# Strategic Interaction, Competition, Cooperation and Observability

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# Chapter 1

# Introduction

The early oligopoly studies by Cournot [13] and Bertrand [8] have shaped economists' thinking using game theoretic methods before the invention of game theory. Nowadays, the standard textbook Cournot model and Bertrand model continue to influence the way economists think of various aspects of market power. Among others, these are phenomena such as cartel formation and implicit collusion or the influence of the number of sellers on the distribution of surplus.

Both the Cournot model and the Bertrand model make important distinctions between buyers and sellers. Sellers are able to influence prices and move first. They choose prices or quantities anticipating the non-strategic reaction of the buyers. The buyers are just represented by a demand function (i.e. act passively) and move second. Given the sellers' choices of quantities or prices, the demand function provides market clearing. This asymmetry in behavior is different from General Equilibrium Theory which treats buyers and sellers symmetrically. There, all agents behave as price takers and make their decisions simultaneously. Since sellers are modelled as strategic players, the Cournot model and the Bertrand model allow to discuss phenomena which can not be analyzed in General Equilibrium Theory: for example, the models provide a framework to study cartel formation and implicit collusion. Due to the asymmetry between buyers and sellers these aspects can be discussed only for the sellers. In addition, they can be studied only under the restriction that the other side of the market, the buyers, behaves non-strategically. In other words: certain roles which agents might play in a given market are fixed exogenously. In a Cournot model, only sellers can develop patterns of implicit collusion sustained by price wars, as modelled with the aid of the theory of infinitely repeated games (Abreu, Pearce and Stacchetti [2]).

The exogenous fixation of roles causes the following difficulty. In the end, we do observe markets in which buyers do not act passively. Many intermediate goods markets today are characterized by market power on both the seller and the buyer side. For example, markets for retailing tend to involve significant market power of retail chains. The power of such chains is a matter of serious policy concern (see Dobson and Waterson [17] and [18]). As a result both producers as sellers and retail chains as buyers exercise market power. Hence theoretic considerations of collusive behavior in such a market should not be based on the textbook story of tit-for-tat in a repeated Cournot oligopoly model. This being said, how robust are our insights on collusion in a framework where sellers and buyers are symmetric?

The present dissertation looks at situations without distinctions between buyers and sellers and discusses these issues in three distinct settings. In all settings all economic agents act strategically. Chapter 2 considers a strategic market game model as originally developed by Shapley and Shubik [52]. I analyze how the distribution of surplus in the market depends on the number of buyers and sellers under different timing structures. Chapter 3 studies patterns of cooperation and collusion in a model of decentralized trading where prices are determined by repeated bilateral negotiations. Chapter 4 considers the functioning of information networks intended to sustain cooperation in a Prisoner's Dilemma with changing partners. As we shall see, the results depend on assumptions on the mechanism of price formation (central exchange versus decentralized negotiations), the timing (one-shot game versus infinitely repeated game) and the observability of moves (perfect information versus imperfect information).

In the remainder of this introduction I discuss the motivation for these papers and their main contributions in more detail.

## 1.1 Sequential Moves and Comparative Statics in Strategic Market Games

In a recent paper, Bloch and Ghosal [10] study the formation of trading groups in a bilateral oligopoly. Their model of bilateral oligopoly is based on a strategic market game model à la Shapley and Shubik [52]. Strategic market games model strategic exchange economies as noncooperative simultaneous moves games. Traders send simultaneously bids and supplies of goods to the market mechanism. The goods' prices are determined by the ratio of bids and supplies for a good. A strategic market game models a market with market power on both sides and a centralized exchange and price formation mechanism. It is a spot market organized as an exchange where each trader obtains a share of the total bids from the other side of the market which is proportional to his share of the total bids from his side of the market. Strategic market games are deemed to be an advantage over Cournot oligopolies since all agents act strategically.

Bloch and Ghosal find that the only strongly stable trading group is the grand coalition where all agents trade on the same market. A trading group is strongly stable if no coalition of traders can deviate such that all of its members obtain at least as high as a utility as before and some members obtain a strictly higher utility. The incentives to form trading groups are derived from two comparative statics results on the Nash equilibrium of the noncooperative model of exchange. First, traders on one side of the market always benefit from an increase in the number of traders on the other side of the market. Second, on any symmetric market, where the number of traders of the two types is equal, the traders' utilities increase with the size of the market. Third, Bloch and Ghosal find that (p.375)

> ...one of the most intuitive results obtained in partial equilibrium oligopoly analysis does not carry over to the case of bilateral oligopoly. In the classical Cournot oligopoly, as the number of firms increases, the profit of each firm is reduced. In the context of bilateral oligopoly, an increase in the number of traders of one type may induce either positive or adverse effects on the traders of this type...

These findings give rise to two problems: first, one has the feeling that in such centralized exchange markets traders would not benefit from more competition. Moreover, not all traders trade on the same market in such markets, i.e. there exist different exchanges where a good can be traded.

We know from Cournot oligopoly theory that, as the number of firms increases, the profit of each firm is reduced.

Moreover, we know from the Cournot oligopoly model (Bloch and Ghosal, p.369)

> ...that traders have an incentive to set up distinct, separate markets on which they behave as monopolists...

There are two differences between strategic market games and Cournot oligopoly models. Strategic market game models exhibit strategic behavior from all traders *and* simultaneity) of moves. I consider a sequential version of the Shapley-Shubik models in which traders on one side of the market, the sellers, move first. The buyers observe the sellers' choices and move second. I discuss an example where sellers are risk-neutral and buyers' utility is quasilinear and quadratic. For the simultaneous moves game of that setting, an increase in the number of sellers may have a positive effect on sellers' utilities. Moreover, the only strongly stable trading structure is the one where all agents trade on the same market. With sequential moves, the following results are obtained: first, as the number of sellers increases, the profit of each seller is unambiguously reduced. Second, on any symmetric market, an increase in the number of traders has ambiguous effects on the sellers' equilibrium utilities! This in turn implies that there might exist strongly stable trading structures in which *not* all agents trade on the same market.

The introduction of sequential moves has a further implication. In simultaneous moves strategic market games there is always a no-trade equilibrium. The strategic market games literature contains many results that predict Walrasian equilibria in the competitive limit. However, they usually come at the expense of ad hoc assumptions that rule out the pathological no trade equilibria. Weyers [56] studies a strategic market game with limit prices to solve this problem.[1] The set of Nash equilibria of this

[1] The limit price mechanism of Mertens [40] is a mechanism for determine prices and final allocation

game converges to the set containing the competitive and the no-trade equilibrium, when players are replicated. Two-rounds of iterated deletion of weakly dominated strategies eliminate the no-trade equilibria. A sequential moves strategic market game proposes another, simpler solution to this problem concerning the no-trade equilibrium. In essence, traders face a coordination problem, where one equilibrium, the trade equilibrium pareto-dominates the no-trade equilibrium. It is well known that in a 2-player coordination game with sequential moves and perfect information the Pareto-dominant equilibrium emerges as the unique subgame perfect equilibrium. Hence only the trade-equilibrium emerges in my sequential strategic market game. I show for an example that this unique subgame perfect equilibrium converges to the Walrasian equilibrium as the market size goes out of bounds.

## 1.2 The Effects of Competition on Bargaining Power in Repeated Bilateral Negotiations

> "We have a saying at this company: our competitors are our friends and our customers are our enemies...God damn buyers [are the enemy]. We've gotta have them. But they are not my friends. You're my friend. I want to be closer to you than I am to any customer because you can make us money."

This quote is from a former executive of Archer-Daniels-Midland (ADM), a lysine producer. Lysine is a food additive designed to put additional meat on hogs. The quote

when agents trade a finite number of goods by placing an arbitrary number of market or limit orders.

was recorded by an FBI mole. Largely on the basis of the audio- and videotapes he provided, a federal jury in Chicago convicted three former ADM executives in September 1998 of engaging in an international conspiracy to fix lysine prices.[2]

For any economist, the meaning of such a statement seems clear: competitors are friends if they help to sustain collusive prices by reducing production. This is the simple mechanism we know from the standard Cournot textbook model.

For most business-to-business goods, Cournot markets or markets similar to the double auction-like scenario of chapter 2 are just the tiny tip of a huge market of such one-to-one deals. Most business-to-business goods are traded through bilateral contracts.[3] That is, we have a situation of bilateral oligopoly with bilateral negotations; both buyers and sellers exercise market power. Then matters are not all that obvious! To see why consider a market with repeated bilateral negotiations, as modelled in chapter 3. Sellers make all the offers and have zero value for the good, the buyers have a commonly known value of one. Buyers can accept or reject an offer. Every trader maximizes the sum of his discounted per-period utilities from transactions. Suppose there is only one seller and only one buyer, the case of bilateral monopoly. The infinite repetition of the stage game does not preclude the buyer from extracting all the surplus. Threats to refuse any slightly unsatisfactory offer - even if this would be better than not to buy at all - are credible if acceptance induces a reputation for softness and implies worst equilibria in the rest of the relation. Hence, in a market where infinitely repeated transactions take place in isolation, the presence of other buyers does not matter for any individual buyer: there is an equilibrium in which the buyer extracts the full surplus from the bargaining. The argument is based on the Folk Theorem of

[2]See http://www.wwnorton.com/mip/ime/varian/28b.htm for more.

[3]See The Economist [19].

repeated game theory and does *not* rely on other buyers providing punishment through price wars as in a Cournot oligopoly. It is hence not clear what the above quote - with interchanged roles of buyers and sellers - means in such a world. I shall argue that "competitors are friends" holds if there are outside options for the buyers in form of alternative trading partners!

Suppose that there are alternative trading partners for each buyer, that is, a scenario with increased seller competition. In contrast to economic intuition, the presence of more sellers *reduces* a buyer's surplus as compared to the bilateral monopoly situation. Outside options yield more benign equilibria than the worst equilibrium in the continuation of the current relationship. The argument presumes that the alternative trading partners have no information about a buyer's trading history. Hence, they cannot be integrated into a system of punishment strategies.

I then allow buyers to visit a seller who is currently matched to another buyer. If this happens, the two buyers compete for the seller. Buyers' competition mitigates the negative effect of the outside option. In particular, competing buyers prevent each other from achieving a favorable equilibrium in a new relation. In other words: the profitability of the outside option is reduced through competition. This in turn yields more severe punishments than in the case without competition. Then, a buyers payoff in a situation with competition exceeds his payoff in the best equilibrium without competition for any discount factor.

The main result of the chapter is that the presence of competitors may be useful while the presence of more agents on the other side of the market is not. The effect of competition to sustain favorable equilibria is well known from infinitely repeated Cournot or Bertrand games. Chapter 3 shows that implicit collusion as suggested by these models also can be modeled in a market with bilateral negotiations. The role of competition, however, is a different one. In the Cournot and in the Bertrand model, competition on the *same* side of the market causes a problem in the first place. The problem for any buyer in the model of chapter 3 is that competition on the *other* side of the market reduces his surplus by introducing an outside option. Buyers' competition limits the value of the outside option for each buyer.

## 1.3 Communication Networks and Cooperation

The Walrasian theory of exchange is a theory of anonymous interaction among many traders. The only social structure is a market, a mechanism which adjusts prices such that supply equals demand. This mechanism is able to allocate scarce means efficiently making any social pattern other than the market itself superfluous. However, we observe that people do hold personal network-like relations to each other. There is an extensive sociological literature documenting various patterns of social relations in networks (see Wellmann and Berkowitz for a survey [55]). Granovetter [27] criticizes the pure "market" approach of economics and proposes the concept of social *embeddedness*: social structures affect economic actions through various mechanisms.

Chapter 4 studies the influence of social structures on market outcomes in form of a communication network. In a community where trust and cooperative behavior cannot be assured, network based mechanisms are important institutions. The mechanisms work through communication and fear of punishment for misbehavior or through anticipation of reward in case of good behavior. For example, ethnic communities such as the Dominicans in New York and the Cubans in Miami use such systems to sustain informal credit channels (see Portes and Sensenbrenner [46]). The well known rotating credit associations among Asian immigrants, for example the Chinese on Java, rely on trust enforcement through network mechanisms as well (see Granovetter [28]).

There are two questions which concern the structure of such a communication network. First, information, judgements and gossip are exchanged only among close friends in such a network. This implies that a network member may use his close friends as an effective threat to enforce cooperative behavior. The mechanism is simple: if the partner with whom I interact today cheats on me, I inform my close friends which in turn cheat on the noncooperator tomorrow. It seems optimal to choose the number of close friends as large as possible. The second question concerns the optimal size of the communication network. If interaction with network members yields cooperation while this is impossible with agents outside the network, it seems best to have a huge network.

In chapter 4 I discuss these issues in a setting of an infinitely repeated Prisoner's Dilemma with changing partners and random matching. Before the repeated interaction starts, agents form a network and each network member chooses a number of contacts to other network members. If two agents meet, they observe the number of contacts their partner has. Then they play the Prisoner's Dilemma. After having played the stage game, they inform their close contacts about the partner's choice of action. In such a setting, the community wishes to choose a huge network. Moreover, it is best if all network members have close contacts to all other network members: this is the most effective threat inducing the most severe punishments. These findings stand in contrast to the following observations. First, people do not entertain close contacts to all people in their network of personal relations[4]. Second, networks with a large number of participants are ineffective.

To provide a rationale for these observations, I introduce noise in the communication among agents. There are many reasons why communication should be noisy. Either I might inform my friends mistakenly about my partner's behavior or there might be some distortion in the technology used to transmit information. There might also be errors in interpretation of messages actually sent correctly. Then a network member has to fulfill his punishment obligation although nobody did defect or a network member might get punished although he behaved cooperatively. It is then optimal not to entertain close contacts to all other network members: it is best to have a number of close contacts which is just enough to deter other agents from defecting. More noise has another detrimental effect: suppose that each agent receives noisy information about a partner not only from the network member who interacted last with that partner, but also from all other network members - *gossip*. Then, as the number of agents in the network gets large, cooperation fails. The reason is that network members cannot distinguish between information with content and pure noisy information stemming from gossip. If the network cannot provide cooperation and if there is a cost to set up the network then the community does not form a network at all.

[4]see Boissevain [11].

# Chapter 2

# Sequential Moves and Comparative Statics in Strategic Market Games

## 2.1 Introduction

Strategic market games have been developed by Shubik [53] and Shapley and Shubik [52] as an alternative to Cournot oligopoly models. In strategic market game models, traders send simultaneously bids and supplies of goods to the market mechanism. The ratio of bids and supplies exchanged for a good determine its terms of trade. Strategic market games are models of exchange in which all players behave strategically. This is deemed to be an advantage over Cournot oligopoly models where the demand side behaves non-strategically.

A recent paper by Bloch and Ghosal [10] suggests that strategic market games à la Shapley and Shubik might have a rather counterintuitive property: an increase in the number of traders on the one side of the market might increase equilibrium utilities for *all* traders, i.e. those on the *same* side of the market as well as those on the other side of the market (Bloch and Ghosal [10], p. 375). The finding contrasts with the familiar Cournot result that an increase in the number of sellers implies lower profits for each one of them. In general, Bloch's and Ghosal's results show that the effect of an increase in the numbers of traders of the *same type* cannot be predicted. In contrast, an increase in the number of traders of the *other* type is always beneficial. For symmetric markets, that is, markets with the same number of traders on each side of the market, Bloch and Ghosal show that all traders benefit from an increase in the number of traders (Proposition 2.2.).

In fact there are two differences between strategic market games à la Shapley-Shubik and Cournot oligopoly models. Strategic market game models exhibit strategic behavior from all traders *and simultaneity of all moves.* The main motivation for the present chapter is to regain comparative statics properties as known from Cournot oligopolies for Shapley-Shubik strategic market games. This chapter considers a sequential version of the Shapley-Shubik models. I present an example where in the simultaneous version traders benefit from an increase in the number of traders of the same type. I show that this counterintuitive result does not hold for the first movers in the sequential version of the game. Hence the finding of Bloch and Ghosal concerns sequential versus simultaneous moves games rather than Shapley-Shubik strategic market games versus Cournot oligopoly models. Both in the simultaneous version and in the sequential version, an increase in the number of traders on the *other* side of the market is beneficial. For symmetric markets, I find that effects on first-movers' utilities are ambiguous as the number of traders increases.

I develop a model similar to the one used in Bloch and Ghosal [10]. There are two goods and two types of traders, buyers and sellers: each buyer's utility is quadratic and quasilinear and sellers' utility is linear. Sellers hence could be interpreted as firms. Each trader type initially owns one unit of one of the goods only. The timing in the model is sequential: the sellers choose their strategy first. The buyers observe the sellers' choices and make their move. I adopt subgame perfect Nash equilibrium as the solution concept.

After setting up the model, I illustrate the main qualitative implication of sequential moves versus simultaneous moves in strategic market games. I compute the Nash equilibrium in the *simultaneous moves* version of the game and show that sellers' equilibrium utilities might increase as the sellers' number increases. Buyers' equilibrium utilities increase always as the number of buyers increases. For both sides of the market the very intuition from Cournot oligopolies does not hold.

I then study the *sequential* version with the sellers moving first and find that an increase in the number of sellers has always the expected negative effect on the sellers' utilities! Intuitively, the buyers' behavior in the second stage forces sellers to decrease their individual equilibrium supplies. The equilibrium price decreases as well so that the overall effect is unambiguous. Buyers still benefit from an increase in the number of buyers! If the market is symmetric, the sellers might benefit from a *decrease* in the number of traders while the buyers benefit from an increase in the number of traders.

Apart from implications for comparative statics, the sequential model produces further results. In their paper, Bloch and Ghosal study the formation of trading groups in bilateral oligopoly. They find that the only strongly stable trading structure is the grand coalition where all agents trade on the same market. A trading group is strongly stable, if no coalition of traders can deviate such that all of its members obtain at least as high as a utility as before and some members obtain a strictly higher utility. I find that with sequential moves *other strongly stable trading structures might exist.* Intuitively, this finding is based on the result that, for symmetric markets, first-movers might prefer markets with a small number of traders.

The introduction of sequential moves has a further implication. In simultaneous moves strategic market games there is always a no-trade equilibrium. The strategic market games literature contains many results that predict Walrasian equilibria in the competitive limit.[1] However, they usually come at the expense of ad hoc assumptions that rule out the pathological no-trade equilibrium. My sequential moves strategic market game eliminates the no-trade equilibrium and I show, albeit only by means of an example, convergence of the unique subgame perfect equilibrium to the Walrasian equilibrium as the number of traders gets large. Weyers [56] proposes a much more complex game to eliminate the no-trade equilibrium. She studies a strategic market game with limit prices.[2] The set of Nash equilibria of this game converges to the set containing the competitive equilibria and the no-trade equilibrium, when players are replicated. Two-rounds of iterated deletion of weakly dominated strategies eliminate the no-trade equilibria in her model.

The organization of this chapter is as follows. Section 2.2 sets up the basic model for the comparative statics analysis. Section 2.3 provides a detailed study of comparative statics effects and compares the outcomes to those obtained in the simultaneous moves setting. In section 2.4 I discuss the implications of my findings for the formation

[1] See Mas-Colell [37] for an overview.

[2] The limit price mechanism of Mertens [40] is a mechanism for determining prices and final allocation when agents trade a finite number of goods by placing an arbitrary number of market or limit orders.

of trading groups. Section 2.5 presents Walras convergence and section 2.6 proves existence of a subgame perfect Nash equilibrium for more general strategic market games with sequential moves.

## 2.2 The Model

The model setup in this section follows roughly Bloch and Ghosal [10], except for the different timing.

There are two finite sets of traders: $\mathcal{B} := \{1, ..., B\}$ and $\mathcal{S} := \{1, ..., S\}$. Traders are indexed $i = 1, ..., B$ with $i \in \mathcal{B}$ and $j = 1, ..., S$ with $j \in \mathcal{S}$. Traders $i \in \mathcal{B}$ are called buyers and traders $j \in \mathcal{S}$ are called sellers.

There are two tradeable goods in the economy, $x$ and $y$. Good $x$ will sometimes be called "money", good $y$ is a consumption good, say wheat. The final allocation of money and wheat for the traders are denoted by $x_i$, $x_j, y_i$ and $y_j$, respectively. Endowments are distributed so that all buyers have one unit of money and all sellers have one unit (or bushel) of wheat.

I assume that the sellers move first (all sellers move simultaneously). Each seller $j$ decides what amount of wheat he wants to supply. The buyers observe the sellers' choices and make their strategy choices (simultaneously ). Each buyer $i$ decides what amount of money he would like to pay.

The sellers' strategy sets are given by

$$A_j := \{s_j \,|\, s_j \in \mathbb{R}_0^+, \;\; 0 \leq s_j \leq 1\}.$$

Let $A := \times_{j \in S} A_j$ and let $\underline{s} \in A$ denote the strategy profile for the sellers, that is

$\underline{s} = (s_1, ..., s_S)$. The buyers' strategy sets are given by

$$A_i := \{b_i(\cdot) \,|\, b_i : A \to [0,1]\}.$$

Again, $\underline{b}(\cdot) = (b_1(\cdot), ..., b_B(\cdot))$ denotes the strategy profiles for the buyers. Let $\overline{s} := \sum_{j \in \mathcal{S}} s_j$ and $\overline{b}(\cdot) := \sum_{i \in \mathcal{B}} b_i(\cdot)$.

The strategy choices $(\underline{s}, \underline{b}(\cdot))$ determine the market clearing price $p$ for one unit of wheat as

$$p = \frac{\overline{b}(\underline{s})}{\overline{s}} \tag{2.1}$$

and the final allocation

$$y_i = \frac{b_i(\underline{s})}{p}, \tag{2.2}$$

$$x_i = 1 - b_i(\underline{s}), \tag{2.3}$$

$$y_j = 1 - s_j, \tag{2.4}$$

$$x_j = s_j \cdot p. \tag{2.5}$$

I define $\frac{0}{0} := 0$.

The corresponding payoffs are

$$U(\underline{b}(\cdot), \underline{s}) = f\left(\frac{b_i(\underline{s})}{p}\right) + (1 - b_i(\underline{s})), \tag{2.6}$$

where

$$f\left(\frac{b_i(\underline{s}}{p}\right) = \alpha\left(\frac{b_i(\underline{s})}{p}\right) - \frac{\beta}{2}\left(\frac{b_i(\underline{s})}{p}\right)^2 \tag{2.7}$$

for all buyers $i \in \mathcal{B}$. We assume that, for all $(\underline{b}(\cdot), \underline{s})$ and for all $\alpha > 0$, there exists $\beta < \alpha, \beta > 0$ such that $U'' < 0$ and $f' > 0$. It is easy to check that the condition $0 < \beta < \alpha$ for all $\alpha > 0$ is in fact sufficient for $U'' < 0$ and $f' > 0$.

The payoffs for the sellers are given by

$$\pi(\underline{b}(\cdot), \underline{s}) = s_j \cdot p + (1 - s_j) \tag{2.8}$$

for all $j \in \mathcal{S}$.

I chose risk neutrality for the sellers to interpret sellers as firms. This allows me to relate the model and its results to Cournot oligopoly models.

## 2.3 Comparative Statics and Timing

In this section, I examine and compare comparative statics effects for the simultaneous moves and the sequential moves case. First, I recapitulate briefly the comparative statics results obtained by Bloch and Ghosal [10] for the simultaneous moves version of the model in section 2.2.

### 2.3.1 Simultaneous Moves

For the simultaneous moves setting the buyers' strategy sets are given by

$$A_i := \{b_i | b_i \in R, 0 \leq b_i \leq 1\}$$

for all $i$.

It is straightforward to compute the unique Nash equilibrium in pure strategies $\left(\widehat{b}, \widehat{s}\right)$,

$$\widehat{b} = \frac{1}{\beta} \frac{S}{S-1} \left( \alpha - \frac{B}{B-1} \frac{S}{S-1} \right) \tag{2.9}$$

and

$$\widehat{s} = \frac{1}{\beta}\frac{B}{S}\left(\alpha - \frac{B}{B-1}\frac{S}{S-1}\right). \tag{2.10}$$

Suppose that

$$\alpha > \frac{B}{B-1}\frac{S}{S-1},$$

so that the existence of an interior solution is guaranteed.

I obtain the Nash equilibrium by differentiating the payoff functions of all traders with respect to their referring strategy. The first-order conditions are necessary and sufficient due to the assumptions on $\alpha$ and $\beta$. A simple argument shows that any Nash equilibrium in pure strategies must by type-symmetric: all sellers employ the same strategy and all buyers employ the same strategy.

The equilibrium price is given by

$$p = \frac{S}{S-1}. \tag{2.11}$$

Equilibrium payoffs are

$$\pi\left(\widehat{b}, \widehat{s}\right) = \frac{1}{\beta}\frac{B}{S(S-1)}\left(\alpha - \frac{B}{B-1}\frac{S}{S-1}\right) + 1 \tag{2.12}$$

and

$$U\left(\widehat{b}, \widehat{s}\right) = \frac{1}{\beta}\left(\alpha - \frac{B}{B-1}\frac{S}{S-1}\right)\left(\alpha - \frac{S}{S-1}\right) \tag{2.13}$$

$$-\frac{\beta}{2}\left(\frac{\alpha}{\beta} - \frac{1}{\beta}\frac{B}{B-1}\frac{S}{S-1}\right)^2 + 1. \tag{2.14}$$

Simple computations yield the following comparative statics result.

**Result 1.** *The Nash equilibrium satisfies*

$$(R1) \quad \frac{\partial U}{\partial B} > 0, \qquad (R2) \quad \frac{\partial U}{\partial S} > 0, \qquad (R3) \quad \frac{\partial \pi}{\partial B} > 0.$$

*Moreover,*

$$(R4) \quad \frac{\partial \pi}{\partial S} > 0$$

*if and only if*

$$\alpha < \frac{B}{B-1}\frac{S}{S-1}\frac{2S}{2S-1}.$$

Results $(R1)$ and $(R4)$ demonstrate the counterintuitive property which a strategic market game à la Shapley and Shubik might have. An increase in the number of sellers might increase sellers' equilibrium utilities and an increase in the number of buyers increases buyers' equilibrium utilities. The results $(R2)$ and $(R3)$ are what one would expect.

### 2.3.2 Sequential Moves: The Sellers as First-Movers

For the sequential moves version with the sellers as first movers, the buyers' Nash equilibrium strategies in the second stage are given by

$$b^* = \alpha\frac{B-1}{B^2}\,\overline{s} - \beta\frac{B-1}{B^3}\,\overline{s}^2. \tag{2.15}$$

The problem in the first stage reduces to finding a Nash equilibrium in a game among the sellers with payoff functions

$$\pi(s_j, s_{-j}) = s_j(D - C\,\overline{s}) + (1 - s_j), \tag{2.16}$$

where

$$D = \alpha \frac{B-1}{B} \qquad \text{and} \qquad C = \beta \frac{B-1}{B^2}. \tag{2.17}$$

As unique subgame perfect equilibrium strategies, I obtain

$$s_1^* = \frac{D-1}{C(S+1)} \tag{2.18}$$

and

$$b_1^* = \alpha \frac{B-1}{B^2} \frac{S(D-1)}{C(S+1)} - \beta \frac{B-1}{B^3} \left( \frac{S(D-1)}{C(S+1)} \right)^2. \tag{2.19}$$

Equilibrium payoffs are

$$\pi_1(b^*, s^*) = \frac{(D-1)^2}{C(S+1)^2} + 1 \tag{2.20}$$

and

$$U_1(b^*, s^*) = \alpha \left( \frac{S(D-1)}{BC(S+1)} \right) - \frac{\beta}{2} \left( \frac{S(D-1)}{BC(S+1)} \right)^2 - \alpha \frac{B-1}{B^2} \frac{S(D-1)}{C(S+1)} \tag{2.21}$$

$$+\beta \frac{B-1}{B^3} \left( \frac{S(D-1)}{C(S+1)} \right)^2 + 1. \tag{2.22}$$

A simple computation gives

**Result 2.** *The subgame perfect Nash equilibrium with the sellers moving first satisfies*

$$(R5) \quad \frac{\partial U_1}{\partial B} > 0, \quad (R6) \quad \frac{\partial U_1}{\partial S} > 0, \quad (R7) \quad \frac{\partial \pi_1}{\partial B} > 0, \quad (R8) \quad \frac{\partial \pi_1}{\partial S} < 0.$$

Result $(R8)$ states the main point of this chapter: the introduction of sequential moves gives us back the intuitive effect of an increase in the sellers' number on their equilibrium utilities - as opposed to $(R4)$. However, an increase in the buyers' number still leads to higher equilibrium utilities for the buyers.

### 2.3.3 Sequential Moves: The Buyers as First-Movers

For the sake of completeness I also analyze the case with the buyers as first movers. Again, a straightforward computation of the subgame perfect equilibrium gives

$$s_2^* = \frac{B(\alpha(S-1)-S)}{\beta S^2} \tag{2.23}$$

and

$$b_2^* = \frac{\alpha(S-1)-S}{\beta(S-1)} \tag{2.24}$$

as unique equilibrium strategies. Equilibrium payoffs are

$$\pi_2(b^*, s^*) = \frac{B(\alpha(S-1)-S)}{\beta S^2(S-1)} + 1 \tag{2.25}$$

and

$$U_2(b^*, s^*) = \alpha\left(\frac{\alpha(S-1)-S}{\beta S}\right) - \frac{\beta}{2}\left(\frac{\alpha(S-1)-S}{\beta S}\right)^2 \tag{2.26}$$

$$+1 - \frac{\alpha(S-1)-S}{\beta(S-1)}. \tag{2.27}$$

**Result 3.** *The subgame perfect Nash equilibrium with the buyers moving first satisfies*

$$(R9) \quad \frac{\partial U_2}{\partial B} = 0.$$

*Moreover,*

$$(R10) \quad \frac{\partial \pi}{\partial S} > 0$$

*if and only if*

$$\alpha > \frac{S^3 - 3S^2 + S}{S^3 - 3S^2 + 3S - 1}.$$

As in the simultaneous moves setting, result ($R10$) asserts that an increase in the number of sellers might increase sellers' equilibrium utilities if they move second. Thus, the order of moves is a relevant aspect for comparative statics results. Result ($R9$) shows that an increase in the buyers' number does not affect their equilibrium utilities at all. This result seems to come from the assumption of sellers' risk-neutrality.

### 2.3.4 Discussion

The results ($R2$) and ($R3$) are what one would expect and will not be further commented on. For ($R1$), note that an increase in the number of buyers induces an increased bidding from the buyers for wheat. The increased bidding lets sellers increase their wheat supplies as well, leading to an increased trading volume in the market. The increased trading volume has no effects on the equilibrium price then because of the constant seller markup. The intuition for ($R4$) is similar. An increase in the number of sellers induces an increased wheat supply. The increased supply leads to an increase in buyers' bids of money. Here the increased trading volume of wheat *has* an effect on the equilibrium price; the increase in the number of sellers leads to a lower equilibrium price. For $\alpha$ small enough, however, the positive effect on quantities overcompensates the negative price effect on sellers' equilibrium utilities.

With sequential trade the increase in the number of buyers still affects the buyers positively (($R5$)). In contrast, result ($R8$) shows that sellers' equilibrium utilities cannot increase as their number increases. Again the increase in the number of sellers leads to increased bids of the buyers. While the aggregate equilibrium quantity of wheat supplied by the sellers increases, each individual seller's wheat supply decreases. So does the equilibrium price for wheat. These are the effects we usually should expect from a Cournot oligopoly model. Only if the sellers move second, competition may be advantageous for them (($R10$)). Thus, there are two points to be noted:

1. The introduction of sequential moves in strategic market game models makes it possible that the very intuition from Cournot oligopolies can be regained. An important difference between Cournot oligopolies and Shapley-Shubik strategic market games concerns the timing, not only that in Shapley-Shubik strategic market games all traders behave strategically.

2. It is important who moves first and who moves second. Traders still find it advantageous to be more numerous if they move second.

What accounts for the differences between results ($R8$) and ($R4$)? In the sequential moves framework, (cf.($R8$)), the sellers face a *given* reaction from the buyers. The buyer's strategy choice in stage two restricts the possible price-quantity combinations for the sellers. In my model the buyers' strategy choices play the restrictive role of the demand curve in Cournot oligopoly models and the relevant aspects for the sellers' revenues are the elasticities along that curve. For ($R4$) these restrictions are such that the individual wheat quantity cannot increase for a seller if the number of sellers increases.

In the simultaneous moves framework the buyers' strategy choices do not play this restrictive role. The simultaneity of moves may even cause buyers to adjust their strategy choices such that they are willing to buy a much larger quantity of wheat than before. If buyers buy a much larger quantity, sellers increase their individual wheat supplies as well, which finally might result in higher equilibrium utilities.

How general are these comparative statics results? In particular, how general is the result that first-movers never benefit from an increase in the number of traders on the own side of the market? For more general utility functions for the buyers, the setting with risk-neutral sellers moving first is very similar to a Cournot oligopoly. Seade [51] has shown for a Cournot oligopoly with a general inverse demand function that firms never benefit from more competition if firms' payoff functions are concave and if a certain equilibrium stability condition holds. It is not obvious if the same sufficient conditions are relevant here and how to transfer these insights to the sequential moves Shapley-Shubik game. While concavity could simply be imposed in the sequential Shapley-Shubik game, it is not clear how to formulate a dynamic that selects the subgame perfect equilibrium and how the stability condition would have to look like.

## 2.4 Stable Trading Structures

In their paper Bloch and Ghosal characterize stable trading structures. They show that the only strongly stable trading structure is the grand coalition where all agents trade on the same market. I shall argue that there might be other strongly stable structure if moves are sequential. To save on notation I keep the discussion informal.

Let me first introduce some notions from Bloch's and Ghosal's analysis. A *trading group* is a collection of traders of both types who agree to trade with one another. Agents can only trade inside their trading group. Different trading groups hence correspond to separate markets. A *trading structure* is a collection of trading groups on which agents trade. Bloch and Ghosal focus on trading structures where all traders participate to a market, no agents are excluded from trading. A trading structure is *strongly stable* if no coalition of traders can form a trading group in which all its members obtain at least as high a utility as before and some traders obtain a strictly higher utility.

Bloch and Ghosal show that the only strongly stable trading structure is the structure in which all agents trade on the same market (Proposition 3.2). The proof relies on two comparative statics results: Proposition 2.2 in their paper says that in symmetric markets, i.e. markets with the same number $N$ of buyers and sellers, all agents prefer $N$ to be large. Proposition 2.3 states that all agents prefer an increase in the number of agents on the other side of the market. Suppose now that $B = S = N$ and that $\alpha = 2, \beta = 1$.[3] Then, in the simultaneous moves version, Proposition 2.2 of Bloch and Ghosal [10] holds. Since Proposition 2.3 of their paper holds as well, the only strongly stable structure is the grand coalition where all agents trade on the same market. Note that this result does not depend on the values of $\alpha$, as long as $N$ exceeds 4. Consider now the sequential moves version. Sellers' equilibrium utilities are given by

$$\pi_s(b^*, s^*) = \frac{(N-2)^2}{(N-1)(N+1)^2}. \tag{2.28}$$

A quick computation shows that this function has a unique maximum at $N = N^* \approx 6.733$! Hence it is increasing for $N < N^*$ while it is decreasing for $N > N^*$ To understand this result, note that an increase in $N$ has two effects on a seller's equilibrium utility. First, as $N$ increases buyers bid less strategically and increase their bids which is beneficial for the sellers.

This is the usual effect in strategic market game models as the number of agents gets large. On the other hand, we have the effect known from Cournot oligopolies: an

[3] The values for $\alpha$ and $\beta$ are chosen for the sake of exposition. The finding of this section also holds for other values of $\alpha$ and $\beta$.

increase in the number of sellers lowers individual quantities sold and the equilibrium price. For $N$ small enough, the first effect dominates the second one. For $N$ large enough the negative effect dominates! Hence, independent from the effect of an increase in $N$ on the buyers' equilibrium utilities, sellers have an incentive of forming small trading groups. Proposition 2.2 from Bloch and Ghosal does not hold in the sequential model! The buyers' equilibrium utility is

$$U_s(b^*, s^*) = \frac{(N-2)(3N^4 - N^3 + 2N^2 + 2N - 2)}{2(N^2-1)^2(N+1)}, \tag{2.29}$$

which is strictly increasing in $N$. These findings have the following implication. Suppose that the overall number of agents is $kN^*, k \geq 2$. Then a trading structure with $k$ separate submarkets with $N^*$ buyers and sellers in each submarket might be strongly stable. The reason is as follows. In Bloch and Ghosal (Proposition 3.2.) any such trading structure is not stable since all traders would agree to form one single market. This holds since all traders prefer large symmetric markets to small symmetric marktes if traders move simultaneously. However sellers do prefer small symmetric markets if they move first and we cannot be sure if there are other strongly stable trading structures apart from the grand coalition. Any statement about the formation of trading groups in the sense of Bloch and Ghosal hence rests on the particular utility functions used if one considers a sequential moves model.

## 2.5 Elimination of the No-Trade Equilibrium and Walrasian Convergence

Suppose that

$$U_i(\underline{b}(\cdot), \underline{s}) = \sqrt{b_i(\underline{s}/p)} + (1 - b_i(\underline{s})) \tag{2.30}$$

and

$$\pi_j(\underline{b}, \underline{s}) = \sqrt{s_j p} + (1 - s_j). \tag{2.31}$$

Moreover, I set $B = S = N$ and label the game $G$.

If buyers and sellers move simultaneously, there are in fact *two* Nash equilibria. First, there is the no-trade equilibrium, that is, $\hat{b} = \hat{s} = 0$. Second, there is a Nash equilibrium with trade given by

$$\hat{b} = \hat{s} = \frac{(N-1)^2}{4N^2}. \tag{2.32}$$

The *Walrasian equilibrium* for the given economy, with $B = S = N$, is given by $x_i = 3/4, x_j = 1/4, y_i = 1/4, y_j = 3/4, p = 1$. From previous results (e.g. Shapley and Shubik [52]), we also know that the following holds.

**Result 4.** *As $N \to \infty$, the Nash equilibrium with trade coincides with the Walrasian equilibrium. Final allocations $x_i, y_i, x_j, y_j$ and equilibrium prices are the same.*

As Weyers [56] argues this result comes at the expense of some ad hoc assumption that rules out the pathological no trade equilibrium. Weyers [56] proposes a strategic market game with limit prices to solve this problem.[4] The set of Nash equilibria of this game converges to the set containing the competitive equilibria and no-trade, when players are replicated. Two-rounds of iterated deletion of weakly dominated strategies eliminate the no-trade equilibria. A sequential moves strategic market game proposes another, simpler solution to the problem of no-trade equilibria. In essence, traders face a coordination problem, where one equilibrium, the trade equilibrium pareto-dominates the no-trade equilibrium. It is well known that in a 2-player coordination game with sequential moves and perfect information the Pareto-dominant equilibrium emerges as the unique subgame perfect equilibrium. Hence only the trade-equilibrium emerges in my sequential strategic market game.

I now compute the unique subgame perfect Nash equilibrium with *sequential* moves and start in the second stage of the game. Given any choice $\overline{s}$ from the sellers, the buyers solve

$$\max_{b(\underline{s})} \quad \sqrt{b_i(\underline{s})/p} + 1 - b_i(\underline{s}). \tag{2.33}$$

Letting $\phi_i(\overline{s})$ denote a buyer's equilibrium strategy in each subgame, one obtains

$$\phi_i(\overline{s}) = \frac{N-1}{N^{3/2}}\sqrt{\overline{s}} \quad \forall i \in \mathcal{B}. \tag{2.34}$$

The sellers solve

$$\max_{s_j} \left( s_j \frac{N \cdot \phi_i(\overline{s})}{\sum_j s_j} \right)^{1/2} + 1 - s_j. \tag{2.35}$$

This gives

[4] The limit price mechanism of Mertens [40] is a mechanism for determining prices and final allocation when agents trade a finite number of goods by placing an arbitrary number of market or limit orders.

**Result 5.** *For the sequential moves game, with $B = S = N$, the unique equilibrium strategies are given by*

$$s^* = \left(\frac{N-1}{2N}\right)^{2/3} \left(\frac{2N-1}{4N}\right)^{4/3} \tag{2.36}$$

*and*

$$b^* = \left(\frac{N-1}{2N}\right)^{4/3} \left(\frac{2N-1}{4N}\right)^{2/3}. \tag{2.37}$$

*The equilibrium price is*

$$p = \left(\frac{2N-2}{2N-1}\right)^{2/3}. \tag{2.38}$$

The result states that no-trade is not an equilibrium of the sequential moves version of the strategic market game. Letting $N \to \infty$, gives the result of the following Proposition.

**Proposition 1.** *As $N \to \infty$, the final allocation and the equilibrium prices of sequential moves strategic market game coincide with the allocation and the equilibrium prices of the Walrasian equilibrium.*

Walras convergence has been previously established for simultaneous moves strategic market games.[5] Proposition 1 states that Walras convergence can be obtained for sequential moves strategic market games as well. The robustness of a convergence property with respect to a change in the timing has been noted by Robson [48]. He shows that the equilibria of a Stackelberg oligopoly model converge to the efficient equilibrium as the number of firms tends to infinity.

[5] See Mas-Colell [37] for an overview.

## 2.6 Existence of a Subgame Perfect Equilibrium

In this section, I discuss existence of a subgame perfect Nash equilibrium for a more general strategic market game model with sequential moves.

I modify the model and let the payoff functions be

$$U(\underline{b}(\cdot), \underline{s}) = f(b_i(\underline{s})/p) + (1 - b_i(\underline{s})) \tag{2.39}$$

and

$$\pi(\underline{b}, \underline{s}) = f(s_j \cdot p) + (1 - s_j). \tag{2.40}$$

We assume that the function $f$ is a strictly increasing and strictly concave function,

$$f'' < 0 < f' \tag{2.41}$$

over the whole domain. Moreover, $f$ is twice continuously differentiable.

Strategy sets are defined as in section 2. The following definition formalizes the solution concept for the game.

**Definition 1.** *Subgame perfect equilibrium. The profiles $(\underline{b}^*, \underline{s}^*)$ corresponds to a subgame perfect Nash equilibrium in pure strategies if, for all $i \in \mathcal{B}$ and for all $j \in \mathcal{S}$,*

$$(i) \quad s_j^* \in \underset{s'_j \in [0,1]}{\text{argmax}} \quad \pi_j(s'_j, s^*_{-j}, \underline{b}^*).$$

*and, for all subgames induced by $\underline{s}$,*

$$(i) \quad b_i^*(\cdot) \in \underset{b_i(\cdot)' \in [0,1]}{\text{argmax}} \quad U_i(b_i(\underline{s})', b^*_{-i}(\underline{s}), \underline{s}^*).$$

**Proposition 2.** *In the sequential strategic market game for $S \geq 1, B \geq 2$ there exists a symmetric subgame perfect Nash equilibrium, $s_1^* = ... = s_S^*$ and $b_1^* = ... = b_B^*$.*

*Proof.* See Appendix A. □

Proposition 2 asserts that there exists a subgame perfect Nash equilibrium in pure strategies - even for the case with one seller ($S = 1$). It is well known that Nash equilibria in simultaneous moves strategic market games do not exist for $S = 1$.

## 2.7 Conclusion

This chapter focusses on the implications of sequential moves on comparative statics results in strategic market games à la Shapley-Shubik. I have shown by means of an example that counterintuitive effects of more competition (i.e.more traders on the same side of the market) might vanish if the timing in such games is sequential. Moreover, results on the formation of trading groups and Walrasian convergence were established. I also provided an existence proof of a subgame perfect equilibrium.

Let us consider one last implication of sequential moves strategic market games. Shapley-Shubik games provide a modelling framework for collusion or cartel formation in markets where all agents act strategically. Suppose that we are in the simultaneous moves world of section 2.3 and that one side of the market, the sellers, say, aims to maximize the sellers' joint surplus. Suppose also that it is feasible to enforce implicit collusive behavior.

A quick computation shows that sellers maximize their joint revenue by setting $\overline{s} = 0$. Then the buyers' best response is to bid nothing (see equation (2.15)). Hence, implicit collusion based on joint surplus maximization is not profitable for the sellers! This changes if we are in a world with sequential moves. As shown in section 2.3.2, the situation with sequential moves is very similar to the one where Cournot oligopolists face a linear demand curve. And we do know that a strategy which maximizes joint revenue is available and profitable (i.e. provides higher utility than the utility with individual maximization) in this situation. Hence moving first provides sellers with a profitable strategy which maximizes joint revenue while this is not feasible with simultaneous moves.

Suppose further that moves are sequential but that cartel formation cannot be enforced. Then, standard infinitely repeated game arguments could be used to study self-enforcing implicit collusion. In particular, one could examine how the results by Green [29] and Lambson [36] extend to the case of strategic agents on both sides of the markets. Lambson, in particular, shows whether the folk theorem holds as the number of firms in an infinitely repeated Cournot oligopoly increases without bounds. He shows that the folk theorem holds in the limit iff demand increases at the same rate as the number of firms and if some bound on the Cournot price sequence exists. It could be interesting to discuss a similar question in a setting where all agents act strategically. In particular, what can be said about sustaining collusion as the ratio number of sellers/number of buyers varies? From the previous paragraph, it seems that such an analysis should be based on a model with sequential rather than simultaneous moves.

# Chapter 3

# The Effects of Competition on Bargaining Power in Repeated Bilateral Negotiations

## 3.1 Introduction

> " We have a saying at this company: our competitors are our friends and our customers are our enemies...God damn buyers [are the enemy]. We've gotta have them. But they are not my friends. You're my friend. I want to be closer to you than I am to any customer because you can make us money." (a former executive of Archer-Daniels-Midland (ADM), a lysine producer)[1]

[1]Lysine is a food additive designed to put additional meat on hogs. The quote was recorded by an FBI mole. Largely on the basis of the audio- and videotapes he provided, a federal jury in Chicago convicted three former ADM executives in September 1998 of engaging in an international conspiracy to fix lysine prices. See http://www.wwnorton.com/mip/ime/varian/28b.htm for more.

Many intermediate goods markets today are characterized by market power on both the seller and the buyer side. For example, markets for retailing tend to involve significant market power of retail chains. The power of such chains is a matter of serious policy concern (see Dobson and Waterson [17] and [18]). As a result both producers as sellers and retail chains as buyers exercise market power. The market structure is a bilateral oligopoly.

In many cases, prices in such markets are negotiated by bilateral negotiations. For example, most business-to-business goods are traded through bilateral contracts. Spot markets, organized as exchanges or auctions, are just the tiny tip of a huge market of such one-to-one deals (see The Economist [19]).

Economists' understanding of bilateral oligopoly is very incomplete. What are the prices negotiated, and how does the distribution of surplus depend on the number of buyers and the number of sellers? What is the role of agents' information and strategies? What are the effects of repeated transactions? Is there scope for implicit collusion in such a market as we know it from Cournot or Bertrand markets? The meaning of the quote above seems clear in the context of a Cournot market: competitors may be friends if they help to sustain collusive prices in a given market, for example, by reducing production quantities. In a bilateral oligopoly where prices are negotiated, matters are not all that obvious!

To see why consider the situation of a bilateral monopoly in which the seller makes all the offers. The seller has zero value for the good, the buyer has a commonly known value of one. The buyer can accept or reject an offer. The infinite repetition of the stage game does not preclude the buyer from extracting all the surplus.[2] If the

[2] Every trader maximizes the sum of his discounted per-period utilities from transactions.

buyer accepts any slightly unsatisfactory offer - even if this were better than not to buy at all - he gets "punished" by switching to the worst equilibrium in the rest of the relation. In a market where many transactions as this one take place in isolation, the presence of other buyers does not matter for any individual buyer: the argument based on the Folk Theorem does *not* rely on other buyers providing punishments as in a Cournot oligopoly model. It is then not clear why the above quote - with interchanged roles of buyers and sellers - should hold. I shall argue that "competitors are friends" for the buyers if there are outside options for the buyers in form of alternative trading partners!

This chapter presents a model of bilateral oligopoly and analyzes the effects of repeated interaction and competition on bargaining outcomes in a market with bilateral negotiations. Sellers make all offers and have value zero for the good, the buyers have a commonly known value of one. Every trader maximizes the sum of his discounted per-period utilities from transactions. I look for optimal equilibria from the buyers' perspective under the following three types of trading models: (1) a randomly matched pair of a seller and a buyer repeat their negotiations forever, (2) a buyer has an outside option to negotiate with an alternative seller who has no information about the buyers trading history and (3) a buyer can visit a new seller who is currently matched to another buyer, and if this happens, the two buyers compete for the seller. I show that the order of equilibrium payoffs of a buyer in the three trading models is $(1) > (3) > (2)$. While a buyer can exploit all the surplus in (1), an outside option to trade with a new seller with no information of trading history is - contrary to economic intuition - harmful to the buyer in (2). The presence of other buyers and buyers' competition in (3) limits the value of the outside option. The equilibrium sustained by competition can be interpreted as implicit buyer collusion since it is similar to price war equilibria sustaining implicit collusion in repeated oligopoly game models with centralized trading mechanisms.

Theories of markets with decentralized trade have mainly been developed in models with random matching and one-shot bargaining interaction (see Osborne and Rubinstein [44] for an excellent overview). These are mostly models with a one-shot-bargaining scenario, e.g. Diamond [16], Rubinstein and Wolinksy [49] and [50], Gale [25], Binmore and Herrero [9], Vincent [54] and, more recently, de Fraja and Sákovics [15] and Ponsati [45]. In particular, Rubinstein and Wolinsky [50] and Vincent [54] show how bargaining outcomes in markets depend on the amount of information agents use and how "punishment" mechanisms sustain certain equilibria. I examine the effectiveness of punishments and their dependence on information and strategic options for *repeated* bargaining transactions. I assume less information flow than Rubinstein and Wolinksy [50] (section 2 and section 4 in their paper) and Vincent [54] (section 2 in his paper).[3] For example, traders do not know other traders' identities and do not know the trading history of a new trading partner.

Just as Rubinstein and Wolinsky [50] or Vincent [54] for one-shot bargaining games, the literature on the folk theorem in repeated games (see Fudenberg and Tirole [24], ch.5) discusses how certain equilibria can be sustained by punishment strategies. Repeated games have also formed our understanding of implicit collusion sustained by the threat of competition in markets with centralized trading mechanisms (e.g. Abreu [3], Friedman [23] and Abreu, Pearce and Stacchetti [2]). This chapter shows how Abreu's and Friedman's ideas may be employed in a framework with decentralized

[3] see Muthoo [42], ch.10 for an overview on repeated bargaining with two players

trading mechanisms. I show how competition among buyers enlarges the surplus buyers can extract if other punishment mechanisms are of restricted use due to limited information flow.

The chapter is also related the literature studying market power in bargaining situations, e.g. Gul, Sonnenschein and Wilson [32]. Bargaining games under incomplete information can be interpreted as a market in which a monopoly seller makes sequential offers to buyers with different valuation. As in my paper, sellers make all the offers in these works. In some circumstances, buyers can extract all the surplus from the bargaining, that is, the Coase conjecture holds. Gul [31] and Ausubel and Deneckere [6] show that the presence of a second seller provides for better equilibria for the sellers since they can collude, supported by price wars in case of a deviation. This effect is similar to the one in my model. The difference is that the market studied in this chapter is a bilateral oligopoly rather than the durable goods oligopoly studied in those papers.

To explain the relation of the main arguments in the chapter, I shall provide an informal description of the model and its main results. I introduce the basic model in section 3.2. A seller has a commonly known value of zero for the good, a buyer has a commonly known value of one. In each one-shot transaction the seller makes an offer to the buyer who can accept or reject. The infinite repetition of this stage game does not preclude the buyer from extracting all the surplus. Threats to refuse any slightly unsatisfactory offer - even if this would be advantageous - are credible if acceptance induces a reputation for softness and implies worst equilibria in the rest of the relation.

In section 3.3 I show that outside options in form of alternative trading partners reduce the buyer's surplus. Outside options yield more benign equilibria than the worst equilibrium in the continuation of the current relationship. The argument presumes that the alternative trading partners have no information about a buyer's trading history. Hence, they cannot be integrated into a system of punishment strategies. I characterize the equilibria with the maximal surplus for the buyers by arguments similar to the optimal penal code of Abreu [1].

In section 3.4, I allow buyers to leave their trading partner and to compete for a seller who is currently matched to another buyer. I also allow each trading pair to observe the bargaining history of another buyer. Then, the presence of other buyers is useful since they can be integrated into a system of punishment strategies hence limiting the value of the outside option. One can interpret these equilibria as implicit buyer collusion since they are similar to equilibria sustaining implicit collusion in repeated oligopoly game models. The buyers' payoffs in an equilibrium sustained by competition exceed those in the best equilibrium without competition for any discount factor.

In section 3.5 I compare the model in more detail to the papers by Rubinstein and Wolinsky [50] and Vincent [54]. I conclude in section 3.6.

## 3.2 Powerful Buyers

There are two sets of traders, buyers and sellers. Each set contains two traders.[4] A typical seller will be denoted by $s$, a typical buyer will be denoted by $b$. Time is discrete and indexed by $t = 1, 2, ....$ At each point in time, each seller receives an endowment of one indivisible unit of some good. The good is homogenous, i.e. all sellers' endowments are perfect substitutes. The good is also perishable. It cannot be stored from period to

[4]The numbers are chosen for the sake of exposition

period. Sellers' valuation of their endowment is zero. Sellers derive utility from money. A seller who sells his unit in some period at price $p$ receives utility $p$ in that period. Buyers have no endowment of the good, but they have money. In each period, each buyer has exactly one unit of money which is nonstorable. Buyers are interested in buying at most one unit of the good. Every buyer's valuation of the good is equal to 1. A buyer who pays a price $p$ for the good receives in that period utility $1 - p$.

Both traders receive a period payoff of 0 if the seller's offer is rejected. A trader's overall payoff equals the expected present value of the sums of his discounted per-period payoffs. Sellers and buyers maximize the sums of discounted present values of their period utilities. Sellers' discount factor is $\delta_s \in (0, 1)$. Buyers' discount factor is $\delta_b \in (0, 1)$.

Initially all traders are in a matching pool. At time $t = 1$, the two buyers get matched to the two sellers. Each buyer is assigned to one seller, and no seller is matched to more than one buyer. I denote a relation as $(bs)$. In the relation $(bs)$, seller $s$ makes a take-it-or-leave-it offer to buyer $b$ by proposing a price $p_s(t) \in \mathbb{R}$ to $b$. The buyer may accept this price, $(Y)$, or reject the offer, $(N)$. Denote this action as $r_b(t) \in \{Y, N\}$. All this continues indefinitely. Each trader observes all the events in his relation. Strategies are functions from these histories to action sets.

This scenario might resemble a market for an intermediate good. Sellers are the upstream firms, buyers the downstream firms which buy in each period a fixed amount of the intermediate good. Since buyers have a positive valuation for the good, this might be a scenario where buyers have market power on the final good sector which allows them to make positive profits on any unit they buy from the input good. The model focusses on the interaction in the intermediate goods market this assuming that profits (and hence valuations) which buyers can obtain in the final market are not affected by whatever happens in the intermediate goods market.

**Proposition 3.** *For $\delta_b \geq 1/2$ there is a subgame perfect equilibrium such that in each relation the buyer and the seller trade for the price $p_s(t) = 0$.*

Proposition 3 says that in each period any buyer can extract all the surplus from the bargaining in a subgame perfect equilibrium. A buyer supports this equilibrium by building up a reputation for being a tough bargaining partner and by rejecting all prices above zero. Any soft behavior, as the acceptance of a higher price, leads to the harshest punishment possible (see Abreu [3], [1]). Buyers cannot avoid the punishment and, anticipating this, have an incentive to stay tough for $\delta_b \geq 1/2$. Each seller recognizes his partner as a tough bargaining partner and does not demand a price higher than zero.

## 3.3 Disadvantageous Outside Options

Suppose there is a third seller. We shall see how the presence of that additional trading partner reduces a buyer's ability to extract surplus from the bargaining given the amount of information sellers use. Outside options as introduced in this section create a difficulty for the buyers which competition among buyers resolves in the next section, section 3.4.

**Remark 1.** *I shall comment on the assumption with three sellers and two buyers below. The numbers are chosen for the sake of exposition and the results do not depend upon them.*

The sequence of events is as follows: in $t = 1$, there is random matching. Each buyer is assigned to one seller, and no seller is matched to more than one buyer. One seller remains unmatched and receives a per-period payoff of zero. In each relation $(bs)$ the seller $s$ proposes a price which buyer $b$ can accept or reject. Then, $b$ has two options which I denote $\alpha_b(t) \in \{\text{stay, leave}\}$:

(*i*) $\alpha_b(t) = \text{stay}$: Stay with seller $s$. In that case their relation continues, in $t = 2$ seller $s$ makes a proposal which $b$ can accept or reject and so on

(*ii*) $\alpha_b(t) = \text{leave}$: Leave seller $s$. Then buyer $b$ and seller $s$ return to the matching pool where they wait to get matched anew in the next period.

In time $t = 2$, there is again random matching. Any unmatched buyer gets matched. If there is no unmatched buyer, no new match is formed. Then the same bargaining game as before is played: each seller proposes a price to the buyer with whom he is matched. The buyer can accept or reject this price and so forth. All this continues indefinitely.

In each period in which he has a partner each trader only observes the price proposed, the response to this price, and the decision of the buyer whether or not to separate. Traders do not observe what happens in matches in which they are not involved. Traders do not observe the identities of their trading partners.[5] Summarized, I impose

**Assumption 1.** *All agents' strategies can only be conditioned on the history of play*

[5]This assumption might seem unrealistic in a setting with 5 traders. But recall that I chose the small number for the sake of exposition. The assumption seems quite realistic in a market with more traders.

*with the current partner.*

To formalize this assumption, let $b(t)$ denote the buyer with whom a given seller is matched in period $t$ and define $s(t)$ similarly. A seller's private bargaining history is

$$H_s(t) = \{p_s(1), r_{b(1)}(1), \alpha_{b(1)}(1), ..., p_s(t-1), r_{b(t-1)}(t-1), \alpha_{b(t-1)}(t-1)\} \quad (3.1)$$

and a buyer's private bargaining history is

$$H_b(t) = \{p_{s(1)}(1), r_b(1), \alpha_b(1), ..., p_s(t-1), r_b(t-1), \alpha_b(t-1)\}. \quad (3.2)$$

Buyer $b$'s acceptance decision is given by

$$r_b(t) : H_b(t) \times \mathbb{R} \longrightarrow \{Y, N\} \quad (3.3)$$

and the outside option strategy for $b$ is given by

$$\alpha_b(t) : H_b(t) \times \mathbb{R} \times \{Y, N\} \longrightarrow \{\text{leave}, \text{stay}\}. \quad (3.4)$$

A buyer's strategy set is given by

$$\Sigma_b = \{r_b(t) : H_b(t) \times \mathbb{R} \to \{Y, N\}\} \times \{\alpha_b(t) : H_b(t) \times \mathbb{R} \times \{Y, N\} \to \{\text{leave}, \text{stay}\}\}. \quad (3.5)$$

The formal description of a strategy for the seller is given by

$$p_s(t) : H_s(t) \longrightarrow \mathbb{R}. \quad (3.6)$$

I impose a *stationarity assumption* on the sellers' strategies: the seller employs the same strategies in each buyer-seller relation no matter when the relation starts.

**Assumption 2.** *Each seller employs the same strategy in each relation - independent from the time $t$ in which the relation starts.*

A seller "has no watch" and does not distinguish between a relation that starts in period 1 and a relation that starts in, say, period 10. In other words, any seller who starts a new relation is not able to reconstruct his new partner's bargaining history just from the number of the period that relation starts.

Assumption 2 implies that traders cannot condition their behavior on the bargaining history of a new trading partner, similar to the papers by Ghosh and Ray [26] and Datta [14].[6] In those papers the sets of agents are large and there is either the possibility that a match breaks down for exogenous reasons or that new agents constantly flow in the matching pool. One or both of these modelling assumptions guarantees in those models that it is impossible for any agent in the pool to know if a new partner is new in the market or if he broke up an old relation. Assumption 2 restricts strategies in the same way: a seller cannot condition his behavior on the fact that a new trading partner broke up an old relation. I chose to work with the stationarity assumption rather than to work with more agents to simplify the exposition. All my results extend to the case of a large number of buyers and sellers and constant flow of new traders in the matching pool.

Denote the set of strategies for a seller as

$$\Sigma_s = \{p_s(t) : H_s(t) \longrightarrow \mathbb{R}\} \tag{3.7}$$

As usual in the study of extensive games I want to use an equilibrium concept that is stronger than Nash equilibrium and embodies sequential rationality. Given that the game has no subgames and that sellers' strategies are continuous variables the usual definitions of subgame perfect equilibrium or sequential equilibrium do not apply.

[6] These are papers on sustaining cooperation in Prisoner's Dilemma situations when agents are randomly matched and when there is limited information flow.

Even so, the notion that no player wants to deviate from the equilibrium strategy at any point in time given that all other traders stick to their equilibrium strategy is well defined. This is because assumption 1 makes it possible to treat behavior in any one period *as if* the pair played a genuine subgame. I do not give a formal definition of this notion of sequential rationality.

I impose further restrictions on the strategy profiles and consider strategy profiles which are *characterized* by $(i)$ a sequence of *target prices* $\{p^E(t)\}_{t=1}^{\infty}$ which the buyers would like to sustain and $(ii)$ a sequence of *punishment prices* $\{p^D(t)\}_{t=\tau+1}^{\infty}$ indicating the continuation following the acceptance of prices *higher than the target price* in any period $\tau$. This restriction excludes, for example, strategies which "reward" a buyer for not accepting prices higher than the target price.

Given two sequences $\{p^E(t)\}_{t=1}^{\infty}, \{p^D(t)\}_{t=\tau+1}^{\infty}$ the *associated strategies* are specified as followed.

- in the first period of any relation, the seller proposes $p^E(1)$ and the buyer accepts any $p \leq p^E(1)$ and rejects any $p > p^E(1)$
- regardless of the history, the buyer never leaves
- if in all periods $t' < t$ the buyer has rejected any offer $p > p^E(t)$ or if no such offer has been made, the seller proposes $p^E(t)$ and the buyer accepts any $p \leq p^E(t)$ and rejects any $p > p^E(t)$ for all $t > t'$
- if in periods $\tau < t$ the buyer has accepted any offer $p > p^E(\tau)$, then the seller proposes $p^D(t)$ from period $\tau + 1$ on, which the buyer accepts, for all $t > \tau$.

Strategies for all buyers and all sellers which generate the two sequences

$\{p^E(t)\}_{t=1}^{\infty}, \{p^D(t)\}_{t=\tau+1}^{\infty}$ are referred to as an **equilibrium** if for any $t$ and any history up to $t$ it is sequentially rational for any trader to follow his prescribed strategy given that all other traders follow their strategy.

**Proposition 4.** *In an equilibrium with $p^E(t) := p^E$ for all $t = 1, 2...$, we must have $p^E(t) = 1$ for all $t$ and for all matched pairs.*

*Proof.* Suppose there is an equilibrium with price sequences $p^E(t) \in [0, 1)$ satisfying $p^E(t) = p^E(t+1) := p^E$ for all $t = 1, 2....$ Suppose that the seller proposes a price $p, p^E < p < 1$ to a buyer. If $p^E$ is an equilibrium the buyer has to reject $p$. If the buyer follows this prescription, his payoff is

$$0 + \frac{\delta_b}{1-\delta_b}(1-p^E). \tag{3.8}$$

If the buyer accepts $p$ his payoff is

$$(1-p) + \delta_b U^P, \tag{3.9}$$

where $U^P$ denotes maximum continuation payoff a buyer can achieve after a deviation given that he gets punished for his deviation. From assumptions 1 and 2, the punishment sequence has to be $p^D(t) = p^E$ in any equilibrium and hence

$$U^P = \frac{(1-p^E)}{1-\delta_b}. \tag{3.10}$$

Then accepting $p$ is strictly profitable. This argument holds for all $p^E \in [0, 1)$. □

Proposition 4 asserts that stationary target prices combined with an outside option in form of new trading partners cause a complete loss of the buyers' bargaining power. The market provides buyers with no incentives to stay tough in bargaining: buyers can always leave their partner if they face a punishment from the next period on. The new trading partner does not know the deviant buyer's trading history and allows the buyer to extract surplus by trading to the same price as if no deviation had occurred. But then there is no incentive for the buyer to build up a reputation for being tough in his first relation and he is willing to accept higher prices. Sellers anticipate this behavior and propose a price of one right away.

For non-stationary target prices buyers can extract surplus from the bargaining.

**Proposition 5.** *If $\delta_b \in (1/2, 1)$ there is an equilibrium which generates the following decreasing sequence of prices: $p^E(1) = 2$ and $p^E(t) = 0$ for all $t = 2, 3, ....$ The equilibrium is sustained by $\{p^D(t)\}_{t=\tau+1}^{\infty} = \{p^E(t)\}_{t=1}^{\infty}$ if a buyer accepts an offer $p(\tau) > p^E(\tau)$ in period $\tau$. This equilibrium yields strictly positive utilities for all buyers.*

*Proof.* I propose the following equilibrium strategies:

- Let $p^E(1) = 2$ and $p^E(t) = 0$ for all $t = 2, 3, ....$

- For any match $(bs)$: $s$ proposes $p^E(t)$, $b$ accepts all $p(t) \leq p^E(t)$ and rejects all $p(t) > p^E(t)$. Moreover, $b$ never leaves $s$.

- If $b$ accepts any $p(\tau) > p^E(\tau)$ in period $\tau$, restart the price sequence, that is, $p^D(\tau + 1) = p^E(1) = 2$ and $p^D(\tau + k) = 0$ for all $k = 2, 3, ...$

The only interesting point is when $s$ proposed $p(t) > p^E(t)$ for some $t \geq 2$. The incentive constraint for $b$ reads

$$0 + \frac{\delta_b}{1 - \delta_b} \geq (1 - p(t)) + \delta_b \left( -1 + \frac{\delta_b}{1 - \delta_b} \right) \tag{3.11}$$

for all $p(t), t \geq 2$. Since this incentive constraint has to hold for all $p(t)$, it has to hold for $p(t) = 0$. Rearranging terms for $p(t) = 0$ yields

$$\delta_b \geq \frac{1}{2} \tag{3.12}$$

It is straightforward to check that all other equilibrium conditions are satisfied as well. In particular, it is not profitable for a buyer to leave his current trading partner and try to escape from his punishment. Buyers' equilibrium utilities are strictly positive. □

Proposition 5 demonstrates how a decreasing target price sequence yields a strictly positive utility level for a buyer. The market provides buyers with incentives to stay tough in bargaining if it is costly to build up a new reputation for being tough. A high initial price keeps a buyer from leaving his current partner since any deviation triggers a restart of the decreasing price sequence with the same trading partner. If the buyer takes his outside option, the new partnership starts with the high first period price as well and hence taking the outside option is not a profitable deviation.

Next, I look for *optimal equilibria* from the perspective of the buyer. An equilibrium is optimal if it yields the highest payoffs for the buyer for a *given* discount factor $\delta_b$. This does not follow the Folk-Theorem approach which gives results on feasible payoffs for discount factors "sufficiently large".[7] Rather, I suppose that traders are not absolutely patient.[8] Denote an optimal equilibrium (target) price sequence by $\{p^*(t)\}_{t=1}^{\infty}$ and the equilibrium utility from $\{p^*(t)\}_{t=1}^{\infty}$ as $U(p^*)$. I omit the dependence of the optimal target price sequence on $\delta_b$ and write simply $\{p^*(t)\}_{t=1}^{\infty}$. As the previous proposition suggests an optimal equilibrium target price sequence is decreasing and is

[7] see Fudenberg and Tirole [24], ch. 5, for a discussion of various versions of the Folk-Theorem for perfect and imperfect information.

[8] see Abreu [3], [1] for a seminal analysis in this spirit.

supported by the threat of restarting the sequence in case of a deviation.

**Proposition 6.** *For all $\delta_b \in (1/2, 1)$, the target price sequence*

$$p^*(1) = \frac{1}{\delta_b}, p^*(2) = p^*(3) = ... = 0 \tag{3.13}$$

*is optimal and yields*

$$U(p^*) = \left(\frac{2\delta_b - 1}{1 - \delta_b}\right)\frac{1}{\delta_b}. \tag{3.14}$$

*Proof.* See Appendix B. □

**Remark 2.** *The next section introduces information leakage: after the buyer's acceptance decision each trading pair $(bs)$ receives information about the bargaining history of buyer $b' \neq b$. Note that all results of this section continue to hold even under this additional assumption. Assumption 2 implies that in a new relation sellers would not use any of this information, even if it were obtained in a previous match.*

## 3.4 The Effect of Buyer Competition

This section introduces an additional action for a buyer: he can leave his trading partner and visit the other seller who is currently trading. We shall see how this assumption combined with some information flow implies more severe punishments for a deviant buyer. Then, for any discount factor, buyers can extract more surplus than in the best equilibrium of the previous section (Proposition 6). This section thus highlights the role of competition in a market with bilateral negotiations: similar to markets with centralized trading systems, competition may be used as an effective punishment system to increase market power of one side of the market. This works in particular if other punishment mechanisms are useless due to limited information flow, as shown in the previous section. Hence, in this setting with bilateral negotiations buyer competition mitigates the negative effects' from seller competition. In Cournot or Bertrand oligopoly models, competition mitigates negative effects from competition on the same side of the market.

The sequence of events is as follows. Again, at time $t = 1$, the two buyers get matched to two sellers, one seller remains unmatched. Each buyer is assigned to one seller, and no seller is matched to more than one buyer. In a match between two agents, say $(bs)$, seller $s$ makes a take-it-or-leave-it offer to buyer $b$ by proposing a price $p_s(t) \in \mathbb{R}$ to $b$. The buyer may accept this price, $(Y)$, or reject the offer, $(N)$. Then the pair $(bs)$ receives information about the bargaining history of buyer $b' \neq b$. I call this event *information leakage* and shall describe it in detail below. Both seller $s$ and buyer $b$ receive the same information. Then, $b$ has three options which I denote with $\alpha_b(t) \in \{\text{stay, leave, b'}\}$:

$(i)$ $\alpha_b(t) = \text{stay}$: stay with $s$

$(ii)$ $\alpha_b(t) = \text{leave}$: leave $s$ and return to the pool to get matched anew

$(iii)$ $\alpha_b(t) = \text{b'}$: visit the seller who is matched to buyer $b'$.

When $b$ stays, $(bs)$ play the same bargaining game again. When $b$ leaves, that is, if $\alpha_b(t) = \text{leave}$, then both $b$ and $s$ return to the matching pool. I shall explain in detail below what happens if $\alpha_b(t) = \text{b'}$. All this continues indefinitely.

The sequence of events is depicted in figure 3.1.

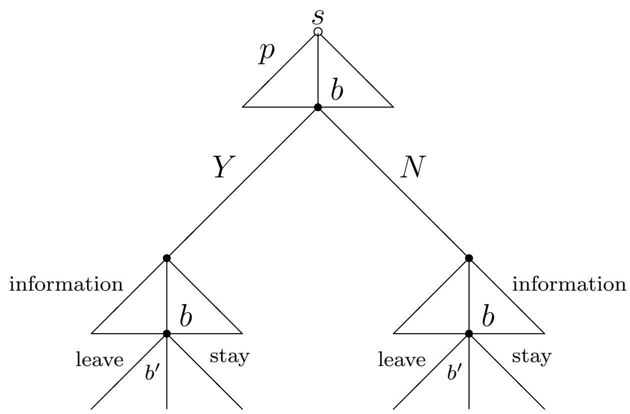


Figure 3.1: Sequence of Events

**Information Leakage:**

Denote by $\mathcal{H}_b(t)$ the set of *trading histories* for any buyer $b$ up to period $t$ with $h_b(t) \in \mathcal{H}_b(t)$. An element $h_b(t) = (\{p_{s(\tau)}(\tau)\}, \{r_b(\tau)\}, \{\alpha_b(\tau)\})_{\tau=1}^t$ of $\mathcal{H}_b(t)$ is a sequence of price offers, possibly from various sellers, $\{p_{s(\tau)}(\tau)\}_{\tau=1}^t$, a sequence of action choices $\{r_b(\tau)\}_{\tau=1}^t$,$r_b(\tau) \in \{Y, N\}$, and a sequence of action choices $\{\alpha_b(\tau)\}_{\tau=1}^t$ , $\alpha_b(\tau) \in \{$stay, leave, b'$\}$,for buyer $b$.

In each period each matched pair $(bs)$ observes the full experienced trading history from period 1 to $t$ of the other buyer, that is $(bs)$ observes that element $h_{b'}(t) \in \mathcal{H}_{b'}(t)$ which actually occurred. An observation in period $t$ includes price offers and responses from the same period $t$. This is possible since information leakage occurs after the trading stage. The main result does not depend on this assumption which is made for simplicity. Denote the information which $(bs)$ receive as $I_{(bs)}(t)$.

Only matched trading pairs receive information. The seller who is not matched receives no information and any seller who returns to the pool forgets any previously obtained information about any buyer's bargaining history.

**Visiting other Sellers:**

Suppose $b$ chooses $\alpha_b(t) = b'$ and that $b'$ currently trades with $s'$. Then the situation is as follows. Seller $s'$ proposes a price to buyers $b'$ which $b'$ accepts or rejects. If buyer $b'$ accepts then buyer $b$ returns to the pool and both $b'$ and $s'$ learn the full experienced trading history of buyer $b$. Then, $b'$ has his three options: leave, stay or choose $b$, and so on, that is, $(b's')$ continue their relation.

If buyer $b'$ rejects, he has to return to the pool and seller $s'$ and buyer $b$ start a new relation in which $s'$ makes a proposal to $b$. Buyer $b$ can reject or accept this proposal. If $b$ rejects or accepts the proposal of $s'$, the relation between $b$ and $s'$ continues with the information leakage stage and $b$ and $s'$ learn the full trading history of buyer $b'$. Then, $b$ has his three options: leave, stay or choose $b'$, and so on.

All this is common knowledge among $b', s'$ and $b$. Note that there is always some seller with whom buyer $b'$ is matched: even if $b'$ has left his partner in the last period, he is matched again with probability 1. Figure 3.2 depicts the situation.

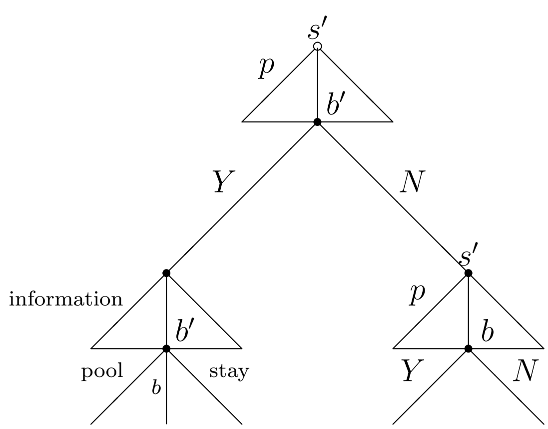


Figure 3.2: Competing for Sellers

**Strategies and Information:**

Apart from information leakage, traders do not observe what happens in matches in which they are not involved. For example, while $b$ and $s$ are matched, $s$ learns nothing about the trading history of previous matches of buyer $b$. As already noted traders in any given relation do not use any information obtained through information leakage in any previous match. However, $b$ and $s$ learn each period the trading history of buyer $b'$. In each period in which he has a partner each agent only observes the price proposed, the response to this price, the information transmitted by information leakage and the decision of the buyer whether to return to the pool, to look for a new trading partner or to stay with his current partner. Traders do not observe the identities of their trading partners.

Let $b(t)$ denote the buyer with whom a given seller is matched in period $t$ and define $s(t)$ similarly. A seller's private bargaining history is

$$H_s(t) = \{p_s(1), r_{b(1)}(1), \alpha_{b(1)}(1), I_{b(1)s}(t), ..., p_s(t-1), r_{b(t-1)}(t-1), \alpha_{b(t-1)}(t-1), I_{b(t-1)s}(t-1)\} \tag{3.15}$$

and a buyer's private bargaining history is

$$H_b(t) = \{p_{s(1)}(1), r_b(1), \alpha_b(1), I_{bs(1)}(1), ..., p_s(t-1), r_b(t-1), \alpha_b(t-1), I_{bs(t-1)}(t-1)\}. \tag{3.16}$$

Buyer $b$'s acceptance decision is given by

$$r_b(t) : H_b(t) \times \mathbb{R} \longrightarrow \{Y, N\} \tag{3.17}$$

and the outside option strategy for $b$ is given by

$$\alpha_b(t) : H_b(t) \times \mathbb{R} \times \{Y, N\} \longrightarrow \{b', \mathrm{pool}, \mathrm{stay}\}. \tag{3.18}$$

A buyer's strategy set is given by

$$\Sigma_b = \{r_b(t) : H_b(t) \times \mathbb{R} \to \{Y, N\}\} \times \{\alpha_b(t) : H_b(t) \times \mathbb{R} \times \{Y, N\} \to \{b', \text{pool}, \text{stay}\}\}. \tag{3.19}$$

The formal description of a strategy for the seller is given by

$$p_s(t) : H_s(t) \longrightarrow \mathbb{R}. \tag{3.20}$$

Again, I impose the *stationarity assumption 2* on the sellers' strategies. Denote the set of strategies for a seller as

$$\Sigma_s = \{p_s(t) : H_s(t) \longrightarrow \mathbb{R}\} \tag{3.21}$$

**The Effect of Buyer Competition:**

I shall now explain how buyer competition affects the bargaining outcome. Suppose that buyers want to sustain a target price sequence $\{p^E(t)\}_{t=1}^{\infty} := \{p^{\circ}(t)\}_{t=1}^{\infty}$. A deviation occurs if a seller proposes a price $p(t) > p^{\circ}(t)$ which is accepted by his trading partner. If buyer $b$ deviates, I construct a punishment such that buyer $b$ has to pay the high period 1 price for all future periods to his current trading partner, say $s$, whom he does not leave. If he should try to avoid the punishment by returning to the matching pool, the *other* buyer $b'$ visits any new trading partner of buyer $b$ in any future period. This induces competition among the two buyers. Competition allows any seller to propose a price of 1 to $b$ and to start a new relation with $b'$ if $b$ should reject the proposal. The deviant buyer $b$ rejects this price of 1 and misses the trading opportunity and looks for a new partner each period. The threat of competition prevents $b$ from looking for a new, unmatched trading partner and $b$ accepts willingly his punishment from $s$. Since the punishment constructed through competition is more severe than the most severe punishment in section 3.3, buyers can extract a higher surplus for any discount factor.

**Lemma 1.** *For a decreasing target price sequence* $\{p^E(t)\}_{t=1}^{\infty} := \{p^{\circ}(t)\}_{t=1}^{\infty}$ *there exist equilibrium strategies such that* $\{p^{\circ}(t)\}_{t=1}^{\infty}$ *is supported in an equilibrium through* $\{p^D(t)\}_{t=\tau+1}^{\infty} = p^{\circ}(1)$*, for* $\delta_b$ *large enough.*

*Proof.* Each equilibrium strategy is described as a collection of states and rules of transition between them. Most of the states are defined for any trading pair $(bs)$. One state has to be defined for the situation when a buyer, say $b'$, competes for the seller $s$ who is matched with buyer $b$. The equilibrium strategies and the states are described in Table 3.1. Transition rules are given in Table 3.2.

The state $E_{(bs)}$ dictates traders' behavior for any $(bs)$ as long as $b$ does not deviate. The state $P_{(bs)}$ dictates traders' behavior for any $(bs)$ if $b$ did deviate. The state $A_{(bs)}$ is an intermediate state in which play only rests for one period. It triggers competition among buyers: play switches to this state only if $b'$ is in a punishment phase (state $P_{(b's')}$) and chooses "leave". Play leaves state $A_{(bs)}$ in any case after one period. The state $B_{(bsb')}$ denotes the situation in which buyer $b$ has deviated, left his previous trading partner, is matched to seller $s$ and where buyer $b'$ competes for seller $s$. All states are defined for any buyer $b$, any seller $s$, any pair $(bs)$ and for any triple $(bsb')$.

Not all transitions from the state $B_{(bsb')}$ are described since there are numerous possibilities.

Two points are critical for sequential rationality and must be dealt with:

| Strategies/States | $E_{(bs)}$ | $P_{(bs)}$ | $A_{(bs)}$ | $B_{(bsb')}$ |
|---|---|---|---|---|
| $s$ proposes | $p^{\circ}(t)$ | $p^{\circ}(1)$ | - | $p = 1$ to $b$, $p^{\circ}(1)$ to $b'$ |
| $b$ accepts | $p \leq p^{\circ}(t)$ | $p \leq p^{\circ}(1)$ | - | nothing |
| $b$'s action $\alpha_b(t)$ | stay | stay | $b'$ | - |

Table 3.1: Equilibrium strategies

| switch from | to | if |
|---|---|---|
| $E_{(bs)}$ | $P_{(bs)}$ | $b$ accepts $p(t) > p^{\circ}(t)$ |
| $E_{(bs)}$ | $A_{(bs)}$ | if $b'$ is in a punishment phase (state $P_{(b's')}$) and chooses "leave" |
| $E_{(bs)}$ | $E_{(bs)}$ | otherwise |
| $P_{(bs)}$ | $P_{(bs)}$ | forever (absorbing) |
| $A_{(bs)}$ | "restart" $E_{(bs)}$ | If $b$ is supposed to compete for the seller of $b'$ but did not leave his current partner $s$. |
| $A_{(bs)}$ | $B_{(b's'b)}$ | otherwise |
| $B_{(b's'b)}$ | $E_{(b's')}$ | if $b'$ accepts the first proposal from $s'$ |
| $B_{(b's'b)}$ | $E_{(bs')}$ | otherwise |

Table 3.2: Transition Rules

$(i)$ Buyer $b$, after having defected, tries to avoid his punishment by running away from seller $s$.

$(ii)$ Buyer $b'$, having observed the deviation of buyer $b$ does not want to participate in the punishment of buyer $b$.

The above described strategies circumvent these problems:

($i$) The prescribed strategies are sequentially rational. If $b$ tries to avoid his punishment he finds a new partner and trades for the price of $p^{\circ}(1)$ for one period with, say, seller $s'$. But since his decision to go back to the pool is noticed by buyer $b'$, buyer $b'$ competes then for the new seller of $b$ for the next period and is willing to start a new relationship. Knowing this, seller $s'$ demands a price of one from $b$. This causes $b$ to take his outside option and to leave $s'$ the period before (after trade with $s$ has occurred at the price $p^{\circ}(1)$). But this happens in all future periods: $b$ always has to pay the price $p^{\circ}(1)$ if he leaves $s$; buyer $b'$ does not allow him to get to low price periods. Since according to the proposed equilibrium strategies $b$ has to pay $p^{\circ}(1)$ in his relationship with $s$, a deviation, that is, leaving $s$, is not profitable for $\delta_b$ sufficiently large.

($ii$) To make sure that buyer $b'$ does actually play his role in the punishment of defector $b$, I require that the equilibrium price sequence $p^{\circ}(t)$ gets restarted in the relation $(b's')$ whenever $b'$ does *not* follow his prescribed strategy. If $b'$ leaves his partner and competes for the new seller $s$ of $b$, he will be the new trading partner of $s$ since seller $s$ makes his "old" buyer $b$ a price offer of one which this buyer rejects. Since $b'$ will be the new trading partner of $s$ and since this is a new relation $(b's)$ start with the high first period price and lower prices follow. According to the equilibrium strategies it is not a profitable deviation for $b'$ to stay with his old partner. It is also not profitable for $b'$ to return to the matching pool since there he has to start a new relationship as well with a decreasing price sequence.

It is easy to check that the proposed strategies satisfy all other equilibrium conditions, too. $\square$

**Proposition 7.** *There exists a target price sequence* $\{p^\circ(t)\}_{t=1}^{\infty}$ *such that*

$$U(p^\circ(t)) > U^*(p^*(t)) \quad \textit{for all} \quad \delta_b \in (0,1)$$

*in equilibrium.*

*Proof.* Let $p^\circ(1) > 0$ and $p^\circ(t) = 0$ for all $t = 2, 3, ....$ Fix $\delta_b$. I determine $p^\circ(1)$. By Lemma 1, the incentive constraint for a buyer if a seller proposes a price $p(t) > p^\circ(t)$ in excess of $p^\circ(t)$ is given by

$$0 + \frac{\delta_b}{1-\delta_b} \geq (1 - p(t)) + \frac{\delta_b(1 - p^\circ(1))}{1-\delta_b} \quad \text{for all } p(t) \tag{3.22}$$

for period 1. Since this constraint has to hold for all $p(t)$ it certainly has to hold for $p(t) = p^\circ(t)$. Solving for $p^\circ(1)$, I obtain $p^\circ(1) \geq (1-\delta_b)/\delta_b$. The incentive constraints for periods $t = 2, 3, ...$ are given by

$$0 + \frac{\delta_b}{1-\delta_b} \geq (1 - p(t)) + \frac{\delta_b(1 - p^\circ(1))}{1-\delta_b}. \tag{3.23}$$

for all $p(t)$. In these cases, I set $p(t) = 0$. Solving then for $p^\circ(1)$, I obtain $p^\circ(1) \geq (1-\delta_b)$. To satisfy both incentive constraints, $p^\circ(1) \geq (1-\delta_b)/\delta_b$ has to hold. Since I want to determine the maximal utility buyers can achieve, I let $p^\circ(1) = (1-\delta_b)/\delta_b$.

If $p^\circ(1) = (1-\delta_b)/\delta_b$, then

$$U(p^\circ(t)) = \frac{(2\delta_b - 1)(1-\delta_b) + \delta_b^2}{\delta_b(1-\delta_b)} > \frac{2\delta_b - 1}{\delta_b(1-\delta_b)} = U^*(p^*(t))$$

for all $\delta_b \in (0,1)$. □

**Proposition 8.** *Maximum utility for buyers in a collusive equilibrium is given by*

$$U(p^{\circ}(t)) = \frac{(2\delta_b - 1)(1-\delta_b) + \delta_b^2}{\delta_b(1-\delta_b)}.$$

*The optimal equilibrium price sequence that implements this utility level is given by*

$$p^{\circ}(1) = \frac{1-\delta_b}{\delta_b}, p^{\circ}(2) = p^{\circ}(3) = ... = 0.$$

*Proof.* I only need to show that there exists no other equilibrium price sequence $\{p^a(t)\}_{t=1}^{\infty}$ which yields buyers a higher utility given the available punishments. So suppose that $\{p^{\circ}(t)\}_{t=1}^{\infty}$ is an equilibrium price sequence and that there exists another equilibrium price sequence $\{p^a(t)\}_{t=1}^{\infty}$ with $p^a(1) < p^{\circ}(1)$ and $p^a(t) > 0$ for at least one $t \geq 2$. Suppose that $U(p^a(t)) > U^{\circ}(p^{\circ}(t))$.

Since $\{p^{\circ}(t)\}_{t=1}^{\infty}$ is an equilibrium price sequence, by Lemma 1 the following incentive constraint holds for the buyers in period $t = 1$ if a seller proposes a price higher than $p^{\circ}(1)$ :

$$0 + \frac{\delta_b}{1-\delta_b} \geq (1 - p^{\circ}(1)) + \delta_b \left[\frac{1}{1-\delta_b}\left(1 - \frac{1-\delta_b}{\delta_b}\right)\right]. \tag{3.24}$$

Choose $p^{\circ}(1)$ such that this incentive constraint holds with equality.

The appropriate incentive constraint in $t = 1$ for the alternative equilibrium price sequence $\{p^a(t)\}_{t=1}^{\infty}$ is

$$0 + \delta_b U_c^a \geq (1 - p^a(1)) + \delta_b \left[\frac{1}{1-\delta_b}\left(1 - \frac{1-\delta_b}{\delta_b}\right)\right], \tag{3.25}$$

where $U_c^a$ is the equilibrium continuation payoff from period 2 on for the equilibrium price sequence $\{p^a(t)\}_{t=1}^{\infty}$. But if $p^a(1) < p^{\circ}(1)$, the last constraint cannot hold:

since equation (3.24) holds with equality, the LHS (RHS) of (3.25) is smaller (larger) than the LHS of (3.24). Hence the price sequence $\{p^a(t)\}_{t=1}^{\infty}$ cannot be an equilibrium price sequence.

The optimal equilibrium price sequence has a positive first period price and is zero thereafter. For the closed form solution for $p^{\circ}(1)$, consult the proof of Proposition 7. $\square$

Let us discuss the role of the various assumptions play for the results of this paper: in section 3, the assumption that traders do not use any information about a new trading partner's history and the assumption that buyers can look for alternative trading partners reduce the surplus a buyer is able to extract - as compared to the situation of a bilateral monopoly.

In the present section, *both* information of, say, $(bs)$ about the history of $b'$ *and* the strategic option which creates competition among the buyers offset the negative effect which the outside option induces. It is important to note that without the possibility of competition all results from section 3 continue to hold *even under the assumption that traders can observe events in other relations*! This comes from the stationarity assumption for sellers' strategies, assumption 2. If traders had only information about the other buyer's history, a buyer still could avoid a harsh punishment by running away and looking for a new, unmatched trading partner who has no information about the buyer's history. The start of a new relation of the deviant buyer would be observed, but not be punished!

It is hence the additional strategic option for competition which increases a buyer's surplus in section 4. While the strategic option to look for a new trading partner reduces the buyers' surplus, the option to look for a new trading partner who is currently matched - competition - increases the surplus again.

While all results from sections 2 and 3 can be obtained for one buyer, the result of section 4 relies on the presence of more than one buyer! To understand more the role of the presence of another buyer in providing punishments, it is helpful to compare the present work to the literature on norm enforcement in games with large populations (see Kandori [34] and Ellison [21]). This literature focuses on the enforcement of norms in situations of common interest among all agents, for example cooperation in a Prisoner's Dilemma. In common interest situations, *all* agents may contribute to the most severe punishment for an agent who deviates from the norm. In my model, the situation is not one of common interest: buyers try to sustain a norm which is not in the interest of sellers. Still sellers help in sustaining a good equilibrium for the buyers by providing punishments. This comes out most clearly in the bilateral monopoly case: whenever a buyer accepts a price higher than the target price he is punished by the stage game equilibrium with the seller proposing a price of one for all future periods. This sort of punishment does not seem to have a natural interpretation and has the feature that sellers even prefer to be in a punishment stage. But the sellers' possible help as punishers is reduced if they cannot condition their behavior on information about a new trading partner. In that situation buyers can achieve better equilibria - even with full information flow - only if other buyers contribute to a more severe punishment, if "competitors are friends".

## 3.5 Relation to the Bargaining Literature

In Vincent [54], buyers make sequential offers to a single seller. The first buyer makes an offer which, if it is accepted, ends the game. If not, the second buyer gets to make an offer and so on. All agents observe all events in the market. Note that a single buyer could extract all the surplus given that he makes the offer. Vincent shows that the presence of many buyers still allows the buyers to extract some surplus: each buyer proposes a price of zero and the seller gives the good to each buyer with equal probability. This equilibrium is sustained by the threat of competition: if one buyer proposes a price higher than zero, the seller rejects this offer and the next buyer(s) makes a much higher offer which the seller then accepts. Hence, buyer competition is not necessarily disadvantageous for the buyers. In contrast, I show that the presence of other buyers is actually needed if individual bargaining power is reduced by the lack of credible commitment to "accept" punishments. Moreover, since I model a market with repeated one-shot transactions I can explicitly study the trade-off between a "realized" short term gain and the long term loss of a deviation as it is known from other infinitely repeated games. Finally, sellers and buyers cannot be integrated that easily in a punishment system as in Vincent's paper since each agent possesses only information about his private bargaining history. Due to the stationarity assumptions sellers are not able to reconstruct a buyer's bargaining history.

Since the paper by Vincent is closely related to the paper by Rubinstein and Wolinsky [50], most of these comments apply also to the paper by Rubinstein and Wolinsky. Rubinstein and Wolinsky study a random matching model where the proposer in a simple ultimatum bargaining game is selected randomly. If two agents agree on a price, they trade and depart from the market. If they don't agree they leave and get matched anew. Rubinstein and Wolinsky have two cases under which buyers can extract all the surplus (in the case without discounting). In the first case, agents observe the full history of the whole game and sellers observe buyers' identities. In the second case, each agent only observes his private history but sellers still do observe the buyers' identities. My informational assumptions reduce each seller's role in providing "punishments" for a defecting buyer. This enables me to highlight the role of buyer competition. The presence of other buyers matters since they have to be integrated in the system of punishment strategies.

## 3.6 Conclusion

I presented a model of bilateral oligopoly with bilateral negotiations, random matching and an option to continue trade. The model allowed me to study the effects of competition in a bilateral oligopoly which is characterized by such features. Increased seller competition turns out to be detrimental for buyers if sellers do not observe a buyer's private bargaining history. Buyer competition mitigates this negative effect induced by the outside option since buyer competition limits the value of the outside option by punishing buyers who use that option.

The main result of this chapter is that the presence of competitors may be useful while the presence of more agents on the other side of the market is not. The effect of competition to sustain favorable equilibria is well known from infinitely repeated Cournot or Bertrand games. I argue that implicit collusion as suggested by these models also can be modeled in a market with bilateral negotiations. The role of competition, however, is a different one. In the Cournot and in the Bertrand model, the problem is that competition on the *same* side of the market might cause a problem in the first place. The problem for any buyer in the model this chapter is that competition on the *other* side of the market reduces his surplus by introducing an outside option. Buyers' competition limits the value of the outside option for each buyer.

Repeated game models usually are plagued by a multiplicity of equilibria. I focused on equilibria which seem to be natural. In particular, the equilibria which model implicit buyer collusion in section 4 are interesting since they model an institution which we also observe in real world economies. Still, there *are* other equilibria and any equilibrium is associated with a certain distribution of surplus between buyers and sellers. However, in real life markets we do not observe any distribution of surplus in bilateral oligopolies. In a given market, one or the other side is the dominant side of the market and is able to dictate the terms of trade. In other words: it would be interesting to have a model which explains endogenously which side of such a market obtains a higher surplus. Such a model would necessarily have to narrow the set of equilibria in such a repeated game model. To do so, there have to be restrictions on the strategies players use in equilibrium. To find reasonable restrictions on strategies one could look at institutional details and certain competition policy aspects. One could think, for example, of a model where competition policy is modeled as a player who is able to exclude certain strategies for one side of the market or the other. This in turn would influence the distribution of surplus in the market.

# Chapter 4

# Communication Networks and Cooperation

## 4.1 Introduction

Network based mechanisms are important institutions for the enforcement of trust and cooperative behavior in communities. The mechanisms work through communication and fear of punishment for misbehavior or through anticipation of rewards in case of good behavior. For example, ethnic communities such as the Dominicans in New York and the Cubans in Miami use such systems to sustain informal credit channels (see Portes and Sensenbrenner [46]). The well known rotating credit associations among Asian immigrants, for example the Chinese on Java, rely on trust enforcement through network mechanisms as well (see Granovetter [28]).

In such a network, information, judgements, gossip and so on are exchanged only among *close* friends. The number of close friends network members have among each other is hence an important input to foster cooperation. A dense network, that is, a network where each network member has many close ties seems beneficial since each network member may use these ties as an effective threat to enforce cooperative behavior. It is less clear what the costs of a dense network are. For example, an exacerbation of the obligations within a network can conspire exactly against the network. Portes and Sensenbrenner display interesting examples like faulty assaults or constraints on freedom. Boissevain [11] studies the structure of relations inhabitants of Malta have. He shows with an example of two inhabitants that people generally do not maintain close ties to all members in their network. What is then the optimal number of contacts or close friends for each network member, and what might constrain the number of close friends in one's network?

Another important dimension with direct consequences for economic behavior is the network size, that is, the overall number of participants in a network. Granovetter [28] notes the success of the overseas Chinese on Java, which is based on rotating credit associations. The Javanese failed in establishing such a mechanism. Being immigrants, the Chinese are a small community in relation to the Javanese: in a Javanese town dubbed "Modjokuto" they numbered 1.800 out of a total of approximately 18.000. Granovetter offers the following explanation. Successful Javanese face demands for a piece of the cake achieved from an unlimited number of other Javanese (relatives, kins, etc.). The Chinese immigrants did not suffer from such excessive claims, since their immigrant status simplified the process of "decoupling" from relatives and kins. This chapter suggests that there is an additional detrimental effect of large numbers. On the one hand, cooperating network members benefit from a large number of network members since it is more likely that they interact with other cooperating network members. On the other hand, large communities are plagued by gossip which makes monitoring more difficult. In particular, the number of close friends which is required to sustain cooperation can then be prohibitively high.

This chapter addresses these two issues in the setting of a repeated Prisoner's Dilemma with changing partners. In a world with noisy communication I show that it is in general not socially optimal to have close contacts to all other network members. However, private incentives differ and network members choose to have close contacts to all other network members. Moreover, as the size of the network gets large, cooperation fails; if network formation is costly, no network sustaining cooperation might form at all.

The basic framework is that of a repeated game with changing partners à la Kandori [34]. Before the repeated interaction starts, a subset of all agents form a network and each network member chooses a number of other network members - close contacts - to whom he communicates the events in each of his per-period interactions. If communication is frictionless harshest punishments are achieved by choosing as many contacts as possible. Moreover, it is optimal to have as many network members as possible. If communication is noisy, it is no longer socially optimal to have a maximum number of contacts possible. The reason is that noisy communication might lead to sanctions with positive probability although there was no misbehavior. This lowers the overall benefit from cooperation if the number of contacts is too large. The message is that communication and information flow among all members is not socially optimal. However, private incentives to maintain close contacts might significantly differ from social incentives, so that an overinvestment in relations may occur. The noise in communication may be interpreted as noise in the technology which is used to transmit information. Another interpretation is that each agent, with positive probability, mistakenly reports as compared what actually happened.

In a community there is not only communication among close friends, but also communication among *all* network members. Hence, individuals receive messages with some informational content from their close friends while they receive pure noisy messages - gossip - from all other network members. Then, as the number of network members gets large, cooperation cannot be sustained anymore. Intuitively, if the number of network members gets large, agents cannot distinguish between gossip information and truthful information; cooperation fails. If network formation were costly no network would form! The finding suggests also why in many communities we do *not* observe communication networks but other, more centralized institutions, such as courts which rely not only on communication as a means of investigation.

Milgrom, North and Weingast [41] model the role of courts in enforcing cooperation in a basic setting similar to the one in this chapter. In their paper, agents may inform a court about a partner's misbehavior and they also may obtain information about a partner's previous behavior from the court. Hence information flows through a central institution whereas here information flows are decentralized. Greif's [30] remarkable work on the Maghribi trader's coalition studies an efficiency wage based mechanism which the Maghribi used to enforce honest behavior from their agents. A wage which has to be paid to enforce honest behavior is decreasing in the probability of future hiring. Hence a merchant prefers to hire an honest agent. Such a mechanism also rests on the assumption of information flow among the Maghribis. In contrast to the present work, Greif does not model the details of the communication network. Another paper which explicitly models network benefits from a communication network is Boorman's [12] analysis on the impact of communication networks on job search.

There is a also growing literature on strategic noncooperative network formation which focusses on individual incentives to form links, e.g. Bala and Goyal [7] or Jackson and Wolinsky [33].[1] Bala and Goyal interpret their model as a model of information flow. An agent's benefits and costs in a network in their paper is given by the number of agents he is linked to. The authors show that specific network architectures, e.g. a star or a ring, emerge as a Nash equilibrium in a strategic game of link formation. I contrast to Bala and Goyal I do not study the pattern of a network but examine the number of links or contacts agent must have to sustain cooperation. Benefits and cost from a contact are determined directly from the repeated interaction that follows the stage of contact formation. In particular, I model costs from network formation endogenous: the same mechanism which supports cooperation through communication in the first place might be detrimental to the community if communication in the network is noisy. The aspect that frictions are present in networks is missing in most of the literature mentioned (section 5 in Bala and Goyal [7] is a notable exception).

Kranton and Minehart [35] also analyze patterns of strategic network formation. Network benefits are given by an economic environment of trade among buyers and sellers. Buyers must form links to sellers before they can buy one unit of a good. After the stage of link formation an efficient trading mechanisms (e.g. a generalization of an ascending-bid auction) allocates the goods among the buyers. Kranton and Minehart show that the efficient network structure, i.e. a network structure which maximize overall surplus, is always an equilibrium. In contrast, I show that if communication is

[1]See Aumann and Myerson [5] and van den Nouweland [43] for approaches using cooperative game theory.

noisy, private and social incentives for network formation differ.

The rest of the chapter is organized as follows. Section 4.2 sets up the model. Conditions for a network equilibrium without noise are derived in section 4.3. Section 4.4 contains the result on the optimal number of contacts with noise, while section 4.5 presents the result on cooperation failure when the number of network members gets large. Section 4.6 discusses how the results of the model are affected if certain assumptions are changed and section 4.7 concludes.

## 4.2 The Model

I shall first describe the situation *without* communication network. There are $i = 1, ..., I > 2$ agents, where $I$ is even. The number $i \in \{1, ..., I\}$ is also called the *identity* of agent $i$. Time is discrete, $t = 1, 2, ...$, and runs from one to infinity. In each period, all agents get matched pairwise. If a match forms, the two agents play the following stage game.

| | $C$ | $D$ |
|---|---|---|
| $C$ | $a, a$ | $b, c$ |
| $D$ | $c, b$ | $0, 0$ |

with $c > a > 0 > b$.

Each agent only observes his private history but does not observe the events in other matches. For each agent, a strategy is a function from his private history to his action set. Each agent maximizes his average discounted payoff. Let $\delta \in (0, 1)$ denote the discount factor for all agents. This setting is equivalent to those studied by Kandori [34] or Ellison [21]. Whereas Kandori and Ellison sustain cooperation through punishments relying on contagion effects coming back to hit a noncooperator, I shall consider a more direct punishment mechanism.[2] This more direct punishment mechanism comes from a communication network.

**The Game with Communication Network**

Suppose that $N \leq I$ agents have decided to set up a **communication network**. The game looks then as follows.

(N1) all agents inside the network know and recognize the identities of other network members. Let $\mathcal{N}$ denote the set of network members.

(N2) in period 0 - *before* the repeated interaction starts - all network members choose simultaneously $L_i \leq N-1$ contacts or close friends. Let $\mathcal{L}_i$ denote the set of contacts of agent $i$. These contacts will sometimes be called *outgoing contacts*. We say that $i$ has a contact to $j$ if $j \in \mathcal{L}_i$. Let $\overline{\mathcal{L}}_i$ be the set of agents who have a contact to $i$, that is $\overline{\mathcal{L}}_i = \{j \in \mathcal{N}, j \neq i | i \in \mathcal{L}_j\}$. Let $\overline{L}_i$ denote the number of network members who have a contact to $i$, that is, $\overline{L}_i = \#\overline{\mathcal{L}}_i$. These contacts will sometimes be called *incoming contacts*. A network $\mathcal{L}$ is defined as the pattern of contacts agents have among each other. Each network $\mathcal{L}$ implies, for each network member $i$, a number of contacts $i$ has to other agents, $L_i$, and a number of contacts other agents have to $i$, $\overline{L}_i$. Let $L_N(\mathcal{L}) \in \mathbb{R}^{N \times 2}, L_N = \{(L_1, \overline{L}_1), ..., (L_N, \overline{L}_N)\}$ be the matrix of these numbers. For the rest of the paper only these numbers, not the exact pattern of contacts of agents, are relevant and I write $L_N$. [3] Agents

[2] I shall explain the relation of this chapter to their papers in more detail below.

[3] Hence two networks $\mathcal{L}$ and $\mathcal{L}'$ are equivalent if $L_N(\mathcal{L}) = L_N(\mathcal{L}')$

who are not network members cannot choose contacts.

(N3) Relations among agents are not symmetric, that is $j \in \mathcal{L}_i \nRightarrow i \in \mathcal{L}_j$ for all $i, j \in \mathcal{N}$. Hence, for agents $i \in \mathcal{N}$ and $j \in \mathcal{N}$, $i$ has a contact to $j$ *and* $j$ has a contact to $i$ if and only if $j \in \mathcal{L}_i$ and $i \in \mathcal{L}_j$. Moreover, relations among agents are not transitive, that is, $i \in \mathcal{L}_j \wedge z \in \mathcal{L}_i \nRightarrow z \in \mathcal{L}_j$ for all $i, j, z \in \mathcal{N}$.

(N4) The repeated interaction itself is as follows: in any period $t = 1, 2, ...$, agents get matched pairwise. All agents can get matched both with agents in the network and with agents outside the network. If two network members $i$ and $j$ meet they observe the number of contacts $L_i$ and $L_j$ of their partner and then choose their action, $C$ or $D$.

(N5) after network members $i$ and $j$ have played the stage game, both $i$ and $j$ inform their respective contacts ($L_i$ contacts for $i$ and $L_j$ contacts for $j$) of their partners' actions in the match. Hence the information of $i$ given to his $L_i$ contacts contains an element of the set $\{C, D\}$ and the identity (i.e. a number $i \in \{1, ..., I\}$) of his partner. This information about $i$ is available to the $L_j$ contacts of $j$ at the beginning of the next period.

Assumptions (N1)-(N5) define a *game with communication network*.

**Equilibrium Strategies in the Repeated Game:**

In the repeated game with network formation, strategies for a network member $i$ are now functions from private histories *and* from the information $i$ receives from the $\overline{L}_i$ agents who chose to have contacts to $i$ in period 0. For the periods $t = 1, 2, ...$, that is, for the infinitely repeated interaction following any choice of a communication network $\mathcal{L}$, I consider equilibria involving the following strategies:

(E1) agents outside the network choose $D$ in each period.

(E2) agents inside the network choose $C$ if they have the information that their partner has always chosen $C$. If there is no information since the partner is not a network member an agent chooses $D$.

(E3) agents inside the network choose $D$ if they have the information that their partner has chosen $D$ in any previous period. An agent who has chosen $D$ meeting an agent who plays $C$ chooses $D$ forever after.

(E4) if a network member $i$ deviated in a match with network member $j$, then $j$ informs all his $L_j$ contacts about this. That is, he sends the message $D$ and $i$'s identity to all his $L_j$ contacts. Agent $i$ sends the message $C$ and the identity of $j$. From the next period on those $L_j$ contacts punish $i$ forever after. If both $i$ and $j$ chose $C$, then both agents send the message $C$ to their respective contacts.

(E5) once punishment for an agent $i$ takes place, $i$ sends the message $C$ to all his $L_i$ contacts for the rest of the game.

To understand (E5), suppose that network member $i$ did not cooperate when playing with network member $j$ and that $j$ informs all his $L_j$ contacts about this. Assumption (E5) says that noncooperator $i$ does not trigger punishments on his punishers by informing his $L_i$ contacts. The assumption assures that punishing is always a best response in the equilibrium described for the punishers. If assumption (E5) does not hold, an agent who is required to punish another agent might fear further punishment and chooses $C$ when he is supposed to choose $D$. As in Kandori [34] one could circumvent the problem and fix stage game payoffs such that punishing *is* a best response (i.e. $b$ has to be sufficiently small). Then, all the qualitative results of the paper continue to hold even if (E5) is not imposed. I chose to impose assumption (E5) since, in contrast to Kandori's analysis I do not want to focus on contagion effects. I discuss the implications of relaxing (E5) for the results of this paper in more detail in section 4.6.

The equilibrium strategies (E1) to (E4) are considered since they seem natural in a communication network I have in mind. Network members inform their close contacts about any network member choosing $D$. This noncooperator is then punished by the close friends of the agent who was cheated. The strategies resemble in a simple way the communication network mechanisms used by ethnic communities as mentioned in the introduction.

Let $\sigma_n$ denote the strategy profile for network member described above while $\sigma_{-n}$ denotes the strategy profile for all agents not in the network. The strategy profile for all agents is denoted by $\sigma$. The payoff $V_i^n(\sigma)$ for a network member, is

$$V_i^n(\sigma) = \frac{N-1}{I-1}a \tag{4.1}$$

The average expected payoff $V_i(\sigma)$ for an agent not in the network is $V_i(\sigma) = 0$.

I take a network with $N$ members as given and ask wether network members are willing to participate. A network of size $N$ is beneficial if and only if all network members are willing to participate. This participation constrained is required to hold for all histories of play. Hence, to have a network which is beneficial to all participants, $V_i^n(\sigma) \geq 0$ has to hold for all histories of play. Note that this participation constraint is satisfied for any network size $N$. If the payoff from both partners playing $D$ is $d > 0, c > a > d > b$, then $N$ has to be sufficiently large. If there is some exogenous cost for network formation, for example, if it is costly to maintain contacts, then $V_i^n(\sigma)$

has to exceed this cost as well.

## 4.3 Equilibrium and Choice of Contacts

In this section I analyze if and under what conditions the strategies specified in (E1) to (E5) form an equilibrium indeed. Moreover, I determine the number of contacts agents choose in period 0.

I shall first show the optimality of (E4) and (E5).

**Lemma 2.** *For any $\mathcal{L}$, information transmission as specified in (E4) and (E5) is always optimal.*

*Proof.* (E4). Suppose that agents behave according to (E5). Pick agent $i \in \mathcal{N}$. Suppose that in a match with $i$ agent $j$ plays $C$. If agent $i$ informs his $L_i$ contacts that $j$ played $D$, they punish $j$ forever after, which does not increase $i$'s payoff. Suppose that $j$ played $D$. If agent $i$ does not inform his contacts about this, his payoff does not increase either (due to (E5)) . Moreover, any information of $i$ about some other agent $z \neq j$ would not increase $i$'s payoff either.

(E5). With a similar argument, a noncooperator $j$ cannot increase his payoff sending the message $D$ instead of the message $C$. □

Given this lemma, punishing a noncooperator is an equilibrium in the continuation game after any defection, as required in (E3).

To have network members cooperate as required in (E2), the following incentive constraint has to hold for all $i \in \mathcal{N}$:

$$\frac{N-1}{I-1}a \geq \frac{N-1}{I-1}c + \frac{\delta}{1-\delta}\left(\frac{N-L_j-1}{I-1}a\right) \tag{4.2}$$

for all $j \in \mathcal{L}_j$.

The benefit of cooperation is given by the probability of meeting a network member, $(N-1))/(I-1)$ times the payoff of cooperation, $a$. The benefit from defection is the defection payoff $c$ and the continuation payoff. The continuation payoff from defection is zero with probability $L_j/(I-1)$ and with probability $(I-N)/(I-1)$ and $a$ with probability $(N-L_j-1)/(I-1)$. Due to Lemma 1 it is always a best response for $j$'s $L_j$ contacts to punish $i$.

Solving inequality (4.2) yields

$$L_j \geq L^\circ = \frac{(N-1)(1-\delta)(c-a)}{\delta a} \tag{4.3}$$

as the minimum number of contacts each network members needs to have in order to sustain cooperation.[4] Thus it is not necessary that all network members have contacts to *all* other network members. Behavior as specified in (E1) is clearly optimal. Last note that $V_i^n(\sigma) \geq 0$ holds for all histories of play.

To determine the number of contacts agents choose in period 0, I focus on network members' equilibrium choices which induce the maximal threat on their partner.

(E6) network members choose a number of contacts such that their partners payoff is lowest in case of noncooperation.

Contact choices induce *extremal equilibria* in the sense of Abreu, [3] or [1]. Hence, it is optimal for each agent during the stage of contact formation that every agent

[4] This number is actually $[L^\circ]+1$, where $[x]$ denotes the next largest integer to $x$.

chooses $L_i = N - 1$. This number of links provides the harshest punishment for a noncooperator.

If the communication network $\mathcal{L}$ is organized such that it maximizes each network member's utility, it is also an optimal strategy that all network members have $L_i = N - 1$ contacts. This implies also that each network member is contacted by all other network members, $\overline{L}_i = N - 1$. Hence network members' private incentives and social incentives for the network as a whole are the same! Efficient communication network formation is always an equilibrium on the game of network in stage 0, given repeated interaction from period 1 on.

Considering network size, note that equilibrium payoffs are increasing in $N$ so that agents prefer large networks. If $c > a/(1 - \delta)$, the number of contacts required to sustain cooperation would strictly exceed $N - 1$; cooperation based on a network mechanism fails in that case. The following Proposition summarizes these observations.

**Proposition 9.** *Suppose that (E1) to (E6) hold in the game with communication network.*

*(i) Cooperation is sustainable in the network if and only if*

$$L_i \geq \frac{(N-1)(1-\delta)(c-a)}{\delta a}$$

*for all $i \in \mathcal{N}$.*

*(ii) Suppose that contact choices induce extremal equilibria. For all $N^* \geq 2$, it is then optimal to form a network in which each network member chooses $L_i = N - 1$ contacts. Then punishment is maximal and cooperation in the network can be sustained for $\delta \geq (c - a)/c$. The choice of $L_i = N - 1$ is optimal for*

*each individual network member and also optimal if the network maximizes each network member's utility: efficient communication network formation is always an equilibrium.*

*(iii) Since equilibrium payoffs for network members are strictly increasing in $N$, it is optimal to have all agents in the network, that is $N = I$ is optimal.*

*(iv) If $c > a/(1-\delta)$ it is not possible to sustain cooperation.*

### 4.3.1 Cooperation without Institutions ?

In Kandori [34] and Ellison [21] cooperation is sustained by punishments relying on contagion effects coming back to hit a noncooperator. In such a contagion equilibrium, all agent initially cooperate. If an agent ever meets an opponent who defects, he defects from then on. Hence, playing $D$ today will eventually lead all agents to play $D$. If these strategies are an equilibrium depends on how fast contagion spreads, which in turn depends on the number of agents and on the stage game payoffs. The main problem is that agents prefer to continue choosing $C$ even after meeting an agent who plays $D$ in order to slow down the spread of contagion. In particular, Kandori shows that, for any fixed number of agents, the contagion strategies are an equilibrium for discount factors close to 1 if stage game payoffs are such that the payoff to playing $C$ against an agent playing $D$ is sufficiently negative. Ellison extends Kandori's work and assumes that a publicly observable random variable is available. The public randomization allows to adjust the severity of the punishments. Punishments can be tailored such that agents fear a breakdown of cooperation, so they do no deviate first. Moreover, they do not fear the breakdown so much that they do not spread out the play of $D$.

In those papers cooperation is possible without any institutions. However, we do observe communication networks (see the introduction of this paper)! A reason for this could be that the direct punishments in this chapter work for discount factors smaller than the discount factor required to sustain Kandori's or Ellison's mechanism.

**Proposition 10.** *The discount factor necessary to sustain cooperation through network formation is smaller than the discount factor necessary to sustain cooperation through contagion.*

*Proof.* Cooperation through contagion requires a larger discount factor than cooperation through network formation if a noncooperator's continuation payoff after defection is larger.

With network formation, a noncooperator is punished immediately forever after a deviation. Hence, a noncooperator's continuation payoff after a deviation is zero.

In punishments relying on a contagion effects and it lasts at least $(I-2)/2$ periods until a noncooperator is punished with probability 1 in every period. Hence, in the first $(I-2)/2$ periods after a deviation there is always a strictly positive probability that a noncooperator is not punished. This implies that the infimum of a noncooperator's continuation payoff is strictly bounded away from zero. $\square$

Hence this more effective way of punishment might be a reason why we observe institution such as communication networks or courts (see Milgrom, Weingast and North [41]).

## 4.4 Noisy Communication

In the previous section it was optimal for each agent to choose as many contacts as possible. This choice was optimal since contacts never became active in equilibrium. A large number of contacts is a very effective threat! However, in many networks, the same mechanism which supports cooperative actions may also be detrimental to the agents in the network. In particular, if communication is noisy, those contacts might not cooperate with me although I myself did cooperate. It may then be optimal if my partner has fewer than $N-1$ contacts. Moreover, it could be that I have to punish other network members even though they did cooperate. Hence, I prefer that $\overline{L}_i$, the number of other network members who have a contact to me, is not too high. I shall analyze the socially optimal choice of a communication network $\mathcal{L}$ when there is noise in the transmission of information.

### 4.4.1 Noise and Equilibrium Restrictions

I model the presence of noise as follows. Suppose that there is noise in the stage of the game where each agent informs his $L_i$ contacts about the behavior of his partner $j$ in a given period. Two things can happen: the partner $j$ of agent $i$ did deviate, but none of $i'$s contacts received the message and all of $i'$s contacts continue to believe that $j$ did cooperate. Or, $j$ did not deviate, but the $L_i$ contacts of agent $i$ did receive the message that agent $j$ did deviate.

Formally, each message between two agents generates one of two signals at each period. The signal space is given by $X = \{\mathcal{C}, \mathcal{D}\}$ and is the same for all matches. The signal $\mathcal{C}$ is interpreted as a "good" signal, the signal $\mathcal{D}$ is interpreted as a "bad" signal.

Let, for all matches between $i$ and $j, i, j \in \mathcal{N}$,

$$\begin{aligned}
\alpha &= \Pr(L_i \text{ contacts receive signal } \mathcal{C} \mid j \text{ reports } C) \\
1-\alpha &= \Pr(L_i \text{ contacts receive signal } \mathcal{D} \mid j \text{ reports } C) \\
\beta &= \Pr(L_i \text{ contacts receive signal } \mathcal{C} \mid j \text{ reports } D) \\
1-\beta &= \Pr(L_i \text{ contacts receive signal } \mathcal{D} \mid j \text{ reports } D).
\end{aligned}$$

I assume $\beta < \alpha$. It is more likely that signal $\mathcal{C}$ results if $C$ was reported than if $D$ was chosen. All contacts of a network member receive the same signal. Each agent receives $\overline{L}_i$ signals and each agent can identify the network member to whose behavior a given signal is related. Moreover, after a match between $i$ and $j$, it is common knowledge for $j$ and all the network members in the set $\mathcal{L}_i$ which signal the agents in the set $\mathcal{L}_i$ observe. Similarly for $i$ and the network members in the set $\mathcal{L}_j$.

The game is now as follows: assumptions (N1)-(N5) from section 4.2 hold. The only difference is that information transmission is noisy. For the repeated interaction in periods $t = 1, 2, ...$, for any network choice $\mathcal{L}$, I consider the following equilibria.

($\mathcal{E}1$) agents outside the network choose $D$ in each period

($\mathcal{E}2$) agents inside the network start the repeated interaction by playing $C$. An agent inside the network continues to choose $C$ if he receives the signal $\mathcal{C}$ about the previous behavior of his new partner. If an agent does not receive any signal about his new partner, he chooses $D$.

($\mathcal{E}3$) if an agent $i$ receives a signal $\mathcal{D}$ about the previous behavior of an agent $j$, then agent $j$ is punished by $i$. The punishment of $i$ lasts forever after a deviation, that is, all agents who receive signal $D$ about $j$ punish $j$ whenever they meet $j$.

($\mathcal{E}4$) information transmission: if in a match between $i$ and $j$ both agents choose $C$, then both agents send the message $C$ to their respective contacts. If $i$ deviates while $j$ plays $C$ then $i$ sends the message $C$ while $j$ sends $D$. Once punishment for an agent $i$ takes place, $i$ sends the message $C$ to all his $L_i$ contacts for the rest of the game. Messages produce signals according to the signal technology described afore.

($\mathcal{E}5$) equilibrium strategies for all network members are symmetric, that is, all agents use the same strategies.

Requirement ($\mathcal{E}4$) contains the analogue to assumption (E4) in section 4.2: punishments are not contagious. Moreover, I assume that (E6) from the previous section holds: agents choose contacts which induce the most severe punishment on their partners. I also assume that all network members choose the same number of contacts, $L_i = L$ for all $i \in \mathcal{N}$. In the rest of this section I analyze the conditions for $(\mathcal{E}1) - (\mathcal{E}5)$ to be an equilibrium in the repeated game. I also characterize the communication network $\mathcal{L}$ which maximizes the network members' overall surplus and compare the result to the communication network $\mathcal{L}$ which arises from network members' private incentives under assumption (E6).

### 4.4.2 Analysis of Equilibria with Noisy Communication

In contrast to the previous section, not only the number of contacts each agent $i$ has is important for the analysis. Since each network member may have to punish other agents in each period with positive probability, it is also important, how many other agents have contacts to agent $i$. Recall that $\overline{L}_i$ denotes the number of agents in the network who have contacts to $i$, that is $\overline{L}_i := \{\# \cup_j \mathcal{L}_j | i \in \mathcal{L}_j\}$ where $\#$ denotes the cardinality of the set $\cup_j \mathcal{L}_j$. Let $p(\overline{L}_i)$ denote the probability that agent $i$ receives a signal $\mathcal{C}$. Given that these other network members did actually cooperate, $p(\overline{L}_i)$ is a function of $\alpha$. The exact expression for $p(\overline{L}_i)$ is complicated and depends on $\mathcal{L}$, that is, the exact pattern of contact choices in period 0. However, it clearly holds that $p'(\overline{L}_i) < 0$ and $0 < p(\overline{L}_i) < 1$ for $\overline{L}_i > 0$. That is, the more other agents chose agent $i$ as a close friend, the lower the probability that agent $i$ does *not* have to fulfill any punishment obligations.

Denote by $V_i^+$ the payoff from the proposed equilibrium strategy profile. It is given by

$$V_i^+ = (1-\delta)\frac{N-1}{I-1}a + \delta\alpha p(\overline{L}_i)V^+ + \delta(1-\alpha)p(\overline{L}_i)\frac{N-L_j-1}{I-1}a, \tag{4.4}$$

where $L_j$ are the contacts each partner $j \in \mathcal{N}$ of $i$ has. Using symmetry, $L_j = L$ for all $j \in \mathcal{N}$, this can be rewritten as

$$V_i^+ = \frac{[(1-\delta)(N-1) + \delta p(\overline{L}_i)(1-\alpha)(N-L-1)]a}{(I-1)(1-\delta\alpha p(\overline{L}_i))}. \tag{4.5}$$

It is easy to show that this expression is strictly monotonically decreasing in $L$ and in $\overline{L}_i$. The more network members have contacts to $i$, the more often $i$ has to punish another network member. The larger the number of contacts $i$'s partners $j \neq i$ have to *other* agents, the more often $i$ is punished. In both cases, an increase in the respective number of contacts lowers $i'$ s payoff.

Moreover, the following incentive constraint has to hold to fulfill ($\mathcal{E}2$):

$$V_i^+ \geq (1-\delta)\frac{N-1}{I-1}c + \delta\beta p(\overline{L}_i)V_i^+ + \delta(1-\beta)p(\overline{L}_i)\frac{N-L_j-1}{I-1}a. \tag{4.6}$$

Note that the probability $p(\overline{L}_i)$ is the same on both sides of that equation: $p(\overline{L}_i)$ denotes the probability that agent does *not* have to punish other agents he gets matched to and thus obtains a positive payoff from the interaction with them. Using the expression for $V_i$, we obtain that

$$L \geq \frac{(N-1)(c-a)(1-\delta\alpha p(\overline{L}_i))}{(\alpha-\beta)\delta a p(\overline{L}_i)} \tag{4.7}$$

has to hold to sustain cooperation, given the stage of contact formation in period 0. Note that $L$ is increasing in $\overline{L}_i$. The more network members have contacts to $i$, the higher is his deviation payoff relative to the payoff from cooperation. Hence a larger number of contacts outgoing from network members, $L$, is required. Note that $(\mathcal{E}3), (\mathcal{E}4)$ and $(\mathcal{E}1)$ are satisfied by the same arguments as in section 4.3.

If the network aims to maximize overall surplus of its members, it maximizes in the game of contact formation in period 0 each network member's utility. Since all that matters are the numbers of ingoing and outgoing contacts $L_N$ induced by a network $\mathcal{L}$, the network maximizes

$$\max_{L_N} \quad V_i^+ \qquad \text{for all } i \in \mathcal{N} \text{ subject to (4.7).}$$

We know that the objective function $V_i^+$ is strictly decreasing in $L_j, j \neq i$ and $\overline{L}_i$ for all $i$. On the other hand, the incentive constraint (4.7) has to hold. Suppose, for simplicity, that the network maximizes overall welfare of its members by choosing $L_N$ such that $\overline{L}_i = L_j = L, j \neq i$. This means that each network member has the same number of incoming and outgoing contacts. This is the case, for example, if all network members have contacts to all other network members. Then, the incentive constraint reads as

$$\frac{ap(L)L}{(1-\delta\alpha p(L))} \geq \frac{(N-1)(c-a)}{(\alpha-\beta)\delta}. \tag{4.8}$$

If $L = 0$, the left-hand side of equation (4.8) is zero, hence there must be a choice of contacts with $L > 0$. On the other hand note that the left-hand side of the equation (4.8) is strictly increasing in $L$! Hence there exists a unique $L^*$ with $0 < L^* < N - 1$ such that this incentive constraint is satisfied indeed for any strictly decreasing function $p(L)$. Moreover, since each network member's utility is decreasing in $L$, it is optimal to choose $L^*$ as the number of contacts agents have and agents receive.

**Proposition 11.** *Suppose that the network maximizes the surplus of its members by choosing $\overline{L}_i = L_i = L$ for all $i$.*

*(i) The strategies $(\mathcal{E}1) - (\mathcal{E}5)$ form an equilibrium in the infinitely repeated game in periods $t = 1, 2, ...$ if every agent has $L$ contacts, where $L$ satisfies*

$$\frac{ap(L)L(1-\delta)}{(1-\delta\alpha p(L))} \geq \frac{(N-1)(1-\delta)(c-a)}{(\alpha-\beta)\delta}$$

*Punishment is induced by bad signals, $\mathcal{D}$, providing incentives for cooperation which occurs on good signals, $\mathcal{C}$.*

*(ii) Equilibrium payoffs are equal to*

$$V_i^+ = \frac{[(1-\delta)(N-1) + \delta p(L)(1-\alpha)(N-L-1)]a}{(I-1)(1-\delta\alpha p(L))}. \tag{4.9}$$

*for all $i \in \mathcal{N}$ and are monotonically decreasing in $L$.*

*(iii) The optimal number of contacts for each network member $L^*$ is given by the unique number $L^*$ that solves*

$$\frac{ap(L^*)L^*(1-\delta)}{(1-\delta\alpha p(L^*))} = \frac{(N-1)(1-\delta)(c-a)}{(\alpha-\beta)\delta}. \tag{4.10}$$

The proposition states that it may well be optimal to restrict the number of contacts and close contacts each agent has. In particular,

$$L^* < N - 1 \quad \Leftrightarrow \quad a < \frac{(c-a)(1-\delta\alpha p(N-1))}{p(N-1)(\alpha-\beta)\delta}.$$

In that case it is optimal to have fewer than $N-1$ contacts in the network.[5] As one easily checks, the following comparative statics results hold.

$$\frac{\partial L^*}{\partial a} < 0, \quad \frac{\partial L^*}{\partial \delta} < 0, \quad \frac{\partial L^*}{\partial c} > 0, \quad \frac{\partial L^*}{\partial N} > 0, \quad \frac{\partial L^*}{\partial \alpha} < 0, \quad \frac{\partial L^*}{\partial \beta} > 0 \quad (4.11)$$

The optimal number of contacts decreases as $\alpha$ increases ($\beta$ decreases). The intuition for this would be that less noise in communication reduces the need for powerful punishments so less close contacts are necessary to sustain cooperation. On the other hand, network size $N$ increases, many contacts are needed: as $N$ increases, a noncooperator finds more easily partners for future cooperation. This increases the incentives to deviate from the cooperative equilibrium. Then, more contacts are needed to induce more severe punishment. As the incentives for deviation, $c$, increases, the number of contacts must be higher as well.

The *private incentives* for network formation differ significantly from the social incentives. It is easy to see that, for each network member, a choice of $N-1$ contacts is again an equilibrium of the network formation game in period 0 if the choice of contacts is supposed to induce extremal equilibria. This holds since each network members payoff function $V_i^+$ does *not* depend on the number of contacts agent $i$ has to other agents. The payoff of network member $i$ is decreasing in the number of contacts other network members have to $i$ and is also decreasing in the number of contacts other partner have. But the payoff of $i$ does not depend on $L_i$.

[5] In the language of graph theory, the optimal network is not connected, i.e. there is not a path between every pair of agents. Connectedness is a standard assumption in much of recent work on social learning and local interaction; see e.g. Anderlini and Ianni [4], Ellison [20] or Ellison and Fudenberg [22].

## 4.5 Gossip

In the previous section, the distribution of the signal depended only on the outcome of any given match. In this section I add more noise to the situation and model network gossip. I assume that each agent is informed about the behavior of a friend's partner not only through a more or less informative signal which is generated from any given match. Rather, each agent receives a noisy message from each other network member. The interpretation is that even network member who did not observe an agent's behavior gossip about what that agent did.

Formally, for any given agent $i$, the partner matched to $i$ receives now

- one signal about $i$'s behavior which is informative according to the above defined probabilities $\alpha$ and $\beta$. It still holds that $\alpha > \beta$. Suppose for simplicity that $\alpha = 1$ and $\beta = 0$.

- and also signals from all the other $N - 2$ agents in the network. Those agents did not observe what $i$ did in the previous period. So, they gossip and transmit their gossip to the new partner of $i$: each of the $N - 2$ signals can be $\mathcal{C}$ with probability $\gamma$ and $\mathcal{D}$ with probability $1 - \gamma$. The probability $\gamma$ is not conditioned on the action agent $i$ and his partner took in the previous period. An agent does not know the source of the signal: he does not know if any of the $N - 1$ signals is generated by gossip or by the last match of agent $i$. The signal technology is hence anonymous in the sense of Green [29].

Hence, each agent receives $N - 1$ signals about the behavior of his new partner. Each agent then uses a simple rule to evaluate the $N - 1$ signals and to condition his future actions on the signals.

- If a sufficiently large share $K \in (0,1)$ of the signals is $\mathcal{C}$, an agent cooperates with his new partner. Otherwise, he chooses action $D$.

Let $|\mathcal{C}|$ denote the number of signals with the value $\mathcal{C}$ an agent received about his new partner and define, for all $i, j \in \mathcal{N}$,

$$\begin{aligned}
\widehat{\alpha} &= \Pr(L_i \text{ contacts receive at least } (N-1)\cdot K \text{ signals } \mathcal{C} \mid j \text{ reports } C) \\
\widehat{\beta} &= \Pr(L_i \text{ contacts receive at least } (N-1)\cdot K \text{ signals } \mathcal{C} \mid j \text{ reports } D).
\end{aligned}$$

I assume $\widehat{\alpha} > \widehat{\beta}$. It is more likely that signal $\mathcal{C}$ results relatively more often if $C$ was chosen than if $D$ was chosen. All network members receive the same signals. I shall for further use rewrite these probabilities as

$$\widehat{\alpha} = \Pr(|\mathcal{C}|/(N-1) \geq K|C)$$

and

$$\widehat{\beta} = \Pr(|\mathcal{C}|/(N-1) \geq K|D).$$

Since these probabilities are the same for $i, j \in \mathcal{N}, j \neq i$, subscripts are omitted. Basically, we have now a situation similar to the one in section 4.4, but with "more" noise.

Given the probabilities $\widehat{\alpha}$ and $\widehat{\beta}$, one can repeat the exercise from the previous section and solve for the number of $L$ contacts each agent has. I consider the same equilibria as in the previous section and impose the same assumptions on contact choice.

I denote with $q(\overline{L}_i)$ the probability that a network member does receive enough signals $\mathcal{C}$ so that he does not have to punish another network member.

I now analyze the possibilities for network cooperation as $N$ gets large. Recall that the incentive constraint is given by

$$L \geq \frac{(N-1)(c-a)(1-\delta\widehat{\alpha}p(\overline{L}_i))}{(\widehat{\alpha}-\widehat{\beta})\delta a p(\overline{L}_i)} \tag{4.12}$$

for all $i \in \mathcal{N}$. This can be rewritten as

$$(\widehat{\alpha}-\widehat{\beta}) \geq \frac{(N-1)(c-a)(1-\delta\widehat{\alpha}q(\overline{L}_i))}{aq(\overline{L}_i)L\delta}. \tag{4.13}$$

for all $i \in \mathcal{N}$. I use this incentive constraint to show that cooperation fails as $N$ gets large.

**Proposition 12.** *There exists $\overline{N}$ such that for all $N \geq \overline{N}$ cooperation cannot be sustained.*

*Proof.* Note that

$$\widehat{\alpha} = \Pr(|\mathcal{C}|/(N-1) \geq K|C) = \Pr(|\mathcal{C}|/ \geq K(N-1)-1|C).$$

This follows from the fact that in case of the partner's action being $C$, any new partner of any of the agents involved in the match obtains one signal $\mathcal{C}$ for sure (Recall that $\alpha = 1$). This lowers the critical bound of other signals an agent has to obtain to continue with cooperation.

On the other hand, since $\beta = 0$, we have

$$\widehat{\beta} = \Pr(|\mathcal{C}| \geq K(N-1)|D) = \Pr(|\mathcal{C}| \geq K(N-1)|D).$$

The argument is now that $\lim_{N\to\infty} |\widehat{\alpha} - \widehat{\beta}| = 0$ while the right hand side of (4.13) tends to infinity as $N \to \infty$. To see this, note that

$$\widehat{\beta} = 1 - \sum_{k=1}^{K(N-1)} \binom{K(N-1)}{k} \gamma^k (1-\gamma)^{K(N-1)-k}$$

and

$$\widehat{\alpha} = 1 - \sum_{k=1}^{K(N-1)-1} \binom{K(N-1)-1}{k} \gamma^k (1-\gamma)^{K(N-1)-1-k}.$$

Then, $\widehat{\alpha} - \widehat{\beta}$ is given by

$$\sum_{k=1}^{K(N-1)} \binom{K(N-1)}{k} \gamma^k (1-\gamma)^{K(N-1)-k} -$$

$$\sum_{k=1}^{K(N-1)-1} \binom{K(N-1)-1}{k} \gamma^k (1-\gamma)^{K(N-1)-1-k}.$$

Note that

$$\sum_{k=1}^{K(N-1)} \binom{K(N-1)}{k} \gamma^k (1-\gamma)^{K(N-1)-k} =$$

$$\sum_{k=1}^{K(N-1)-1} \binom{K(N-1)-1}{k} \gamma^k (1-\gamma)^{K(N-1)-1-k} + \gamma^{K(N-1)}.$$

Hence, $|\widehat{\alpha} - \widehat{\beta}| = \gamma^{K(N-1)}$ and $\lim_{N\to\infty} |\widehat{\alpha} - \widehat{\beta}| = 0$. So there exists $\varepsilon > 0$ such that $|\widehat{\alpha} - \widehat{\beta}| < \varepsilon$ for all $N \geq \overline{N}$. Moreover, as $N \to \infty$ the right hand side of (4.13), tends to infinity. This holds since all other expressions in the denominator of the expression on the right hand side of that equation cannot tend to infinity as well as $N \to \infty$. Moreover, no expression in the numerator can go to zero as $N \to \infty$. But since the left hand side of the equation cannot be larger than $\varepsilon$, there exists $\overline{N}$ such that for all $N \geq \overline{N}$ equation (4.13) does not hold.

It is straightforward to extend the Proposition for any $\alpha \in (0, 1)$ and any $\beta \in (0, 1), \alpha > \beta$.

$\square$

If there is a cost for network formation, the network would not form if $N \geq \overline{N}$! The finding also suggests why in many communities we do *not* observe decentralized cooperation networks. If the number of community members gets large, other institutions such as courts might be needed (see Milgrom, North and Weingast [41]). Courts use a huge legal apparatus which does not rely on decentralized communication alone.

## 4.6 Discussion

I briefly discuss the robustness of these results with respect to assumption (E5). This assumption prevents contagious punishments. Without this assumption, it is not clear if agents fulfill their punishment obligations. I shall demonstrate how the qualitative results of this chapter hold even without assumption (E5).

Consider first the analysis and results of section 4.3, the situation without noise. The relevant condition is that the payoff from punishing a noncooperator has to be higher than the payoff from playing $C$:

$$\begin{aligned} 0 + \frac{\delta}{1-\delta}\left[\frac{I-N}{I-1}0 + \frac{L_j+1}{I-1}0 + \frac{N-L_j-2}{I-1}a\right] \geq \\ b + \frac{\delta}{1-\delta}\left[\frac{I-N}{I-1}0 + \frac{1}{I-1}0 + \frac{N-2}{I-1}a\right]. \end{aligned} \tag{4.14}$$

There are two possibilities to satisfy this constraint without making assumption (E5). First, the payoff $b$ from playing $C$ when my partner plays $D$ can be made small enough to provide the correct incentives. This is the same requirement as in Kandori [34] who needs this assumption to make his contagion mechanism work (see section 4.3.1). A second possibility would be the incentive constraint for $L_j$ and derive an upper bound $\widehat{L}$ on the number of contacts. Then, we would have two restrictions on the number of contacts. First, as shown in section 4.3, the number of contacts has to be high enough to ensure cooperation:

$$L_j \geq L^{\circ} = \frac{(N-1)(1-\delta)(c-a)}{\delta a}. \tag{4.15}$$

Second, the number of contacts has to be less than $\widehat{L}$ to ensure network member's participation in a punishment stage of the game. Hence, $L^{\circ} < \widehat{L}$ would have to hold for such an equilibrium to exist. Suppose that this condition holds and that network members choose contacts which induce extremal equilibria (assumption (E6)). This numbers of contacts is then given by $\widehat{L}$. This is also a number which is optimal from a social point of view. Hence, private and social incentives overline as in section 4.3.

Consider now the model with noise. Recall that network members' payoffs are decreasing in the number of contacts their partners have. The number of contacts has to be high enough to ensure cooperation. On the other hand, punishment has to be ensured. Again, this can be achieved by making $b$ sufficiently small. Or, the number of contacts has to be low enough to ensure network members' participation in the punishment but high enough to sustain cooperation in the first place. Clearly, since network members payoffs are decreasing in the number of contacts of their partners, it is socially optimal to choose the lower number of the two which is just enough to support cooperation. Private incentives would again go in the other direction and agents would choose too much contacts in the game of contact formation in stage 0. In other words: the basic results from section 4.4 continue to hold. This is also valid for the main result in section 4.5.

## 4.7 Conclusion

This paper models a communication network and shows how the optimal number of contacts in such a network is affected when communication is noisy. If communication is noisy, it is not optimal to have close contacts to all network members. Moreover, as the number of network members gets large, cooperation in the network fails. If network formation is costly, we cannot expect a network to form.

There are many other institutions sustaining cooperation in such situations, for example courts. It would be interesting to examine under which circumstances we observe decentralized institutions such as communication networks and under which circumstances we observe the existence of centralized institutions such as courts. Moreover, when would we expect a mechanism without any institution as the contagion mechanism proposed by Kandori and Ellison?

Another line of research could focus on communication networks in a setting where there is additional source of externalities among network members. While in my setting honest information transmission is always a best response, one can easily imagine a situation where an agents payoff strictly increases if an other agent gets punished. This could be the case if there is not only a double-sided moral hazard problem but also if there is the situation that a network member benefits from another network member being punished. Suppose, for example, that there is some competitive relation among some or all network members. Hence, there is a direct incentive to assault another network member even if that network member did cooperate. The network would then serve not only as a means to support cooperation but information flow is then also a means to hurt a potential competitor.

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# Appendix A

# Appendix to Chapter 2 (The Proof of Proposition 2)

I shall prove the Proposition by establishing four Lemmata. I introduce some further notation. Given an equilibrium in stage two of the game, the correspondence $\underline{s} \to b_i^*(\cdot)$ will be denoted as $\phi_i(\cdot)$. The vector $(\phi_1(\cdot), ..., \phi(\cdot))$ will be denoted as $\phi$. Let $\Phi(\underline{s}) := \sum_{i \in \mathcal{B}} \phi_i(\underline{s})$.

**Lemma 3.** *(i) In each of the subgames in stage 2, a unique symmetric Nash equilibrium in pure strategies $(b_1^*(\underline{s}), ..., b_B^*(\underline{s}))$ exists. It is given by*

$$b_i^* := b^* = f'(\overline{s}/B)\frac{\overline{s}(B-1)}{B^2}. \tag{A.1}$$

*(ii) The correspondence $\Phi(\underline{s})$ is a continuous function in $\overline{s} := \sum_{j \in \mathcal{S}} s_j$.*

*Proof.* $(i)$. It is straightforward to show that any Nash equilibrium in stage must be symmetric, that is $b_1^*(\underline{s}) = ... = b_B^*(\underline{s})$ for all $\underline{s}$. To find the Nash equilibrium, I compute the first-order condition and use symmetry to find a unique solution

$$b_i^* := b^* = f'(\overline{s}/B)\frac{\overline{s}(B-1)}{B^2}. \tag{A.2}$$

for all $i$. Due to concavity of $f$ this is a maximum indeed.

$(ii)$. This follows immediately from inspection of equation (A.2) and from the assumption that $f$ is twice continuously differentiable. $\square$

Given these results, the problem amounts to show existence of a Nash equilibrium for the game between the sellers, where now,

$$\pi(s_j, s_{-j}) = f\left(s_j p(\overline{s})\right) + 1 - s_j \tag{A.3}$$

with

$$p(\overline{s}) = \frac{\Phi(\underline{s})}{\overline{s}} = f'(\overline{s}/B)\frac{B-1}{B}, \tag{A.4}$$

from Lemma 3.

**Lemma 4.** $p'(\overline{s}) < 0 \qquad \forall\, \overline{s}$.

*Proof.* Recall that $f'' < 0$. The result follows then from equation (A.4). $\square$

We cannot prove that $\pi$ is quasiconcave, let alone be concave in $s_j$ for any seller $j$. However, note that $\pi$ is a continuous function.

Let $R$ be the reaction correspondence of any seller for *that* game, that is

$$R : [0, (S-1)] \rightarrow [0, 1] \tag{A.5}$$

and $r := \inf R$.

Using standard arguments, one can prove

**Lemma 5.** *The correspondence $R$ satisfies*

*(i) $R$ is nonempty*

*(ii) $R$ is closed*

*Proof.* Omitted. $\square$

Given these results, I complete the proof with an argument building on Roberts and Sonnenschein [47] - of which the original idea appeared in McManus ([38] and [39]). Roberts and Sonnenschein show existence of a symmetric Nash equilibrium in pure strategies for the Cournot model with general downward sloping demand functions and symmetric firms.[1] Notice that, from Lemma 4, the "inverse demand function" slopes downward as a result of the model! Roberts and Sonnenschein [47] (Lemma, p. 114) prove that a symmetric pure strategy Nash equilibrium exists, if the best response correspondences in the game between the sellers are nonempty, closed from the right and exhibit only upward jumps. They might be sloping downward, though. [2] Since, from Lemma 5, $R$ is nonempty and closed, existence of a symmetric Nash equilibrium is assured if I can establish that best response correspondences jump only upwards.

The last step shows hence that the function $r$ defined by $r = \inf R$ is upper semicontinuous from the left, that is $s^n_{-j} \to s_{-j}, s^n_{-j} < s_{-j}$ implies $\limsup r(s^n_{-j}) \leq r(s_{-j})$. As a last piece of notation, let $\sigma = \sum_{k \in \mathcal{S} \setminus \{j\}} s_k$. Then, $\overline{s} = \sigma + s_j$,

**Lemma 6.** *The function $r(s_{-j}) := \inf R(s_{-j})$ is upper semicontinuous from the left.*

[1] Symmetry refers to the firms' cost functions.

[2] I will not state this result formally, but the reader may convince himself by a drawing for the one-dimensional case when best responses are single-valued.

*Proof.* I proceed in three steps.

*Step 1.* I show that if, for any $s_{-j}$, $s_j$ is such that for all $\tilde{s}_j \geq s_j$, I have $\pi_j(s_j, s_{-j}) \geq \pi_j(\tilde{s}_j, s_{-j})$, then this implies $r(s_{-j}) \leq s_j$. Assume not, that is $r(s_{-j}) > s_j$. Then $\pi_j(s_j, s_{-j}) \geq \pi_j(r(s_{-j}), s_{-j})$ would imply $s_j \in R(s_{-j})$, a contradiction, since $s_j < r(s_{-j}) = \inf R(s_{-j})$.

*Step 2.* Assume that $s_j = r(s_{-j})$. Then, for any $\widehat{s}_j$ such that $\widehat{s}_j \geq s_j$, it must hold that

$$f\left(s_j p(s_j + \sigma)\right) - s_j \geq f\left(\widehat{s}_j p(\widehat{s}_j + \sigma)\right) - \widehat{s}_j. \tag{A.6}$$

I omit the initial endowment 1 on both sides. Since the function $\pi$ is continuous and since, from Lemma 4, $p' < 0$, it also holds that, for any $\delta > 0$,

$$f\left((s_j + \delta) p(s_j + \sigma)\right) - (s_j + \delta) \geq f\left((\widehat{s}_j + \delta) p(\widehat{s}_j + \sigma)\right) - (\widehat{s}_j + \delta). \tag{A.7}$$

Rewrite the last inequality as, after proceeding in the same way as for $\widehat{s}_j \geq s_j$,"for any $u \geq (s_j + \delta)$,

$$f\left((s_j + \delta) p((s_j + \delta) + (\sigma - \delta))\right) - (s_j + \delta) \geq f\left(u p(\sigma - \delta + u)\right) - u." \tag{A.8}$$

This reads: $s_j + \delta$ is better than $u$ against $\sigma - \delta$. Moreover, $u \geq (s_j + \delta)$. I shall use this information in the last step, step 3, and apply step 1 on these inequalities.

*Step 3.* Combining step 1 and the last equation in step 2, I get

$$r(s_{-j} - \delta) \leq s_j + \delta. \tag{A.9}$$

Letting $\delta := s_{-j} - s^n_{-j}$, this expression is equal to

$$r(s^n_{-j}) \leq s_j + (s_{-j} - s^n_{-j}). \tag{A.10}$$

Taking lim sup on both sides, I finally get

$$\limsup_{s_{-j}^n \to s_{-j}} r(s_{-j}^n) \leq s_j. \tag{A.11}$$

□

# Appendix B

# Appendix to Chapter 3 (The Proof of Proposition 6)

I look for prices $p(1), p(2), ...$ that maximise the buyer's objective function *and* that are sustainable as an equilibrium. In this equilibrium, trade always occurs and buyers never leave their sellers. Moreover, the buyer always rejects any prices higher than the ones specified by the equilibrium price sequence. If a buyer accepts a higher price the equilibrium price sequence gets restarted. By this construction I impose the most severe punishment on a defector. The most severe punishment *is* the restart of the sequence since the buyer can always leave the seller and start a new partnership. Hence, any punishment that the buyer's trading partner imposes cannot be harsher than a restart of the equilibrium price sequence and any optimal equilibrium must be supported by the most severe punishment.

The incentive constraints for the buyers that must be satisfied by any optimal equilibrium price sequence $\{p^*(t)\}_{t=1}^{\infty}$ are given by

$$\sum_{\tau=t+1}^{\infty}(1-p^*(\tau))\delta_b^{\tau-t}-\delta_b\left[\sum_{t=1}^{\infty}\delta_b^{t-1}(1-p^*(t))\right]\geq(1-p(t)) \tag{B.1}$$

for all $t$ and for all $p(t)$.

Equation (B.1) compares, at each point in time, the trade-off between a one-period gain from a deviation from the prescribed equilibrium, which is $1-p(t)$, to the discounted future losses which are given by the left hand side of equation (B.1). The first term on the left hand side is the expected discounted payoff from following the equilibrium strategy whereas the second term on the left-hand side resembles the restart from the price sequence $\{p^*(t)\}_{t=1}^{\infty}$. Incentive compatibility requires that discounted future losses exceed the one-period deviation gain at each point in time. Let $E(t,\delta_b)$ be the set of all price sequences that satisfy the incentive constraints.

An optimal price sequence is a solution to problem $(\mathcal{P})$:

$$(\mathcal{P}) \qquad \max_{p(t)\in E(t,\delta_b)}\sum_{t=1}^{\infty}\delta_b^{t-1}(1-p(t)) \tag{B.2}$$

In words: if I look for a price sequence that maximizes buyers' expected discounted utility subject to $p^*(t)$ satisfying the incentive constraints for all $t$, I must end up with $\{p^*(t)\}_{t=1}^{\infty}$ itself. Since $\{p^*(t)\}_{t=1}^{\infty}$ depends on $\delta_b$, I denote buyers' utility from an optimal equilibrium sequence as $U^*(p^*(t))\equiv U^*$.

Suppose, I wish to implement a price sequence with $p^*(1)>0, p^*(2)=p^*(3)=\ldots=0$. Then, in equilibrium, the following incentive constraints have to be satisfied for the buyers after each proposal from a seller.

I have, for $t=1$,

$$0+\frac{\delta_b}{1-\delta_b}\geq(1-p(t))+\delta_b U^* \qquad \text{for all} \quad p(t)\geq p^*(1). \tag{B.3}$$

For all $t = 2, 3, ...,$

$$0 + \frac{\delta_b}{1-\delta_b} \geq (1 - p(t)) + \delta_b U^* \qquad \text{for all} \quad p(t) \geq 0. \tag{B.4}$$

To satisfy the requirement of sequential rationality within each relationship , I have to find the incentive constraints which put the hardest restrictions on $U^*$ for any given $\delta_b$. The other constraints will be satisfied as well. It is easy to see that he crucial constraint is the one from period 2 onwards. Sequential rationality requires that I set $p(1) = p^*(1)$ and $p(t) = 0$ for all $t = 2, 3, ....$

This yields, from equation (B.4),

$$U^* \leq \left(\frac{2\delta_b - 1}{1-\delta_b}\right)\frac{1}{\delta_b}, \tag{B.5}$$

which I let hold with equality.

I show next that the proposed price sequence is optimal indeed. Let $p_A(t)$ denote any arbitrary price sequence with $0 \leq p_A(t) \leq 1$ for all $t$. Let $U^A(p_A(t))$ be the utility from an arbitrary sequence and let $U^A(t)$ be the continuation utility from period $t$ on for all $t$.

**Lemma 7.**

$$U^*(p^*(t)) \geq U^A(p_A(t))$$

*for all sequences $p_A(t)$.*

*Proof.* Suppose that there exists an alternative price sequence $\{p_A(t)\}_{t=1}\infty$ such that $U^*(p^*(t)) < U^A(p_A(t))$ and that $p_A(t) \neq 0$ for at least one $t \geq 2$ and $p_A(1) < p^*(1)$.

Fix the equilibrium with the sequence $\{p^*(t)\}_{t=1}^{\infty}$. In particular, choose $p^*(1)$ such that the incentive constraint for the equilibrium in period 1 holds with equality:

$$0 + \frac{\delta_b}{1-\delta_b} = (1 - p^*(1)) + \delta_b U^*. \tag{B.6}$$

Suppose that there exists a sequence $\{p_A(t)\}_{t=1}^{\infty}$ with $p_A(t) \neq 0$ for at least one $t \geq 2$ and $p_A(1) < p^*(1)$ such that $U^A(p_A(t)) > U^*(p^*(t)$. For $\{p_A(t)\}_{t=1}^{\infty}$ to be an equilibrium, I require in period $t = 1$,

$$0 + \delta_b U^A(2) \geq (1 - p_A(1)) + \delta_b U^A. \tag{B.7}$$

Suppose that this constraint is satisfied. Note that, by assumption, $U^A > U^*$ and $p_A(1) < p^*(1)$. Then, the LHS of (B.7) is $<$ than the LHS of (B.6) and that the RHS of (B.7) is larger than the RHS of (B.6). But then, given $\delta_b$, if $\{p^*(t)\}_{t=1}^{\infty}$ is an equilibrium, (B.7) cannot hold. This in turn implies that $\{p_A(t)\}_{t=1}^{\infty}$ cannot be an equilibrium, yielding a contradiction. □

The derivation of $p^*(1) = 1/\delta_b$ follows immediately from solving the equation

$$(1 - p^*(1)) + \frac{\delta_b}{1-\delta_b} = \left(\frac{2\delta_b - 1}{1-\delta_b}\right)\frac{1}{\delta_b}.$$

Hiermit erkläre ich, dass ich die Dissertation selbständig angefertigt und mich anderer als der in ihr angegebenen Hilfsmittel nicht bedient habe, insbesondere, dass aus anderen Schriften Entlehnungen, soweit sie in der Dissertation nicht ausdrücklich als solche gekennzeichnet und mit Quellenangaben versehen sind, nicht stattgefunden haben.

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