the predominant behavioral pattern in both figures. (However, the lowest boundary of net valuations is zero.)

4.3 Effect of outside options on bidding

4.3.1 Public outside options

In each of the three experimental sessions for treatment B, subjects faced three different randomly drawn values as their public outside options. Including treatment A with $w = 0$, we obtained bidding data

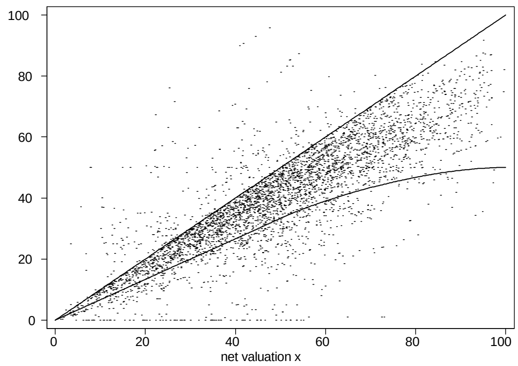


Figure 8: Bidding data for treatment C (all bids)

for $w \in \{0.00, 10.51, 12.79, 14.59, 23.49, 25.82, 25.96, 37.28, 44.24, 46.07\}$. The following figure clearly shows that observed bidding behavior is affected by the value of public outside options. In the left panel, median splines for bidding data conditional on the outside option's value w are plotted. Median splines for treatment B are constructed from 1,000, 1,040 and 1,080 data points, respectively. The median spline for treatment A is constructed from 2,760 data points. As the value of the outside option increases, e.g. from zero to levels between 10 and 15 ECU, median bids pronouncedly decrease. The right hand panel illustrates the theoretical equilibrium predictions. The prediction for a given valuation v corresponds to a range of bids since similar outside options are grouped in the same way as in the left panel. In the figure, bid prediction ranges are indicated by the shaded areas. It is easy to see that the qualitative comparative static effect found in the data is in line with the behavior of equilibrium bidding functions. Intuitively, as the opportunity cost of bidding increases (=value of foregoing the outside option), winning the auction becomes less important and thus leads to less aggressive bidding. This implies that the quality of the equilibrium prediction for first-price bidding data that ignores outside options deteriorates as public outside options gain importance.

From Figure 9 it appears that median splines are "steeper" than the corresponding equilibrium predictions. However, it is difficult to see if the predicted comparative static effects due changes in w are quantitatively reproduced in the laboratory. In order to resolve both issues, we estimate linear bidding functions for treatments A and B separately and pooled. Calculations of standard errors take into account that observations might be correlated within matching groups but not across matching groups; Rogers, 1993). The regression results are summarized in table 4. The estimated coefficient of the public outside option value w suggests that it has a strong negative impact on bidding behavior. However, the negative impact of public outside options on bidding data observed in the data, $\hat{\beta}_w = -0.77$, is significantly weaker than theoretically predicted, $\beta_w = -1$ ($t_5 = 9.1647$, $p = 0.000$). The slopes of all equilibrium bidding functions are equal to $1/2$. Though estimated slopes are positive, they significantly exceed 1/2 ($t_5 = 105.27$, $p = 0.000$).

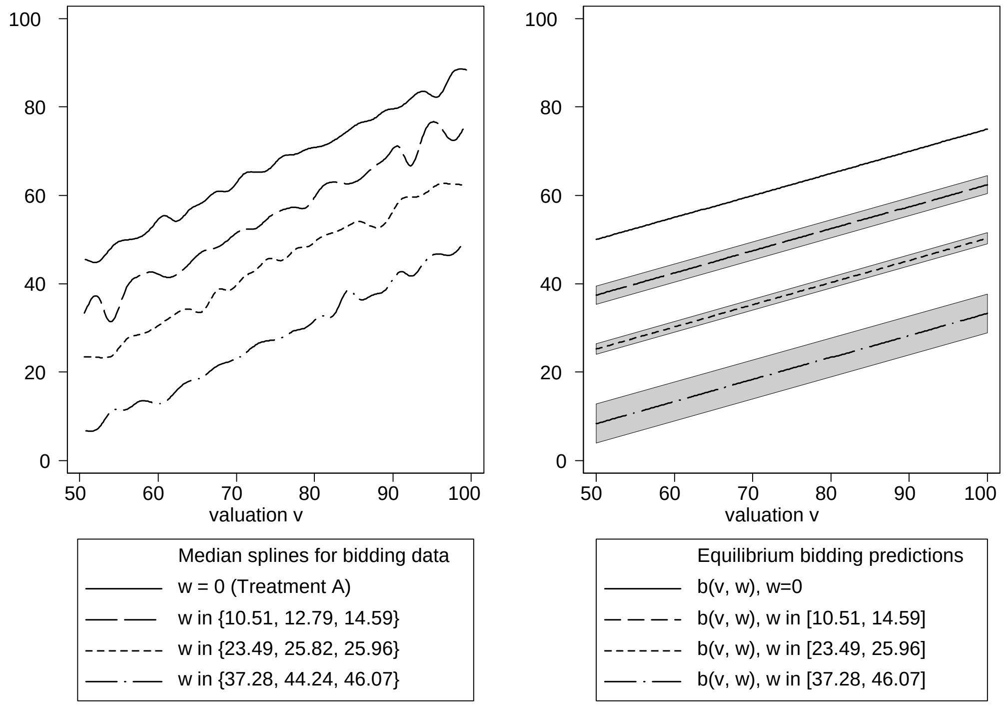


Figure 9: Bidding behavioral effects of public outside options

The result that individuals do not fully expropriate their outside option by bidding too high is interesting in light of the debate on the declining price anomaly (see Ashenfelter and Graddy, 2003, and the references therein). If this expropriation failure arises with endogenous outside options, too, then one might expect a series of falling prices as observed in the field.

4.3.2 Private outside options

In order to get a first impression if there is any effect of the private outside option on bidding behavior, the bidding data is classified into five groups according to the value of the outside option. The five ranges for outside options are (0,10), (10,20), (20,30), (30,40), and (40,50). In the left panel of figure 10, median bids conditional on the outside option value range and over individuals and rounds are plotted for each of the valuations in {50, 60, 70, 80, 90, 100} for which subjects keyed in their bids. Median bids are constructed from 152-204 entered bids, the precise number depends on the particular outside option range. One pattern in the data deserves particular attention: bids tend to decrease in the value of the outside option as in the case of public outside options. To compare the bidding data to the theoretical benchmark, the corresponding equilibrium predictions are provided in the right panel of the figure. Since we consider median bids for small ranges of outside options, the theoretical benchmark is a range of bidding functions. In the figure, the equilibrium bidding function for the expected value of the outside option

Table 4: Regression results

Treatment	expl. var.	coeff. $\hat{\beta}_x$	robust σ_{β_x}	t	$P > \lvert t \rvert$	95% conf.	interval
A	constant	0.2770354	2.494552	0.11	0.919	-7.661742	8.215813
	v	0.8626416	0.0248474	34.72	0.000	0.783566	0.9417172
B	constant	1.8715710	1.1418200	1.64	0.162	-1.06357	4.806713
	v	0.7964621	0.0123155	64.67	0.000	0.7648041	0.82812
	w	-0.7668116	0.0254442	-30.14	0.000	-0.8322181	-0.7014051
A+B	constant	2.3330420	1.532425	1.52	0.162	-1.133545	5.7996280
	v	0.8275259	0.0179942	45.99	0.000	0.7868202	0.8682317
	w	-0.8527819	0.0253570	-33.63	0.000	-0.9101434	-0.7945203

conditional on the particular outside option range is drawn as a patterned line. It is embedded in an area of equal shading that contains all equilibrium bidding functions given the particular outside option range. By construction, every shaded area represents a different theoretical benchmark that corresponds to the observed median bids. It can be seen that the strong outside option pattern in the data is qualitatively predicted by theory. Median bidding functions appear to lie above their theoretical counterparts.

Table 5 reports the results from a linear regression that explains observed bids with the equilibrium bidding prediction and the value of outside options.[7] If subjects were to bid according to the theoretical benchmark, the coefficient of the equilibrium prediction would equal unity. For treatment C, $\hat{\beta}_{b^{\mathrm{C}}(x_i)}$ significantly exceeds unity ($t = 5.446$, $p = 0.001$, two-tailed). From this it cannot be directly concluded that there is overbidding on average since other coefficients are significant, too. In particular, the estimate for the constant is negative as is the coefficient on the interaction term. However, it is straightforward to show that there is no valuation pair $(v, w_i) \in [50, 100] \times [0, 50]$ that leads to a bid predicted from the estimated regression model lower than the corresponding theoretical equilibrium prediction.[8]

Theory also predicts that only net valuations, i.e. the difference between object valuation and outside option value, matter for bidding and not their individual levels. If subjects were to bid according to this

[7] Again, we account for the fact that observations are independent between but not within matching groups.

[8] The bidding prediction from the estimated model is $\hat{b}^C = -12.99646 + 1.610504 b^{\mathrm{C}}(x_i) + 0.3347499 w_i - 0.0085359 b^{\mathrm{C}}(x_i) w_i$. There is global overbidding if for all $x_i \equiv v_i - w_i$ the inequality $\hat{b}^{\mathrm{C}} \geq b^{\mathrm{C}}(x_i)$ is not violated, equivalently

$$0.610504 b^{\mathrm{C}}(x_i) + 0.3347499 w_i - 0.0085359 b^{\mathrm{C}}(x_i) w_i \geq 12.99646.$$

Since $b^{\mathrm{C}}(x_i)$ is monotonic increasing, the lowest bid for $x_i \geq 50$ is $b^{\mathrm{C}}(50) = 100/3$. Using this value with the above inequality leads to $7.3537 + 0.05022 w_i \geq 0$.This holds due to nonnegative outside options. It remains to check the above inequality for $x_i < 50$. Substitution of $b^{\mathrm{C}}(x_i < 50) = 2/3(v_i - w_i)$ yields

$$v_i \geq \frac{12.99646 + 0.072\,25 w_i - 0.0056906 w_i^2}{0.407 - 0.0056906 w_i}.$$

The maximum of the RHS is 42.633 (at $w_i = 27.665$) while the lowest valuation v_i is 50. Thus, the inequality is never violated confirming global overbidding.

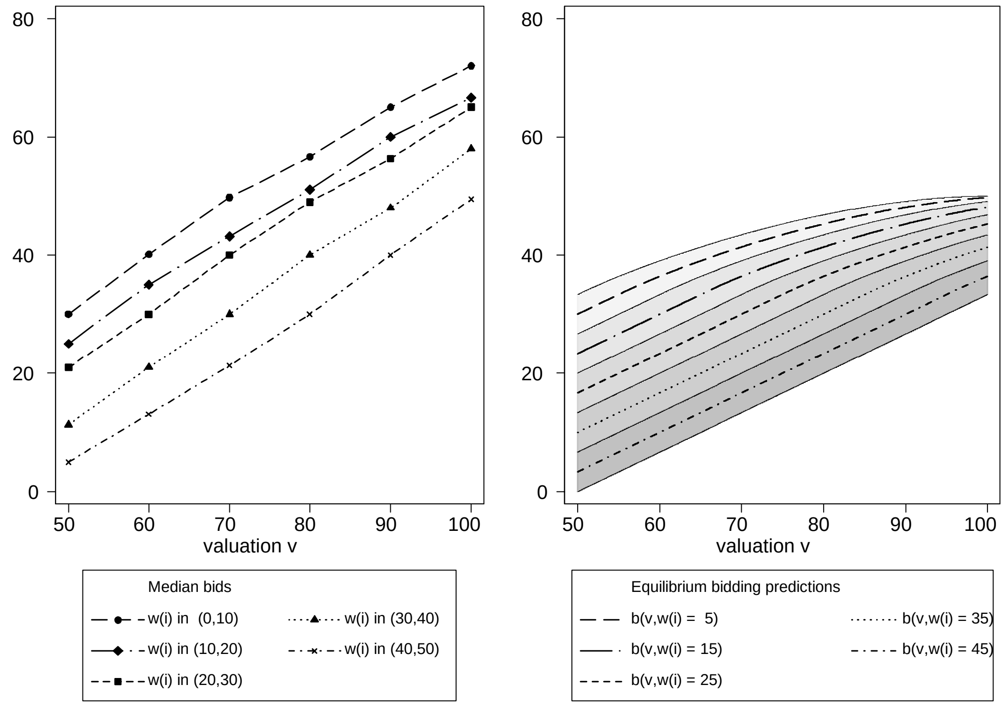


Figure 10: Bidding behavioral effects of private outside options

prediction, then the coefficient of the outside option would equal zero. The estimation results show that the coefficient of the private outside option's value, $\hat{\beta}_{w_i}$, significantly exceeds zero.

Table 5: Regression results: Deviations from Nash bids

Treatment	expl. var.	coeff. $\hat{\beta}_x$	robust σ_{β_x}	t	$P > \|t\|$	95% conf.	interval
C	constant	-12.99646	4.655477	-2.79	0.027	-24.00491	-1.988004
	$b^{\mathrm{C}}(x_i)$	1.610504	0.1120983	14.37	0.000	1.34544	1.875567
	w_i	0.3347499	0.110117	3.04	0.019	0.0743646	0.5951352
	$b^{\mathrm{C}}(x_i) \cdot w_i$	-0.0085359	0.0022247	-3.84	0.006	-0.0137965	-0.0032753

4.3.3 Effects of outside option type

We have reported that both types of outside options, private and public, decisively affect bidding behavior of subjects. The difference between the two types lies in their associated common knowledge sets. With public outside options, each subject knows that any competing bidder is faced with the same outside option value. Whereas under the private outside option regime, each subject is uncertain about it and only knows the underlying distribution. Here, we investigate if the particular type of outside options affects observed bidding behavior. For this we estimate the regression model[9]

$$\begin{aligned} b_j &= \beta_{oB} \cdot B + \beta_{1B} \cdot B \cdot v_j + \beta_{2B} \cdot B \cdot w_j \\ &\quad + \beta_{oC} \cdot C + \beta_{1C} \cdot C \cdot v_j + \beta_{2C} \cdot C \cdot w_j + \varepsilon_j \end{aligned}$$

where B and C are treatment-specific dummy variables with pooled data from treatments B and C. The specification follows from equilibrium bidding in treatment B. The results are reported in table 6. Under the hypothesis that the type of outside options does not influence bidding behavior, the estimated coefficients for treatment B should be no different from the estimates for treatment C. Testing the hypothesis

Table 6: Regression results: Effects of outside option types

Treatment	expl. var.	coeff. $\hat{\beta}_x$	robust $\hat{\sigma}_{\beta_x}$	t	$P > \|t\|$	95% conf.	interval
B+C	B	1.871571	1.081695	1.73	0.107	-0.4652893	4.208432
	$B \cdot v_j$	0.7964621	0.011667	68.27	0.000	0.7712571	0.8216671
	$B \cdot w_j$	-0.7668116	0.0241044	-31.81	0.000	-0.8188861	-0.7147372
	C	-0.9549387	2.989683	-0.32	0.754	-7.413756	5.503879
	$C \cdot v_j$	0.7820828	0.0374816	20.87	0.000	0.7011088	0.8630568
	$C \cdot w_j$	-0.6941989	0.023007	-30.17	0.000	-0.7439024	-0.6444953

that all estimated coefficients for treatment B coincide with their counterparts for treatment C can be significantly rejected ($F_{3,13} = 4.70$, $p = 0.0196$). Pairwise comparisons of the estimates indicate that only the coefficients of the outside options significantly differ from one another ($t = 2.179$, $p = 0.0483$, two-tailed). It appears that the subtle difference in common knowledge stemming from different types of outside options affects bidding behavior. In particular, more valuable outside options lead to stronger relaxations of aggressive bidding under public outside options than under private outside options as theoretically expected.[10] Therefore, the details of common knowledge may matter more than suggested elsewhere.[11]

4.4 Efficiency and outside options

In this section, we investigate if outside options influence the efficiency generated by first-price sealed-bid auctions. An allocation is Pareto-efficient if the bidder with the highest net valuation, $x \equiv v - w$, is awarded the object in the auction and the loser seizes the outside option (in the baseline treatment, the value of the outside option equals zero such that $v = w$). One measure of efficiency is the relative

[9] Again, we take into account that observations are independent between but not within matching groups.

[10] Notice that $\partial b^{\mathrm{B}}/\partial w = -1$ and $\partial b^{\mathrm{C}}/\partial w_i \leq -2/3$.

[11] Güth and Ivanova-Stenzel (2003) report that the manipulation of common knowledge in asymmetric private value auctions (competitor's valuation distribution is know/unknown) "changes behavior only slightly and hardly ever in significant ways." (p. 198f.)

frequency of Pareto-efficient allocations. As table 7 suggests, there do not seem to be large differences between the treatments, though the numbers for treatment B are somewhat lower and relative frequencies for the first six rounds are slightly greater such that efficiency may slightly increase over time.

Table 7: Relative frequencies of Pareto efficient allocations

Treatment	rounds 1-6	rounds 7-12	rounds 1-12
A no outside options	82.5%	86.4%	84.4%
B public outside options	78.5%	83.2%	80.9%
C private outside options	82.4%	85.5%	83.9%

To test for treatment differences and time effects, let $E_{i,r}$ denote the relative frequency of Pareto-efficient allocations with the participation of subject i in round r. Let A, B, and C be treatment dummies and L_A, L_B, and L_C be treatment-specific dummies that indicate if the observation was generated in rounds 1-6 (L=0) or in rounds 7-12 (L=1). The estimation results for the regression model $E = \beta_0 A + \beta_1 B + \beta_2 C + \beta_3 L_A + \beta_4 L_B + \beta_5 L_C$ are summarized in table 8.[12] In all treatments, allocations significantly tend to be more often Pareto-efficient if realized in the second half of the experiments ($p < 0.075$, two-tailed t-tests). However, pairwise tests for differences between coefficient estimates for the treatment dummies A, B, and C reveal no significant differences between treatments A and C and B and C ($p > 0.124$, two-tailed t-tests) in the level of efficiency; however, there is a significant difference between treatments A and B ($p = 0.0989$, two-tailed t-test).

Table 8: Regression results: Efficiency by treatment

expl. var.	coeff. $\hat{\beta}_x$	robust $\hat{\sigma}_{\beta_x}$	t	$P > \|t\|$	95% conf.	interval
A	0.8248175	0.0142926	57.71	0.000	0.7946627	0.8549723
B	0.7851133	0.0176889	44.38	0.000	0.7477930	0.8224335
C	0.8238318	0.0160860	51.21	0.000	0.7898933	0.8577702
L_A	0.0389506	0.0199635	1.95	0.068	-0.0031687	0.0810699
L_B	0.0469380	0.0247687	1.90	0.075	-0.0053194	0.0991954
L_C	0.0307979	0.0082316	3.74	0.002	0.0134307	0.0481650

A disadvantage of the relative frequency as an efficiency measure is that it treats every inefficient allocation in the same way, although it seems reasonable to discriminate between "large" inefficiencies (where the winner of the auction has a much lower net valuation) and "small" inefficiencies (where both bidders have similar net valuations.) One approach to discriminate between "small" and "large" inefficiencies is to compare the outcome of an auction to that of some other allocation mechanism, in particular a random allocation (see Kirchkamp and Moldovanu, 2004). Recall that in our experiment subjects were

[12]The method of estimation accounted for possibly correlated observations within matching groups and used independence between matching groups.

matched pairwise in each round. In each round every subject specified a bidding schedule that allowed her to bid against her matching partner in five unrelated auctions. For each of the five auctions in a round, subjects received a randomly drawn valuation that implied their bids in that auction via the specified bidding schedule. Let $x^{\text{max}} = \sum_a \max\{x_{1,a}, x_{2,a}\}$ where $x_{i,a}$ denotes the net valuation of bidder $i = 1, 2$ in a matching group for auction $a = 1$,...,5 in any round. Let $x^{\text{rand}} = \sum_a (x_{1,a} + x_{2,a})/2$, then the additional surplus that auctions theoretically generate compared to random allocations for any matched bidder pair is $x^{\text{max}} - x^{\text{rand}}$. Let $x^{\text{w}} = \sum_a x_a^{\text{w}}$ where x_a^{w} is the net valuation of the actual winner of auction a and define the auction performance index as $(x^{\text{w}} - x^{\text{rand}})/(x^{\text{max}} - x^{\text{rand}})$ multiplied by 100 that measures the additional surplus generated from auctions compared to random allocations as a fraction of the largest additional surplus that can be obtained from any allocation mechanism. The performance index equals 100 if each auction outcome is Pareto-efficient meaning that auctions realized the maximum surplus that can be generated in addition to expected random allocation surplus. If there is at least one inefficient outcome, the index is lower than 100. The less similar are net valuations underlying an inefficient outcome and the fewer efficient outcomes, the larger the efficiency loss and the lower the performance index. If the outcomes of all five auctions are inefficient, the index equals -100. Table 9 displays the performance indices averaged over matching groups and rounds by treatment.

Table 9: Mean Performance Indices

Treatment	rounds 1-6	rounds 7-12	rounds 1-12
A no outside options	81.83	86.98	84.41
B public outside options	71.46	77.12	74.31
C private outside options	76.93	83.14	80.05

To test for treatment differences, let $P_{i,r}$ denote the performance index of the auctions in which subject i participated in round r. Replacing the dependent variable of the above given regression model with the performance index leads to results that are contained in table 8.[13] With the auction performance index, the time dummies for treatments A and B becomes insignificant whereas that for treatment A differs significantly from zero. Moreover, there is a significant difference between the coefficients of treatment dummies A and B ($p = 0.0241$, two-tailed t-test). There are no significant difference between the dummy coefficient estimates for A and C and B and C ($p > 0.240$, two-tailed t-tests).

Taken together, there appears to be evidence that there the degree of efficiency is higher in treatment A than in treatment B and that seem to be no significant efficiency differences with regard to the type, public or private, of the outside option. Though the share of Pareto-efficient allocations suggest at first that there are time effects, these happen to be insignificant with the Performance index.

Another popular approach to operationalize discrimination between "large" and "small" inefficiencies is to

[13]Again, the method of estimation accounted for possibly correlated observations within matching groups and used independence between matching groups.

Table 10: Regression results: Mean Efficiency Indices by Treatment

expl. var.	coeff. $\hat{\beta}_x$	robust $\hat{\sigma}_{\beta_x}$	t	$P > \|t\|$	95% conf.	interval
A	0.8183129	0.0286358	28.58	0.000	0.7578966	0.8787291
B	0.7146016	0.0305467	23.39	0.000	0.6501536	0.7790496
C	0.7693319	0.0330471	23.28	0.000	0.6996086	0.8390551
L_A	0.0514690	0.0404038	1.27	0.220	-0.0337756	0.1367136
L_B	0.0566345	0.0461333	1.23	0.236	-0.0406983	0.1539673
L_C	0.0620467	0.0197158	3.15	0.006	0.0204500	0.1036435

define the efficiency index as the ratio of the winner's net valuation to the largest net valuation of the two bidders multiplied by 100 (see e.g. Cox et al., 1982). The efficiency rate is 100 if the allocation is Pareto-efficient. If it is lower than 100, the bidder with the highest net valuation has not won the auction. The lower it is, the larger the gap between the net valuations of the bidders. Mean efficiency rates for treatment A: 98.66, B: 96.53, C: 96.51.

4.5 Concavity of bidding functions

An interesting feature of equilibrium bidding functions with private outside options is their concavity in valuations v with uniform distributions that intensifies as the outside option's value decreases (see section 2.2.4. on page 6). One possibility to capture this property is to define a concavity measure that relates bid differences corresponding to high valuations to those corresponding to low valuations, particularly

$$\begin{aligned} K(w_i) \equiv & \left[b^{\mathrm{C}}(60, w_i) - b^{\mathrm{C}}(50, w_i)\right] - \left[b^{\mathrm{C}}(100, w_i) - b^{\mathrm{C}}(90, w_i)\right] \\ & +\delta \left\{ \left[b^{\mathrm{C}}(70, w_i) - b^{\mathrm{C}}(60, w_i)\right] - \left[b^{\mathrm{C}}(90, w_i) - b^{\mathrm{C}}(80, w_i)\right] \right\} \end{aligned}$$

where $0 \leq \delta \leq 1$. The choice of this particular measure is guided by the fact that subjects entered their bids for valuations 50,60,...,100. For a strictly concave bidding function, the concavity measure is positive, for a linear function it is zero. The first line aims at capturing the strength of global concavity assuming the curve to be nonconvex between valuations 60 and 90. The second line revises the concavity measure downward with weight δ if there is some convexity of the bidding schedule between valuations 60 and 90, concavity over this interval revises $K(w_i)$ upward. For the following discussion, we set $\delta = 0.5$. All conclusions drawn below do not depend on the particular choice of δ.

Given the particular equilibrium bidding function $b^{\mathrm{C}}(v_i, w_i)$, the concavity measure strictly decreases as the outside option's value increases. For the largest outside option value we have $K(w_i = 50) = 0$ since $b^{\mathrm{C}}(v_i, 50)$ is a linear function. The theoretical concavity measure is contrasted with its observed realizations represented by a median spline in figure 11 where the median spline is constructed from individual bidding data that is subject-wise averaged over rounds with identical outside options.The figure points to a relationship between the curvature of observed bidding functions and the value of the outside

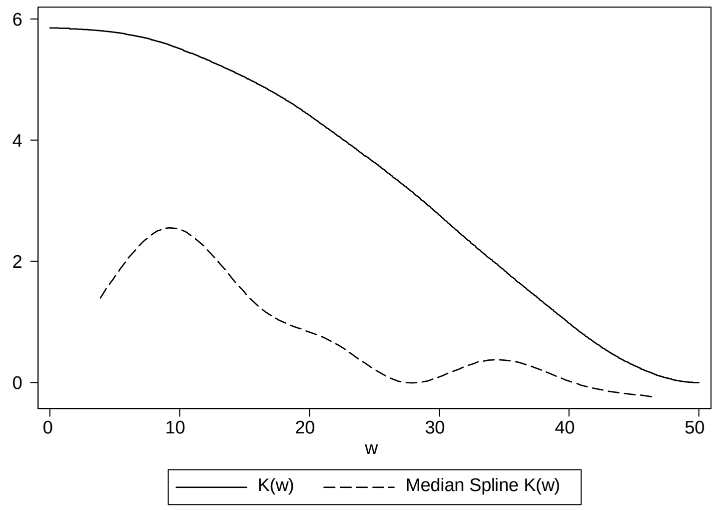


Figure 11: Concavity of observed mean bidding functions for private outside options

option that qualitatively matches the theoretical prediction of more intense concavity for low outside option values. However, the influence of outside options on bidding function concavity seems to be weaker than theoretically expected. Interestingly, aggregate bidding functions are definitely not convex which would be indicated by a negative value of $K(w_i)$. A few bidding schedules that we obtained are provided in the appendix. Robust estimation (that accounts for possibly correlated observations within matching groups) of the linear model $K = \beta_0 + \beta_1 w$ provides evidence for bidding function concavity (positive intercept) and a negative relation between concavity and larger outside option valuation ($\hat{\beta}_0 = 3.56$, $t_7 = 4.15, p = 0.014$ and $\hat{\beta}_1 = -0.089, t_7 = -2.82, p = 0.026$).

The discussed concavity property of equilibrium bidding functions is induced by private outside options. In contrast, equilibrium bidding functions with public outside options are predicted to be linear. The finding in the data for treatment C that lower outside options lead to a more concave bidding function is then only an indicator of prediction quality of theory if there is no such relationship in the data for treatment B. To see this, we robustly estimate the model[14]

$$K = \beta_0 B + \beta_1 B \cdot w + \beta_2 C + \beta_3 C \cdot w$$

with pooled data for treatments B and C where the explanatory variables B and C serve as treatment-specific dummy variables. The results are given in table 11 and indicate that for public outside options neither a significant effect of the outside option value on the concavity measure nor mean concavity (insignificant intercept) can be identified.[15]

[14] The estimation method accounts for possible correlations within matching groups.

[15] If data for treatment B is augmented by data from treatment A that can be viewed as a special case of treatment B, then the coefficient of the outside option value $\hat{\beta}_1$ remains insignificant ($t_{15} = -0.53, p = 0.606$) but the estimate of the intercept $\hat{\beta}_0 = 1.493$ becomes significant ($t_{15} = 3.54$, $p = 0.003$). However, the intercept for treatment C is significantly larger than

Table 11: Regression results: Concavity with private and public outside options

expl. var.	coeff. $\hat{\beta}_x$	robust $\hat{\sigma}_{\beta_x}$	t	$P > \|t\|$	95% conf.	interval
B	1.515362	1.532773	0.99	0.341	-1.795993	4.826718
$B \cdot w$	-0.0059506	0.0380048	-0.16	0.878	-0.088055	0.0761538
C	3.557907	0.8332235	4.27	0.001	1.757837	5.357977
$C \cdot w$	-0.0886455	0.0305614	-2.90	0.012	-0.1546695	-0.0226216

Thus, bidding function concavity appears to be related to the value of outside options only if these are of the private type.

Another prediction related to the curvature of equilibrium bidding functions with private outside options is its linearity in net valuations if $x \leq 50$ and its strict concavity if $x > 50$, see Figure 4 on p. 8. To test if this property is reproduced in the laboratory, we investigate the bidding data for $x \leq 50$ and $x > 50$ separately and fit to both samples a piecewise-linear function with two segments where segments connect at $x = 25$ or $x = 75$ depending on the sample. If the slope over the first segment equals the slope over the second segment, the estimated function is linear. If the slope over the first segment is greater than the slope over the second segment, the estimated function is concave. The two models that we robustly estimate can be compactly written as[16]

$$b_{i,s} = \beta_{0,s} + \beta_{1,s} x^1_{i,s} + \beta_{1,s} x^2_{i,s}$$

where $\beta_{1,s}$ is the slope over the first segment and $\beta_{2,s}$ is the slope over the second segment of net valuations in sample s. If $x_i \leq 50$ then $s = 1$ otherwise $s = 3$. To directly estimate the segment-specific slopes, the explanatory data on net valuations is transformed as follows:

$$x^1_{i,s} = \begin{cases} x_{i,s} & \text{if } x_{i,s} \in [25(s-1), 25s] \\ 25s & \text{if } x_{i,s} \in (25s, 25(s+1)] \end{cases} \qquad x^2_{i,s} = \begin{cases} x_{i,s} - 25s & \text{if } x_{i,s} \in [(25s, 25(s+1)] \\ 0 & \text{if } x_{i,s} \in (25(s-1), 25s] \end{cases}$$

The estimates are summarized in table 12. As predicted, the estimated bidding function in net valuations is linear over the interval $[0, 50]$ since the slopes over the two segment 0-25 and 25-50 do not significantly differ ($t_7 = 1.114$, $p = 0.3019$, two-tailed). In contrast and in line with the prediction, the estimated bidding function over the interval $[50, 100]$ appears to be concave since its slope over the first segment 50-75 ($\hat{\beta}_1 = 0.819$) is significantly larger than its slope over the second segment 75-100 ($\hat{\beta}_2 = 0.502$; $t_7 = 3.889$, $p = 0.003$, one-tailed).

the intercept for treatments A and B ($t_{15} = 2.22, p = 0.0211$,one-tailed).

[16]The estimation procedure allows for possibly correlated observations within matching groups.

Table 12: Regression results: Concavity of estimated bidding function $b^C(x)$

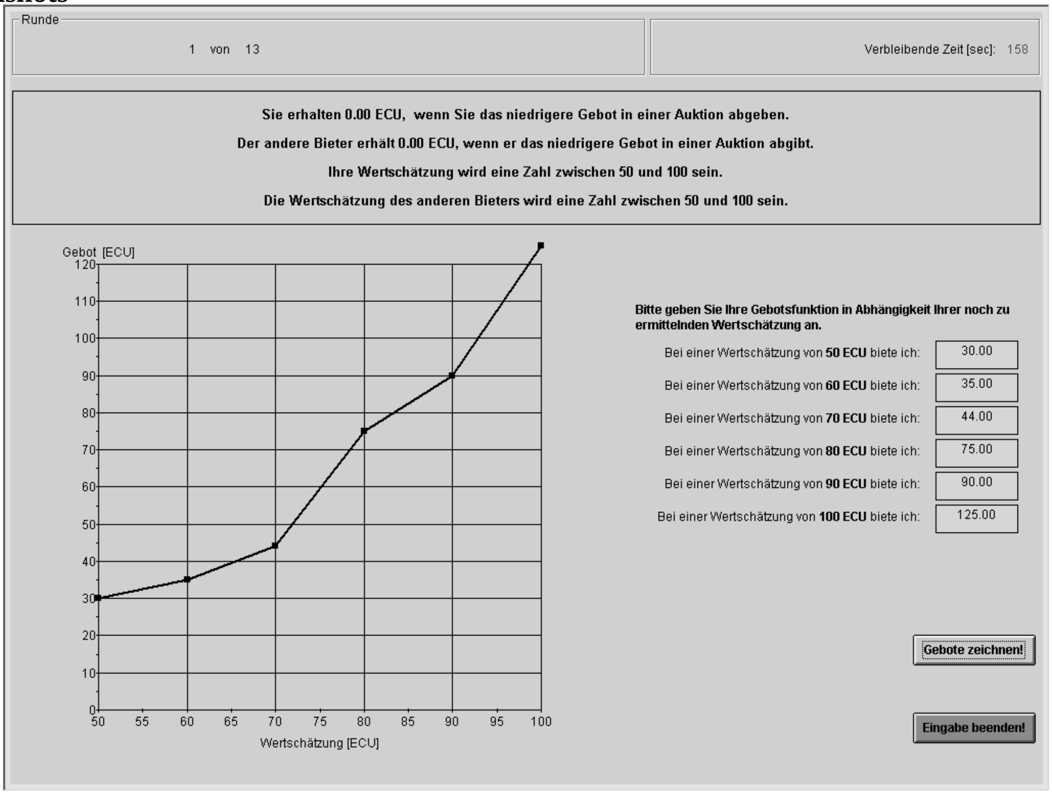
sample	expl. var.	coeff. $\hat{\beta}_x$	robust $\hat{\sigma}_{\beta_x}$	t	$P > \|t\|$	95% conf.	interval
$s = 1$	constant	2.450964	1.916258	1.28	0.242	-2.080265	6.982194
$(x \leq 50)$	$x^{0\text{-}25}_{\leq 50}$	0.7826128	0.054495	14.36	0.000	0.6537527	0.9114729
	$x^{25\text{-}50}_{\leq 50}$	0.7081456	0.0232956	30.40	0.000	0.6530603	0.763231
$s = 3$	constant	-0.7507278	2.050539	-0.37	0.725	-5.599482	4.098026
$(x > 50)$	$x^{50\text{-}75}_{>50}$	0.8189925	0.0278745	29.38	0.000	0.7530797	0.8849053
	$x^{75\text{-}100}_{>50}$	0.5018675	0.07169889	7.00	0.000	0.3323267	0.6714084

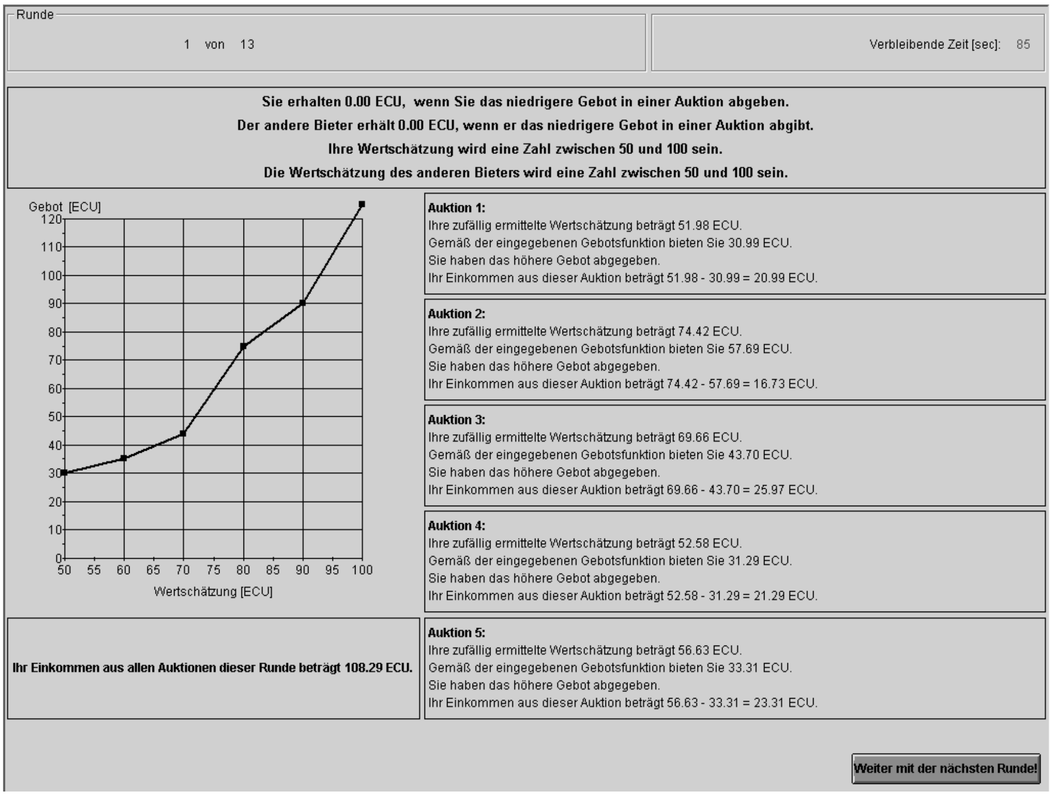
5 Conclusion

We have introduced a bidding model that allows for public and private outside option and experimentally tested it. A key feature of our experimental design is that we collected entire bidding functions. Theoretically, higher-valued outside options lead to less aggressive bidding (ie. lower bids) than the first-price model without outside options. Our experimental data reproduces this prediction. Private outside options differ from public outside options in the set of common knowledge. This difference theoretically implies that bidders respond to an increase of their outside option value with a larger reduction of bids under the public outside option regime. This prediction is confirmed by the data. Private outside options should lead to concave bidding functions, especially for low outside option values. We found this pattern in the data on the aggregate level. However, the degree of concavity that is present in the data is much lower than predicted. The stylized fact of "overbidding" is partially reproduced in our experiments. Taken together, our analysis suggests that outside options crucially influence bidding behavior in a way that is qualitatively predicted by theory and that the particular nature of outside options matters. However, actual bidding behavior seems to deviate from the predictions in important ways.

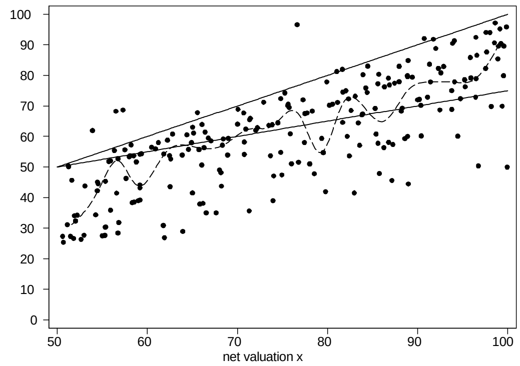
6 Appendix

Screenshots



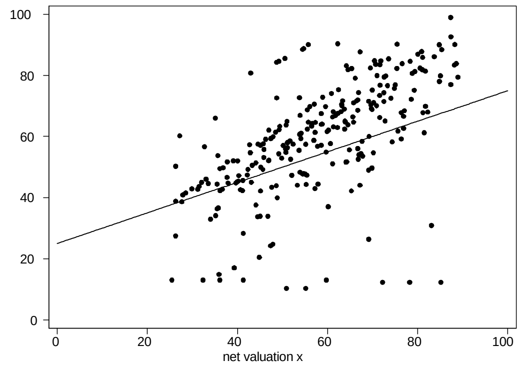


Bidding data for the first round Treatment A with 230 observed bids in the first round:



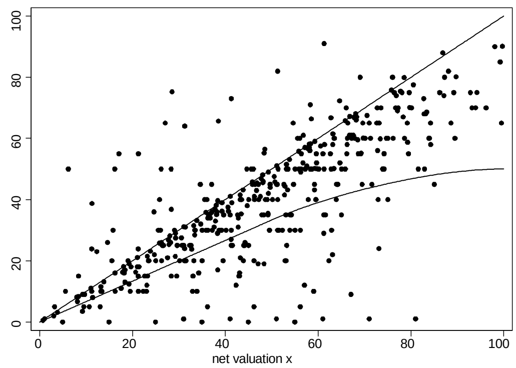
Bidding data for treatment A (inital bids)

Treatment B with bids submitted in the first round:



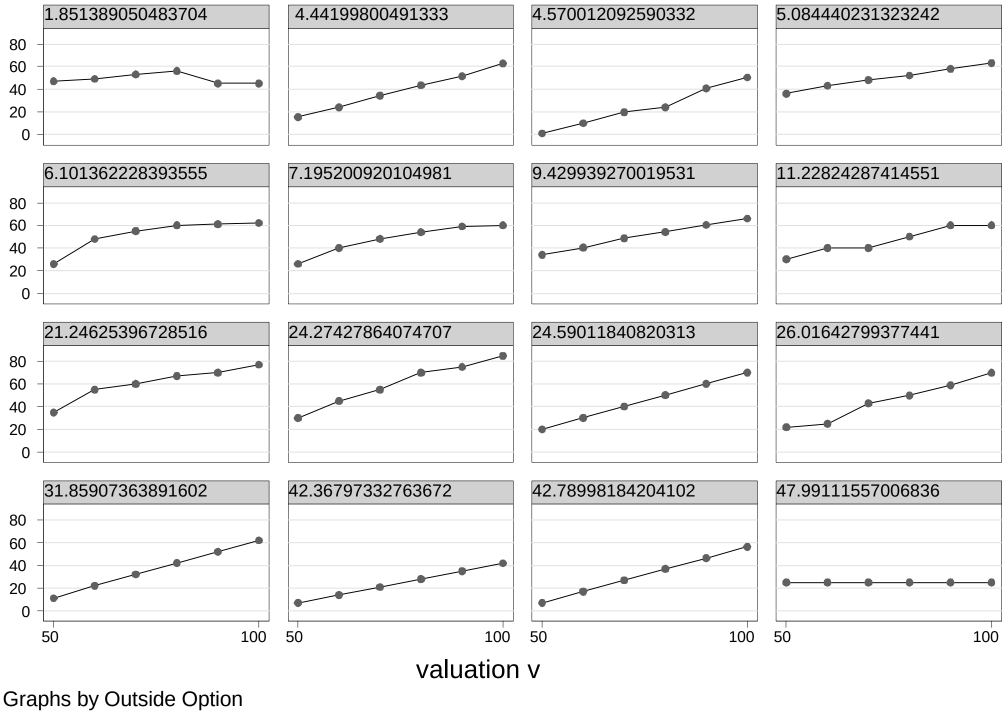
Bidding data for treatment B (initial bids)

Treatment C with bids submitted in the first round:



Bidding data for treatment C (initial bids)

Sample of observed bidding schedules / Treatment C



References

[1] Ashenfelter, O.C., Graddy, K., 2003. Auctions and the price of art. Journal of Economic Literature 41(3), 763-787.

[2] Brosig, J., Reiß, J.P. Entry decisions and bidding behaviour in sequential procurement auctions: an experimental study. FEMM Working paper No. 03023, University of Magdeburg.

[3] Chen, K.-Y., Plott, C.R. Nonlinear behavior in sealed bid first price auctions. Games and Economic Behavior 25, 34-78.

[4] Cox, J.C., Roberson, B., Smith, V.L., 1982. Theory and behavior of single object auctions. Research in Experimental Economics 2, 1-43.

[5] Cox, J.C., Smith, V.L., Walker, J.M., 1988. Theory and individual behavior of first-price auctions. Journal of Risk and Uncertainty 1, 61-99.

[6] Dorsey, R., Razzolini, L., 2003. Explaining overbidding in first price auctions using controlled lotteries. Experimental Economics 6(2), 123-140.

[7] Fischbacher, U., 1999. z-Tree - Zurich Toolbox for Readymade Economic Experiments - experimenter's manual. Working paper No. 21, Institute for Empirical Research in Economics, University of Zurich.

[8] Gale, I.L., Hausch, D.B., 1994. Bottom-fishing and declining prices in sequential auctions. Games and Economic Behavior 7, 318-331.

[9] Güth, W., Ivanova-Stenzel, R., 2003. Asymmetric auction experiments with(out) commonly known beliefs. Economic Letters 80, 195-199.

[10] Güth, W., Ivanova-Stenzel, R., Königstein, Strobel, M., 2002. Bid functions in auctions and fair division games: experimental evidence. German Economic Review 3(4), 461-484.

[11] Güth, W., Ivanova-Stenzel, R., Königstein, Strobel, M., 2003. Learning to bid - an experimental study of bid function adjustments in auctions and fair division games. Economic Journal 113, 477-494.

[12] Isaac, R.M., Walker, J.M., 1985. Information and conspiracy in sealed bid auctions. Journal Economic Behavior and Organization 6, 139-159.

[13] Kagel, J.H., Harstad, R.M., Levin, D., 1987. Information impact and allocation rules in auctions with affiliated private values: a laboratory study. Econometrica 55(6), 1275-1304.

[14] Kirchkamp, O., Moldovanu, B, 2004. An experimental analysis of auctions with interdependent valuations, Games and Economic Behavior 48(1), 54-85.

[15] Kirchkamp, O., Reiß, J.P. The overbidding-myth, Working paper.

[16] Ockenfels, A., Selten, R., 2002. Impulse balance equilibrium and feedback in first price auctions, Working paper, University of Bonn.

[17] Pezanis-Christou, P., Sadrieh, A., 2004. Elicited bid functions in (a)symmetric first-price auctions, Working paper.

[18] Riley, J.G., Samuelson, W.F., 1981. Optimal auctions. American Economic Review 71(3), 381-392.

[19] Reiß, J.P., Schöndube, J.R., 2002. On participation in sequential procurement auctions. FEMM Working Paper No. 02016, University of Magdeburg.

[20] Rogers, W. H., 1993. Regression standard errors in clustered samples. Stata Technical Bulletin 13, 19–23. Reprinted in Stata Technical Bulletin Reprints 3, 88–94.

[21] Selten, R., Buchta, J., 1999. Bidding behavior in first-price auctions with directly observed bid functions. In: Budescu, D., Erev, E., Zwick, R. (Eds.), Games and Human Behavior: Essays in the Honors of Amnon Rapoport, Mahwah, NJ: Lawrenz Associates, 79-102.

[22] Vickrey, W., 1961. Counterspeculation, auctions, and competitive sealed tenders. Journal of Finance 16, 8-37.

[23] Weber, R.J., 1983. Multiple-object auctions. In: Engelbrecht-Wiggans, R., Shubik, M., Stark, R.M. (eds.), Auctions, Bidding, and Contracting: Uses and Theory. New York: New York University Press, 165-191.

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