

University of Mannheim / Department of Economics

Working Paper Series

---

***Parental Time Investment and Intergenerational Mobility***

Minchul Yum

Working Paper 16-06

March 2016

---

---

# Parental Time Investment and Intergenerational Mobility

Minchul Yum\*

University of Mannheim

March 2016

## Abstract

This paper investigates parental time investment in children prior to formal schooling as a source of intergenerational income persistence in the U.S. I develop a dynamic general equilibrium model where lifetime income endogenously persists across generations through multiple channels. My model replicates a series of important untargeted aspects of the data including the U.S. income quintile transition matrix. I find that the parental time investment channel accounts for nearly 40 percent of the observed intergenerational income persistence. Policy experiments suggest that effective ways of improving mobility should focus on narrowing discrepancies in the quantity and quality of parental time investments.

**Keywords:** parental time, human capital investment, intergenerational persistence, college education

**JEL codes:** E24, I24, J22

---

\*Department of Economics, University of Mannheim, Germany, e-mail: minchul.yum@uni-mannheim.de. This paper is based on a chapter from my Ph.D. dissertation at the Ohio State University. I thank Aubhik Khan and Julia Thomas for their invaluable advice. I am also grateful to David Blau for helpful comments. This paper also benefited from discussions with numerous people including Laurence Ales, Nicholas Bloom, Andrew Chen, Antonio Ciccone, Seonghoon Kim, Nobuhiro Kiyotaki, John Leahy, Sang Yoon Lee, Yongseok Shin, Stephen Terry, Todd Schoellman, Michèle Tertilt as well as from seminar participants at Korea Development Institute, Ohio State University, Singapore Management University, Sogang University, University of Cologne, University of Leicester, University of Mannheim, the 9th Economic Graduate Student Conference in Washington University in St. Louis, 2014 European Winter Meeting of Econometric Society in Madrid, Spring 2015 Midwest Macro Meeting in St. Louis, 2015 SED meeting in Warsaw, 2015 Econometric Society World Congress in Montreal, and the 12th Vienna Macro Workshop.

# 1 Introduction

Empirical research has found intergenerational income mobility in the United States to be quite low (Solon, 1999; and Mazumder 2005). A growing empirical literature on the sources of intergenerational mobility, as reviewed in Black and Devereux (2011), suggests that a key determinant of intergenerational mobility in the U.S. is family background. It still remains to be understood which specific family factors are quantitatively relevant for low mobility and through what mechanisms such factors affect intergenerational mobility. For instance, the effect of parental income, an obvious candidate, on child outcomes is often found to be weak (e.g., Blau, 1999; Sacerdote, 2007), suggesting that family income per se may not be a major source of income persistence across generations.<sup>1</sup> The answers to these questions are essential for designing policies to improve intergenerational mobility.

This paper contributes to the literature on sources of intergenerational mobility by investigating the role of parental time investment in children prior to formal schooling. Specifically, I focus on the time spent directly with children in interactive activities such as reading, playing, and talking, all of which can promote development of a child’s human capital in early years. Recent studies have shown that more educated parents spend more time with their children (e.g., Guryan, Hurst and Kearney, 2008; and Ramey and Ramey, 2010) and that active parental time inputs are of first-order importance for human capital development of children at early ages (Del Boca, Flinn, and Wiswall, 2014). Moreover, it has been found that human capital gaps at the beginning of formal schooling tend to persist throughout the childhood (e.g., Heckman, 2008; Cunha, 2013) and that initial conditions of adult human capital around early 20’s are crucial to account for lifetime income inequality (e.g., Keane and Wolpin, 1997; Huggett, Ventura, and Yaron, 2011). As a result, the early parental time investment channel could be an important source shaping how lifetime income persists across generations.<sup>2</sup>

The analysis in this paper is based on a quantitative general equilibrium model with overlapping

---

<sup>1</sup>There is also empirical evidence that shows that income indeed has positive effects on children’s test scores (Dahl and Lochner, 2012). Heckman and Mosso (2014) review the literature on determinants of human capital development and argue that the overall empirical evidence for the importance of income and credit constraints is not strong. See Heckman and Mosso (2014) for more detailed discussions.

<sup>2</sup>See e.g., Knudsen, Heckman, Cameron, and Shonkoff (2006); Cunha and Heckman (2007, 2008); and Cunha, Heckman, and Schennach (2010) for evidence and discussions regarding dynamic complementarity; and Blau and Currie (2006); Heckman, Pinto, and Savelyev (2013) for experimental evidence on how improving disadvantages in early periods could have persistent effects.

generations and incomplete markets. In the model, altruistic parents care about their children's utility. Parents differ in their own human capital and assets as well as the human capital endowment of their children and decide how to split their time across investment in their child's human capital, market work, and leisure. The *quality* of the parental time investments is determined by long-term family factors captured by the parent's human capital as well as the child's human capital endowment. Once children become young adults, they make their own college decision to accumulate human capital. Parents can affect this decision indirectly both through their parental time investment at early ages, as the return to college depends on pre-college human capital, and through financial assets they transfer to their children. Adult human capital is subject to idiosyncratic shocks, which cannot be fully insured since the only available asset is physical capital. Households face not only borrowing limits in each period but also across generations since parents are not allowed borrow against their descendants' income.

I calibrate the model to the recent U.S. economy by minimizing the distance between target statistics from simulated-data and their empirical counterparts. I then evaluate the model as a quantitative theory of intergenerational mobility by examining whether it replicates a series of important untargeted aspects of the data involving the *distribution* of intergenerational income persistence. Although its calibration targets only a single intergenerational mobility statistic (the correlation between the percentile rank of parents' income and that of children's income), I find that my model successfully replicates the most salient features of the U.S. income quintile transition matrix. In both model and data, the intergenerational persistence of income is considerably higher in the bottom and top income quintiles than in the second, third, and fourth quintiles. More specifically, in the U.S., the probability of children remaining in the bottom quintile when their parents' income lies in the bottom quintile is 34 percent, and the probability of children staying in the top quintile when their parents' income belong to the top quintile is 37 percent (Chetty, Hendren, Kline and Saez, 2014a). In my model, these probabilities are 35 and 36 percent, respectively. By contrast, the probabilities of intergenerational income staying in the second, third, and fourth quintiles are 22-24 percent in U.S. data (Chetty, Hendren, Kline and Saez, 2014a) and 21-23 percent in my model. To the best of my knowledge, mine is the first paper to evaluate a candidate model as a quantitative theory of intergenerational mobility by confronting it with the empirical income quintile transition matrix, and to thereby establish its success in explaining those disaggregated

moments. To further corroborate its use, I confirm that my model quantitatively accounts for other regularities relevant to its key mechanisms, such as the widening gaps in human capital over early periods and the positive relationship between college completion and pre-college human capital.

I use my model to investigate the role of the parental time investment channel in shaping the intergenerational persistence of lifetime income. I find that heterogeneity in the quantity of early parental time investment alone can account for nearly 20 percent of the observed intergenerational persistence of income. When I consider both the quantity and quality of parental time investments, I find that the parental time investment channel can account for approximately 40 percent of the observed intergenerational persistence of lifetime income. There are two main reasons for the above findings. First, despite their higher opportunity costs of time, it is parents with higher human capital that choose to invest more time in their young children. This force amplifies the intergenerational correlation of human capital that would arise solely from the factors that determine human capital at birth.<sup>3</sup> This is because a child born into a family with high human capital benefits from parents more than a child with the equal human capital endowment born into a family with low human capital due to both greater quantity and quality of parental time investment. Second, because the parental time investment channel features dynastic smoothing, parents tend to invest more in a child with a relatively low human capital endowment.<sup>4</sup> This dynastic smoothing motive moves children even closer to their parents' status, thereby raising the intergenerational correlation of income further. The above two mechanisms imply that the children who are especially affected by their family background are *able* children born into low human capital families. The low parental time investment they receive hinders mobility and aggregate efficiency. I also examine the role of the other channels such as college education in shaping the intergenerational persistence of lifetime income. I find that the college channel can account for slightly less than 10 percent of the observed intergenerational income persistence. A key reason is that pre-college human capital, formed early in childhood, is a significant factor that determines the college education decision.

I also use my model to consider the efficacy of a series of policy interventions designed to

---

<sup>3</sup>In the model, the pre-birth factors may represent various factors including not only pure nature (genetic transmission) but also assortative marital sorting and prenatal investment. The goal of this paper is to examine specific channels such as parental time investment along with financial transfers and college education in shaping intergenerational persistence while allowing for other channels in a parsimonious way.

<sup>4</sup>This dynastic motive to smooth the marginal value of human capital across generations is tantamount to the infinitely-lived households' motive to smooth consumption (or the marginal utility) intertemporally through savings.

increase intergenerational mobility. I consider each policy's implications not only for intergenerational persistence estimates, but also for aggregate GDP and average welfare in order to evaluate whether those policies that do reduce the intergenerational persistence of inequality are otherwise desirable for the overall economy. In each instance, I take care to account for the implied changes in equilibrium prices, as well as the additional tax burden required to finance the policy by satisfying the government budget constraint. The first set of experiments I conduct aims to facilitate access to college by lowering college costs or relaxing borrowing limits. While these policies raise college completion rates, I find that they do not guarantee greater intergenerational mobility. This is because there is positive self-selection into college completion. Specifically, since those who decide to go to college even before the policy change tend to have higher pre-college human capital as well as higher returns to college than marginal students, facilitating college access for marginal students does not increase intergenerational mobility.

My second set of policy experiments aims to lower the opportunity cost of parental time investment toward encouraging parents to increase time investments in their children. These experiments are motivated by the model's prediction that children born with high human capital endowments to parents with low human capital receive lower parental time investments than the average child in such families despite their potential to accumulate human capital quickly and move up the income ladder. I find that a flat subsidy to parental time investments can increase intergenerational mobility because it disproportionately increases time investment in children whose parents have low human capital and thus low wages. Those rises in mobility are accompanied by sizeable aggregate output gains and average welfare gains. Finally, I consider a policy that approximates universal preschool programs. Because this policy narrows discrepancies not just in the quantity of time investments in young children but also in their quality, I find that it delivers larger rises in intergenerational income mobility and greater output and welfare gains than the parental time investment subsidy.

In addition to the large empirical literature on intergenerational mobility and parental investment mentioned above, my paper is related to a number of theoretical studies involving the intergenerational persistence of inequality.<sup>5</sup> A growing literature investigates different sources of

---

<sup>5</sup>There are also theoretical studies involving efficient parental investments. For example, Aiyagari, Greenwood and Seshadri (2002) study efficient parental investments depending on the market structure and altruism.

intergenerational economic persistence using quantitative dynamic equilibrium models with heterogeneous households (e.g., Restuccia and Urrutia, 2004; Holter, 2014; Lee and Seshadri, 2014; and Herrington, 2015). My paper is distinguished in this literature by the fact that the calibrated model herein matches not only the targeted empirical correlation of income across generations, as is standard in the literature, but also the non-targeted U.S. income quintile transition matrix, as discussed above.

A second important distinguishing feature of my paper is its explicit focus on the parental time investment channel, which has so far received almost no attention in the theoretical literature involving intergenerational mobility to which my paper contributes. Lee and Seshadri (2014) is the one exception that also models parental time investments explicitly. Their quantitative model accounts for various aspects of data such as the intergenerational elasticity and cross-sectional inequality, as does mine. Interestingly, their results suggest that the intergenerational elasticity is largely affected by parental investment, but is nearly *unaffected* by innate ability transmission. My paper complements the Lee and Seshadri’s analysis by stripping away channels other than parental time investment to isolate its implications.<sup>6</sup> It also reconciles the theory with a body of empirical evidence suggesting that pre-birth factors are important to the intergenerational transmission of socioeconomic status.<sup>7</sup> Specifically, unlike their results, my results indicate that pre-birth factors are quantitatively equally as important as parental investments in explaining intergenerational income persistence. The key modeling distinctions explaining our differing results rest in a set of assumptions involving human capital and adulthood market luck. Lee and Seshadri distinguish innate ability (nature in their model) from human capital and assume that heterogeneity in human capital arises in later childhood. By contrast, following Cunha and Heckman (2007), I do not distinguish ability from human capital and model heterogeneity in human capital endowments at birth. This proves to be a key ingredient for economic mobility in my model, due to parents’ dynastic smoothing motives. The degree of intergenerational lifetime income mobility depends not only on childhood events but also on adulthood through intra-generational mobility. Whereas a one-time random draw resolves all adult income uncertainty in the Lee and Seshadri’s model, my

---

<sup>6</sup>Lee and Seshadri (2014) briefly consider the role of parents’ time investments in one experiment by altering the human capital production technology in their model (Table 6).

<sup>7</sup>The empirical literature in economics, behavioral genetics, and sociology often finds that pre-birth factors account for a significant portion of intergenerational persistence. See Sacerdote (2010) for an extensive survey.

model includes idiosyncratic shocks to human capital, period-by-period, over each adult’s working life. This distinction allows my model to incorporate intra-generational mobility while achieving greater consistency with the life-cycle profile of inequality in the data, and avoiding reliance on an empirically questionable negative correlation between innate ability and market luck.

My paper is also related to the literature that uses equilibrium models of human capital investment across generations to study policies designed to raise human capital of children from disadvantaged families (e.g., Fernandez and Rogerson, 1998; Caucutt and Lochner, 2012; Cunha, 2013). So far, this literature has concentrated on parents’ inadequate financial investments in children’s human capital due to credit constraints. In contrast, my paper highlights the role of the quantity and quality of parental time investments in improving human capital of children from disadvantaged families. Finally, my paper also complements the work of Huggett et al. (2011), who show that differences in initial conditions, especially human capital, at age 23 account for a large fraction of lifetime inequality.<sup>8</sup> My contribution is to endogenize the acquisition of human capital before adulthood in order to examine how family-related factors and college education shape the differences in human capital in early adult life.

The paper is organized as follows. Section 2 describes the model. Section 3 explains how the parameters of the baseline model economy are calibrated, and evaluates the baseline model economy as a quantitative theory of intergenerational mobility. Section 4 presents the main quantitative analysis of intergenerational mobility, and Section 5 provides results from policy experiments designed to increase intergenerational mobility. Section 6 concludes.

## 2 Model environment

The economy is populated by overlapping generations of a continuum of households. A household is composed of an adult who lives with a child until the child grows up. An adult lives for eleven model periods (age 20-74) as an economic decision maker. One model period corresponds to five years. In Table 1, I summarize the timeline of the lifecycle events for a pair of overlapping generations for illustration. An adult supplies labor beginning at period  $j = 1$  (age 20) until retirement at  $j = 10$

---

<sup>8</sup>Using structural estimation, Keane and Wolpin (1997) also find that unobserved endowment heterogeneity at age 16 accounts for 90 percent of the variance in lifetime utility whereas exogenous shocks to skills over the lifetime accounts for the rest.

Table 1: Timeline of the life-cycle events

		Parent's age								
		20-24	25-29	...	45-49	50-54	...	65-69	70-74	
Parent	Model age	1	2	...	6	7	...	10	11	
	Market work	← - - - - Labor-leisure - - - - →					Retired - - - - →			
	Human capital investment	College	Parental Time							
Child	Model age	Birth			1	2	...	5	6	
	Market work	← - - - - Labor-leisure - - - - →								
	Human capital investment			College	Parental Time					

(age 65). An adult lives for two periods after retirement and dies at the end of period  $j = 11$ . The next generation is born when parents reach period  $j = 2$ . After 20 years, a child becomes an adult head of a new household facing the same lifetime structure as described above.

Households differ in their human capital, asset holding and age.<sup>9</sup> Human capital endowment at birth is heterogeneous, partially depending on the parent's level of human capital. Human capital investment takes two forms: (i) parental time investment in young child's human capital prior to formal schooling; and (ii) college education. Human capital may also grow due to the mechanical channels such as primary and secondary education, and experience. While an individual works, her human capital is subject to idiosyncratic shocks, as in Huggett et al. (2011). The capital market is incomplete and the only available asset is risk-free physical capital on which the rate of return is  $r$ . Thus the idiosyncratic shocks over the life cycle cannot be fully insured. The household's recursive problems over the life cycle are described in detail in the next subsection.

There is a representative firm which produces output with constant returns to scale technology. Its production function is assumed to be Cobb-Douglas  $Y = F(K, L) = AK^{\alpha_K}L^{1-\alpha_K}$  where  $K$  denotes aggregate capital,  $\alpha_K$  is the output elasticity of capital and  $L$  denotes aggregate efficiency units of labor in the economy. Capital depreciates at the rate of  $\delta$ .

There is a government that taxes labor earnings at the rate of  $\tau_w$  and capital income at the rate of  $\tau_a$ . The revenue is used to provide social security payments to retirees and lump-sum transfer

<sup>9</sup>In my model, human capital is composed of not only factors affected by families and education but also ability. This is in line with Cunha and Heckman (2007) who note "Measured abilities are susceptible to environmental influences, including in utero experiences, and also have genetic components. These factors interact to produce behaviors and abilities that have both a genetic and an acquired character." Therefore, I use the term, human capital (or skills), interchangeably with ability.

$\omega$  to all households. The proportional tax rates,  $\tau_w$  and  $\tau_a$ , along with the transfer  $\omega$  effectively provides an environment with progressive taxation that is found to affect intergenerational mobility (e.g., see Erosa and Koreshkova, 2007; and Holter, 2014). Government balances its budget each period.

## 2.1 Household's decision problems

This paper considers stationary environments in which all prices and aggregate quantities are constant over time. Therefore, the time index for the variables is omitted and I present the household's decision problems recursively.

### College education stage:

A child becomes an independent economic decision maker in the model period  $j = 1$  (20 years old) with a human capital stock of  $\theta$ . As will become clear below, this level of human capital, formed during the early childhood, is influenced by the parent. An important decision to be made at  $j = 1$  is whether to complete college education or not. College completion requires both the random physical cost  $\xi$  and the fixed time input  $\psi$ . After observing the random fixed cost draw  $\xi$ , households make a discrete choice regarding college education. The household's value at  $j = 1$  before the realization of  $\xi$  is given by

$$V_{j=1}(\theta, a) = E_{\xi} \max [\Gamma_0(\theta, a), \Gamma_1(\theta, a, \xi)]. \quad (1)$$

where  $\Gamma_0(\theta, a)$  is the value without a college degree, and  $\Gamma_1(\theta, a, \xi)$  is the value of completing college education.

The household's value if the agent chooses not to go to college is given by

$$\Gamma_0(\theta, a) = \max_{\substack{c \geq 0; a' \geq a_1 \\ n, l \in [0, 1]}} \left\{ U(c, l) + \beta \int V_{j=2}(\theta', a') dG(z') \right\} \quad (2)$$

$$\begin{aligned}
\text{subject to } c + a' &\leq (1 - \tau_w)w\theta n + [1 + (1 - \tau_a)r]a + \omega \\
n + l &\leq 1 \\
\theta' &= \exp(z')\gamma\theta
\end{aligned}$$

where  $c$  is consumption,  $l$  is leisure,  $w$  is the rental price of human capital per unit hours of work (or market wage),  $a$  is the level of assets determined by the previous generation's financial transfer decision, and a variable with a prime denotes its value in the next period. The household's after-tax earnings depend on the individual-specific wage  $w\theta$ , hours of work  $n$ , and the labor tax rate  $\tau_w$ . The capital return  $ra$  is taxed at the rate of  $\tau_a$ . In every period, households receive a lump-sum transfer  $\omega$  from the government.

The level of human capital exogenously increases at the gross growth rate of  $\gamma > 1$  and is subject to the idiosyncratic shock (or market luck)  $z$  of which the cumulative distribution is  $G(z)$ . As in Huggett et al. (2011), I assume that the  $z$  follows an i.i.d. normal distribution. Note that although  $z$  is drawn from an i.i.d. distribution, its effect is persistent over the rest of the life. This is because  $z$  is not a shock to earnings but rather a shock to human capital, which essentially follows a random walk with an age-dependent deterministic drift in logs.<sup>10</sup>

The value of completing college education after the realization of a fixed cost  $\xi$  is given by

$$\Gamma_1(\theta, a, \xi) = \max_{\substack{c \geq 0; a' \geq a_1 \\ n, l \in [0, 1]}} \left\{ U(c, l) + \beta \int V_{j=2}(\theta', a') dG(z') \right\} \quad (3)$$

$$\begin{aligned}
\text{subject to } c + a' + \xi &\leq (1 - \tau_w)w\theta n + [1 + (1 - \tau_a)r]a + \omega \\
n + l + \psi &\leq 1 \\
\theta' &= \exp(z')(\gamma + \Delta)\theta.
\end{aligned}$$

In my model, the benefit of college education is represented by  $\Delta > 0$ , an increment in the growth rate of human capital. As described above, the college costs consist of the direct resource cost  $\xi$  as

<sup>10</sup>For example, taking the log of the law of motion for the human capital in (2), we get

$$\log \theta' = \log \gamma + \log \theta + z'.$$

where a drift term,  $\log \gamma$ , acts as a deterministic trend.

well as the time cost  $\psi$ , implying that the opportunity cost of college education includes foregone earnings. The borrowing limit, faced by an agent whose age is  $j$ , is denoted by  $\underline{a}_j$ .

**Parental time investment stage:**

At the beginning of period  $j = 2$ , each household is endowed with a child. I assume that the child shares the household consumption  $c$  and does not make time allocation decisions relevant to the household's economic status during childhood. The child's human capital endowment at birth  $\theta_0 = \phi(\theta, \zeta)$  depends on both parent's human capital  $\theta$  and the idiosyncratic component  $\zeta$ . This function  $\phi$  generates a positive correlation of the human capital endowment  $\theta_0$  with the parent's human capital  $\theta$ . In other words, high human capital parents are more likely to have high human capital children. The household's state variables also include  $a$ , the level of asset holding determined in the previous period. The value of the household at period  $j = 2$  before the realization of  $\zeta$  is given by

$$V_{j=2}(\theta, a) = E_{\zeta}W(\theta, a, \zeta) \quad (4)$$

The functional equation summarizing a parent's decision problem after observing the child's ability  $\theta_0 = \phi(\theta, \zeta)$  is given by

$$W(\theta, a, \zeta) = \max_{\substack{c \geq 0; a' \geq \underline{a}_2 \\ n, l, h \in [0, 1] \\ s \in \{0, s_l, s_h\}}} \left\{ U\left(\frac{c}{q}, l\right) + \beta \int V_{j=3}(\theta', a', s) dG(z') + \eta \beta^4 V_{j=1}(\theta_c, a_c) \right\} \quad (5)$$

$$\text{subject to } c + a' \leq (1 - \tau_w)w\theta n + [1 + (1 - \tau_a)r]a + \omega$$

$$n + l + h \leq 1$$

$$\theta' = \exp(z')\gamma\theta$$

$$\theta'_0 = f(\theta, h, \phi(\theta, \zeta)), \quad \theta_c = (\theta'_0)^{\alpha_c}$$

$$a_c = \sum_{t=0}^2 [1 + (1 - \tau_a)r]^t s$$

where  $q$  denotes the household equivalence scale,  $\eta \geq 0$  measures the degree of altruism.

The intergenerational link is modeled following the dynastic utility approach in that parents care about their child's utility, which depends on the next generation's utility, and so on. This

recursive structure linked by altruism combines successive generations as a single dynasty as in Becker and Tomes (1986). In contrast to the standard Becker-Tomes type altruism, parents in my model care about the child's utility derived not only from consumption but also from leisure since the utility function is defined over both consumption and leisure. Note that, with a positive  $\eta$ , parents invest their time in their child and have an incentive to make financial transfers since they care about the value of their child's life when she becomes an adult in 20 years, which is represented by the last term of the objective function:  $\eta\beta^4 V_{j=1}(\theta_c, a_c)$ . Optimal parental time investment  $h$  is determined based on equating the marginal value of time in investment, leisure, and market work.

The production function  $\theta'_0 = f(\theta, h, \theta_0)$  describes how a child's developed human capital  $\theta'_0$  at age 5 is formed depending on parental human capital  $\theta$ , parental time investment  $h$ , and the child's human capital endowment at birth  $\theta_0$ , in the spirit of Cunha and Heckman (2007). The intergenerational human capital production function  $f(\theta, h, \theta_0)$  is assumed to have the following properties: (i)  $f_1, f_2, f_3 > 0$ ; (ii)  $f_{22} < 0$ ; and (iii)  $f_{21}, f_{23} > 0$ . Note that the last properties imply that the marginal return on parental time investment increases with parent's ability ( $f_{21} > 0$ ) and child's ability ( $f_{23} > 0$ ). The level of human capital after the primary and secondary school periods,  $\theta_c$ , is assumed to be determined by a simple mapping  $\theta_c = (\theta'_0)^{\alpha_c}$ .

For tractability, I assume that the parental financial transfers decision is discrete and irreversible. Specifically, households are assumed to save an amount  $s \in \{0, s_l, s_h\}$  starting the next period ( $j = 3$ ) until  $j = 5$ . And the total amount of savings in this account is accumulated up to  $a_c$  (with after-tax interests) at the beginning of  $j = 6$ , which is then transferred to the next generation that forms a new household. It is worth noting that this financial transfer effectively serves as parental monetary investment inputs for college education later in children's life since it can help their college decision financially.

It is worth discussing some modeling assumptions at this point. First, I assume that parents can invest time in early periods while they can invest money for their children's college decision. This simplifying assumption not only lessens computational burden but also is reasonable given the recent evidence by Del Boca et al. (2014) that, for the children under 5, active parental time inputs are three to four times as productive as parental expenditures while, their relative importance is completely reversed as children become older. Second, although the inter-vivos transfer decision that helps finance college education is modeled, the decision on bequests is not. Abstracting from

bequests actually provides a better mapping from the data to the model because most empirical studies on intergenerational income (earnings) mobility including the empirical benchmark of the current paper (Chetty et al., 2014a), use matched samples in which the children are relatively young (around or above 30 years old), who are unlikely to be affected by bequests.<sup>11</sup>

**Remaining working stages:**

Households keep making a typical work-leisure decision and consumption-saving decision for periods  $j = 3, \dots, 9$  (age 30-64) until they are retired at  $j = 10$ . The household is composed of a parent and a child until the end of  $j = 5$  when the child forms a new household. Recall that a parent who committed to leave the financial transfer to the child keeps saving  $s$  while living with the child. Therefore, the state variables at periods  $j = 3, 4, 5$  include  $s$  as well. Once the child becomes an adult, the parent's decision to leave the transfer does not affect their choice. Hence, from period  $j = 6$ , the state variables do not include  $a_c$ . The household's problem in these periods can be described recursively as

$$V_j(\theta, a, s) = \max_{\substack{c \geq 0; a' \geq a_j \\ n, l \in [0, 1]}} \left\{ U\left(\frac{c}{q}, l\right) + \beta \int V_{j+1}(\theta', a', s) dG(z') \right\} \quad \text{if } j = 3, 4, 5 \quad (6)$$

$$\text{subject to } c + a' + s \leq (1 - \tau_w)w\theta n + [1 + (1 - \tau_a)r]a + \omega$$

$$n + l \leq 1$$

$$\theta' = \exp(z')\gamma\theta$$

$$V_{j=6}(\theta, a, a_c) = V_{j=6}(\theta, a)$$

and

$$V_j(\theta, a) = \max_{\substack{c \geq 0; a' \geq a_j \\ n, l \in [0, 1]}} \left\{ U(c, l) + \beta \int V_{j+1}(\theta', a') dG(z') \right\} \quad \text{if } j = 6, 7, 8, 9 \quad (7)$$

---

<sup>11</sup>However, it should be noted that it would probably be necessary to model bequest motives if one is interested in intergenerational mobility of *wealth* since bequests play an important role in explaining the distribution of wealth (e.g., De Nardi, 2004). This is left for future work.

$$\begin{aligned}
\text{subject to } c + a' &\leq (1 - \tau_w)w\theta n + [1 + (1 - \tau_a)r]a + \omega \\
n + l &\leq 1 \\
\theta' &= \exp(z')\theta
\end{aligned}$$

Note that, although agents are not allowed to accumulate human capital endogenously after period  $j = 1$ , their human capital accumulates exogenously at the gross growth rate of  $\gamma > 1$  until  $j = 5$ . Thereafter, the growth rate stays constant at one. This structure parsimoniously generates the shape of the empirical age-profile of wage that rises initially and stays flat near the retirement (see e.g., Rupert and Zanella, 2012; Casanova, 2013 for recent evidence).<sup>12</sup>

I assume that the borrowing limit right before the retirement ( $j = 9$ ) is zero (i.e.  $\underline{a}_9 = 0$ ). This guarantees that households cannot borrow against their future social security accounts.

### Retirement stage:

When households retire ( $j = 10, 11$ ), they receive social security payments  $g(\theta)$ . This function is increasing and concave in human capital (or wage) just before retirement in order to approximate progressive U.S. social security in a simple manner. I assume that households are not allowed to be in debt at the retirement stage. The value at the retirement stage is given by

$$V_j(\theta, a) = \max_{c \geq 0; a' \geq 0} \{U(c, 1) + \beta V_{j+1}(\theta, a')\} \quad (8)$$

$$\text{subject to } c + a' \leq g(\theta) + [1 + (1 - \tau_a)r]a + \omega$$

and  $V_{j=12}(\theta, a) = 0$ .

## 2.2 Equilibrium

Let  $x_j \in X_j$  denote the age-specific state space defined according to the household's recursive problems in the previous subsection. A stationary recursive competitive equilibrium is a collection of factor prices  $w, r$ , the household's decision rules  $a_{j+1}(x_j), n_j(x_j), l_j(x_j), h(x_2), s(x_2)$ , value functions  $V_j(x_j)$  and age-specific measures  $\pi_j$  over  $x_j$  such that

---

<sup>12</sup>The hump-shaped earnings profile observed in the data does not need to rely on the hump-shaped wage profile since hours of work, endogenously determined by households, fall near the retirement, as observed in U.S. data.

1. given factor prices,  $a_{j+1}(x_j), n_j(x_j), l_j(x_j), h(x_2), i(x_2), s(x_2)$  solve the household's optimization problems defined in the previous subsection, and  $V_j(x_j)$  are the associated value functions,
2. factor prices are competitively determined:

$$w = F_2(K, L) \quad (9)$$

$$r = F_1(K, L) - \delta, \quad (10)$$

3. markets clear:

$$\sum_{j=1}^{11} \mu_j \int_{X_j} a_j d\pi_j + \sum_{j=3}^4 \mu_j \int_{X_j} \left[ \sum_{t=0}^{j-3} (1+r)^t s_j \right] d\pi_j = K \quad (11)$$

$$\sum_{j=1}^9 \mu_j \int_{X_j} \theta_j n_j(x_j) d\pi_j = L \quad (12)$$

where  $\mu_j$  is the fraction of households living in period  $j$ ,

4. government budget balances:

$$\begin{aligned} & G + \omega + \sum_{j=10}^{11} \mu_j \int_{X_j} g(\theta_j) d\pi_j \\ &= \sum_{j=1}^9 \mu_j \int_{X_j} \tau_w w \theta_j n_j(x_j) d\pi_j + \sum_{j=1}^{11} \mu_j \int_{X_j} \tau_a r a_j d\pi_j + \sum_{j=3}^4 \mu_j \int_{X_j} \left[ \sum_{t=0}^{j-3} r^t s_j \right] d\pi_j \end{aligned} \quad (13)$$

where  $G$  is non-negative,

5. the vector of age-specific measures of households  $\pi = (\pi_1, \pi_2, \dots, \pi_{11})$  is the fixed point of  $\pi(X) = P(X, \pi)$  where  $P(X, \cdot)$  is a transition function determined by the household decision rules and the exogenous probability distributions of  $z, \zeta$  and  $\xi$ ; and  $X$  is the generic subset of the Borel  $\sigma$ -algebra  $B$ , defined over the state space  $\mathbf{X} = \prod_{j=1}^{11} X_j$ .

### 3 Calibrating the baseline model

I calibrate parameter values of the baseline model economy to match relevant U.S. statistics since 1990. There are two sets of parameters. The first set of parameters is chosen externally without

Table 2: Parameters chosen externally

Parameter		Description or related target
$\sigma$	1.5	IES for consumption = 0.67
$\varepsilon$	2.5	IES for leisure = 0.40
$q$	1.3	OECD-modified equivalence scale
$\gamma$	1.06	Lifecycle wage slope $\bar{w}_{j=9}/\bar{w}_{j=2} = 1.26$
$\alpha_c$	1.06	Change in $std(\log(h))$ during K-12 = 6%
$\psi$	0.26	Academic time spent by college students
$\tau_w$	0.27	Tax rate on labor earnings
$\tau_a$	0.4	Tax rate on capital income
$\alpha_K$	0.36	Capital share
$\delta$	0.3	Five-year capital depreciation rate
$A$	5.0	Output scale
$m_\zeta$	$\exp(0.5)$	Human capital scale

using model-generated data while the second set of parameters is determined jointly by minimizing the distance between the statistics from the simulated-model and their counterparts from U.S. data.

### 3.1 Parameters chosen externally

I assume that all households have identical preferences over consumption  $c$  and leisure  $l$ , represented by a standard separable utility function

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + B \frac{l^{1-\varepsilon}}{1-\varepsilon}. \quad (14)$$

The first two parameters,  $\sigma$  and  $\varepsilon$ , in Table 2 govern the household's preference. I set the value of  $\sigma$  equal to 1.5 so that the intertemporal elasticity of substitution for consumption is 0.67 and the value of  $\varepsilon$  equal to 2.5, which implies an intertemporal elasticity of substitution for leisure of 0.4. The implied Frisch elasticity of labor supply is roughly twice as large as this value at the steady state hours of work. These parameter values lie within a broad range of their empirical estimates. As discussed in the previous section, when a parent lives with a child, consumption in the utility function is replaced by  $c/q$ . I set  $q$  to 1.3 according to the OECD-modified equivalence scale which assigns 1 to the first adult and 0.3 to a child.

The gross growth rate of human capital in young periods ( $j = 1, \dots, 5$ ) is calibrated to match the average wage growth over the lifetime. The choice of  $\gamma = 1.06$  implies that the average wage

at  $j = 9$  (age 60-64) is 26 percent higher than the average wage at  $j = 2$  (age 25-29) if lifetime idiosyncratic shocks are set to mean zero. This slope is close to the recent empirical evidence on life cycle profiles in Rupert and Zanella (2012). The slope parameter of the mapping that governs changes in human capital during the primary and secondary education,  $\alpha_c$ , is set to 1.06. This implies that dispersion of human capital, measured by the standard deviation of log human capital, increases by 6 percent during this period (Restuccia and Urrutia, 2004). In the model, human capital investment in college education requires both time and resources. I set the time cost  $\psi = 0.26$  consistent with the empirical finding that academic time invested by full-time college students is about 27 hours per week in 2003 (Babcock and Marks, 2011). The resource costs are random and are discussed in the next subsection.

I set the tax rates on labor earnings ( $\tau_w$ ) and capital income ( $\tau_a$ ) equal to 0.27 and 0.40, respectively (Domeij and Heathcote, 2003). The capital share in the aggregate U.S. data leads to the choice of  $\alpha_K = 0.36$ . The five-year capital depreciation rate is set to  $\delta = 0.3$ . These parameter values are within the range commonly used in the quantitative macroeconomics literature. I set the two scale parameters to  $A = 5$  and  $m_\zeta = \exp(0.5)$  where  $m_\zeta$  is the mean of the idiosyncratic component of ability at birth  $\zeta$  which is defined below. These two parameters set the unit of the output and human capital, respectively. The maximum borrowing limit is set to zero for all ages in the baseline specification.

### 3.2 Parameters chosen jointly using simulation

Table 3 summarizes the remaining 14 parameters that are jointly determined by simulating the model economy. These parameter values are determined as minimizers of the distance between the relevant statistics from the data and those from the model-generated data (see Appendix B for the details). I describe the role of these parameters in the model, and discuss how these parameters linked to the relevant statistics summarized in Table 4.

#### Preference:

First,  $B$  is the parameter which determines the relative weight of leisure compared to consumption. The relevant target is the average weekly hours of work:  $41.3/105 = 0.394$ . Note that I assume that the weekly feasible time endowment is  $15 \times 7$  hours, excluding time for sleeping and basic per-

Table 3: Parameters chosen internally using simulation

Parameter	Description	Functional form
<i>Preference</i>		
$B$	0.361 Utility constant	$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + B \frac{l^{1-\varepsilon}}{1-\varepsilon}$
$\beta$	0.939 Discount factor	
$\eta$	0.259 Degree of altruism	
<i>Human capital dynamics</i>		
$\alpha_1$	0.633 Early human capital	$f(\theta, h, \theta_0)$
$\alpha_2$	0.498 formation technology	$= \theta_0 + (\theta h)^{\alpha_1} \theta_0^{\alpha_2}$
$\alpha_0$	0.176 Initial endowment	$\phi(\theta, \zeta) = \theta^{\alpha_0} \zeta^{1-\alpha_0}$
$\sigma_\zeta$	0.562 Idiosyncratic component	$\log(\zeta) \sim N(m_\zeta, \sigma_\zeta^2)$
$\sigma_z$	0.171 Shocks to adult human capital	$z \sim N(0, \sigma_z^2)$
<i>College education</i>		
$m_\xi$	1.17 College financial costs	$\log(\zeta) \sim N(m_\zeta, \sigma_\zeta^2)$
$\sigma_\xi^2$	0.897	
$\Delta$	0.364 Human capital growth enhancement	
<i>Financial transfer</i>		
$a_h$	1.97 Parental financial transfer	$a_l = 0.5a_h$
<i>Government</i>		
$\omega$	0.509 Lumpsum transfer	
$m$	1.26 Social security	$g(\theta) = m \log(1 + \theta)$

sonal care. All statistics regarding time-use are obtained from the 2003-2012 waves of the American Time Use Survey (ATUS). The next parameter  $\beta$  is the household's discount factor. This parameter largely affects the capital-output ratio, and the relevant target is the ratio of capital to annual output, which is 2.95 in U.S. data (e.g., Huggett et al. 2011). The degree of altruism is governed by  $\eta$ . This parameter increases an incentive to invest time in the next generation. Hence, an additional statistic is the unconditional mean of the parental time investment. To compute statistics regarding parental time investment, I focus on sub-categories such as educational and recreational activities for parents who have a child less than 5 years old (see Appendix C for details). This number is 6.3 hours per week or 0.060 (= 6.3/105) in the model.

### Human capital dynamics:

I now move on to the parameters that affect individuals' human capital dynamics in the model. The intergenerational human capital production function is assumed to be

$$f(\theta, h, \theta_0) = \theta_0 + (\theta h)^{\alpha_1} \theta_0^{\alpha_2}, \quad (15)$$

Table 4: Target statistics from U.S. data and the model-generated data

Target statistics	Data	Model
Average hours worked	0.394	0.393
Capital-output ratio (annual)	2.95	2.92
Average parental time investment	0.060	0.060
Educational gradient in parental time investment ( $\bar{h}_{col}/\bar{h}_{less}$ )	1.35	1.35
Changes in human capital gaps from birth to age 5	2.0	1.97
Intergenerational correlation of percentile-rank income	0.341	0.341
Cross-sectional variance of log wage	0.40	0.402
Slope of variance of log wage from age 25-29 to age 55-59	0.18	0.180
Average college expenses/GDP per-capita	0.176	0.172
Observed college wage gap	1.75	1.76
Fraction with a college degree (%)	30.6	30.5
Average inter-vivos transfers/GDP per-capita	0.047	0.047
Post-govt variance of log wage/Pre-govt variance of log wage	0.60	0.600
Average social security replacement rate	0.40	0.401

which satisfies the three sets of assumptions discussed in Section 2.1. This adds the two parameters ( $\alpha_1$  and  $\alpha_2$ ) which govern productivity with respect to effective parental time ( $\theta h$ ) and the child's initial endowment ( $\theta_0$ ). The first relevant target statistic for these parameters is the gap between the parental time spent by college-educated parents and that spent by less-educated parents. According to the 2003-2012 ATUS data set, parents with a college degree spend 35 percent more time (7.6 hours per week) than parents without a college degree (5.7 hours per week). This educational gradient of 1.35 serves as a target statistic.<sup>13</sup> As for the other relevant statistic, I make use of the changes in the distribution of human capital in the first five years. Specifically, Cunha (2013) finds that the gap in human capital, measured by the mean standardized skills by quartiles of permanent family income, opens up at early ages.<sup>14</sup> Figure 2 in Cunha (2013) shows that the gap between the conditional mean of the second quartile and that of the third quartile nearly doubles between birth and age 5. I use this change of the ratio as the other target for the early human capital production technology parameters.

The initial endowment of human capital is assumed to be determined by

$$\theta_0 = \phi(\theta, \zeta) = \theta^{\alpha_0} \zeta^{1-\alpha_0}, \quad (16)$$

<sup>13</sup>Guryan et al. (2008) provide detailed evidence showing that this positive gradient is a very robust feature.

<sup>14</sup>The mean standardized skills is defined as the average of the normalized human capital ( $\frac{\theta - E(\theta)}{sd(\theta)}$ ). The permanent family income is defined as the discounted sum of parent's lifetime income.

so that the newborn's ability  $\theta_c$  is the weighted average of their parent's ability  $\theta$  and the idiosyncratic component  $\zeta$  in logs. Therefore, this setting adds a single weighting parameter  $\alpha_0$ . The weight  $\alpha_0$  largely affects the degree of association across generations. I set the relevant target for  $\alpha_0$  as the percentile rank correlation of family income of 0.341 (Chetty et al., 2014a), which has a relatively stable trend in recent U.S. data (Chetty, Hendren, Kline, Saez, and Turner, 2014b). As is common in the literature due to the data limitation, Chetty et al. (2014a) estimate intergenerational persistence using the proxy income variable. The percentile rank correlation from the model is also obtained from the proxy incomes equivalently defined (see the next section for the precise definition of the proxy income).

The idiosyncratic component  $\zeta$  is assumed to follow a log normal distribution:  $\log(\zeta) \sim N(m_\zeta, \sigma_\zeta^2)$ . The idiosyncratic shocks to adult human capital  $z$ , following a normal distribution, have mean zero with the standard deviation of  $\sigma_z$ :  $z \sim N(0, \sigma_z^2)$ . Since these two standard deviations,  $\sigma_\zeta$  and  $\sigma_z$ , are the exogenous sources of the cross-sectional dispersion of wages in the model, I choose the cross-sectional variance of log wage as a target statistic. It is important to note, however, that, although the degree of wage inequality monotonically increase with both  $\sigma_z$  and  $\sigma_\zeta$ , their economic mechanism is very different. This is because  $\sigma_z$  affects households over the working life while  $\sigma_\zeta$  affects the variability of the initial condition in human capital. For instance, holding the overall dispersion of wage constant, in the case when  $\sigma_z$  is relatively larger, households would experience more volatile idiosyncratic shocks to human capital, the effect of which accumulates over the life cycle. As a result, the lifecycle profile of wage inequality would become steeper. Therefore, I choose the difference between the variance of log wage at age 55-59 and that of log wage at age 25-29 as an additional target to pin down the relative contribution of each shock process to the overall wage inequality. These statistics on wage inequality in U.S. data for recent periods, obtained from Heathcote, Perri and Violante (2010), are reported in Table 4.

### **College education:**

In addition to the time cost already discussed, there are financial costs following a log normal distribution:  $\log(\xi) \sim N(m_\xi, \sigma_\xi^2)$ . For  $m_\xi$ , I first compute the average ratio of annual college tuition and required fees (excluding room and board) for four-year institutions to the per capita real GDP for the recent periods 1990-2011, which is 0.22 according to the Digest of Education Statistics

(2011, Table 349) and the Bureau of Economic Analysis. In order to approximate actual costs faced by students, I also include the non-tuition expenses such as books, other supplies, commuting costs, and room and board expenses that would not have to be paid by a person who chooses not to go to college, as in Abbott, Gallipoli, Meghir and Violante (2013). These non-tuition expenses amount to approximately 30 percent of the average tuition and fees. In 2000-2001, the average grants (federal, state/local, and institutional) received by full-time students in four-year colleges weighted by numbers enrolled are approximately 50 percent of the average tuition and fees. Based on the above information, the target statistic for the mean of the college fixed cost distribution  $m_\xi$  in the model is set to be the equilibrium ratio of average (tuition and non-tuition) expenses after financial aid to per capita GDP, which equals 0.176.

The variability of the financial cost distribution  $\sigma_\xi$  is chosen in connection with the parameter  $\Delta$ , which captures the enhancement of one-period human capital growth for those who complete college. A greater benefit of college (i.e., higher  $\Delta$ ) would obviously lead to more people who invest in college education. Hence, a natural target is the four-year college completion rate of 30.6 percent obtained from the ATUS samples. At the same time, this parameter  $\Delta$  largely affects the observed college wage gap. However, it is important to note that the observed college wage premium is not only because of the benefit ( $\Delta$ ) but also because of compositional effects due to positive ability selection. In other words, an individual who is more able, measured by pre-college human capital, is more likely to complete college and tends to have even higher wages after completing college. I thus add the ratio between the average wage of those with a college degree and the average wage of those without a college degree (1.75) observed in recent U.S. data (Heathcote et al., 2010), which can pin down the variability of college costs  $\sigma_\xi$  in the model.

### **Remaining parameters:**

In the model, the parental transfers can take one of the three values  $\{0, a_l, a_h\}$ . I set  $a_l = 0.5a_h$ , and calibrate  $a_h$  to match the average parental transfer in equilibrium. Since the role of the inter-vivos transfers in the model is to provide young households with financial resources that help complete college education, I focus on money from parents and college transfers and sum up these transfers for five years (from age 18 to age 22) from Table 4 in Johnson (2013). This leads to the ratio of average parental financial transfers to the five-year GDP per-capita, which is 0.047, which

serves as a target statistic.

There are two remaining parameters regarding government. The first is the lump-sum transfer  $\omega$ . As discussed above, the flat tax rates along with this lump-sum transfer can effectively provide an environment with progressive taxation (Abbott et al., 2013). To measure the degree of progressivity, I add the ratio between the post-government variance of log wage and the pre-government variance of log wage as a target statistic (Heathcote et al., 2010). Social security payments are assumed to be captured by a simple concave mapping:  $g(\theta) = m \log(1 + \theta)$ . A target statistics for  $m$  is set as the average replacement rate of 40 percent in the U.S.

### **3.3 Evaluating the baseline model as a quantitative theory of intergenerational mobility**

Prior to the quantitative exercises in the next sections such as counterfactual and policy experiments, this section evaluates the baseline model economy as a quantitative theory of intergenerational mobility. I consider three measures of intergenerational mobility: (i) the IGE; (ii) the rank correlation; and (iii) the quintile transition matrix. The intergenerational mobility estimates reported below are based on family income in order to be consistent with the U.S. data counterparts from Chetty et al. (2014a). Specifically, in Chetty et al. (2014a), family income is the five-year per parent average of the pre-tax income defined as either the sum of Adjusted Gross Income, tax-exempt interest income and the non-taxable portion of Social Security and Disability benefits (if a tax return is filed) or the sum of wage earnings, unemployment benefits, and gross social security and disability benefits (otherwise). In the model, family income is the five-year per parent sum of labor earnings, interest income, and social security benefits. It is worth noting that family income is the preferred variable for the studies of intergenerational mobility including both sons and daughters (Lee and Solon, 2009), which applies to the gender-neutral model in this paper. The model-implied statistics are based on 100,000 parent-child pairs generated by simulating the baseline model economy.

#### **IGE and percentile rank-correlation:**

The first is the IGE, a conventional way to measure the degree of intergenerational persistence in the literature. The IGE is the slope coefficient obtained by running the following log-log regression

equation:

$$\log y_{child} = \rho_0 + \rho_1 \log y_{parent} + \varepsilon \tag{17}$$

where  $y$  is the permanent income. The IGE provides a straightforward interpretation: a one percent increase in parental permanent income is associated with a  $\rho_1$  percent increase in their children’s permanent income. Thus, a high  $\rho_1$  implies low intergenerational mobility. The second way to measure intergenerational mobility is to use a rank-rank specification instead of a log-log specification, as proposed by Chetty et al. (2014a, 2014b). In other words, I estimate the slope parameter after replacing log income with the percentile rank of income within one’s own generation in (17). The slope coefficient in a rank-rank specification (or the rank correlation) has a similar interpretation: a one percentage point increase in parent’s percentile rank is associated with a  $\rho_1$  percentage point increase in their children’s percentile rank.<sup>15</sup> Unlike the IGE, the rank correlation is known to be less sensitive to the treatment of zero income observations and is relatively robust to the point of measurement in the income distribution (Chetty et al. 2014a, 2014b).

In the literature estimating intergenerational mobility, the biggest challenge is the data requirement: we need a data set which contains career-long earnings histories (or permanent income) for at least two successive generations. Due to the data limitation, in practice, permanent income is replaced with proxy income measured at a point in the life cycle. For purposes of comparison, I present model statistics based on proxy income which is defined similarly to Chetty et al. (2014a) in addition to those based on discounted sum of lifetime income. Specifically, in Chetty et al. (2014a), the child’s income is measured by income when children are around 30 years old, averaged over two years. The parent’s income is averaged over five years when parents are roughly around 45 years old. Accordingly, in the model, the age at which the parent’s income is measured is 45-49 ( $j = 6$ ), and the age at which the child’s income is measured is 30-34 ( $j = 3$ ).

Table 5 reports these first two measures (i.e., slope estimates) from the model and the data. The first column shows estimates from U.S. data in Chetty et al. (2014a). Recall that the rank-rank slope using proxy income from the model matches its value in U.S. data since it has been used as a calibration target. The estimate of the log-log slope (IGE) using lifetime income is 0.377,

---

<sup>15</sup>Note that the rank-rank slope estimate is simply equal to the correlation coefficient in percentile rank since the independent and dependent variables, both of which are normalized by transforming the income level to the percentile ranks, have the same variance.

Table 5: Intergenerational persistence estimates

	U.S. data	Baseline model	
	Chetty et al (2014a)	Proxy income	Lifetime income (discounted)
IGE: log-log slope	0.344	0.320	0.377
Rank corr: rank-rank slope	0.341	0.341	0.361

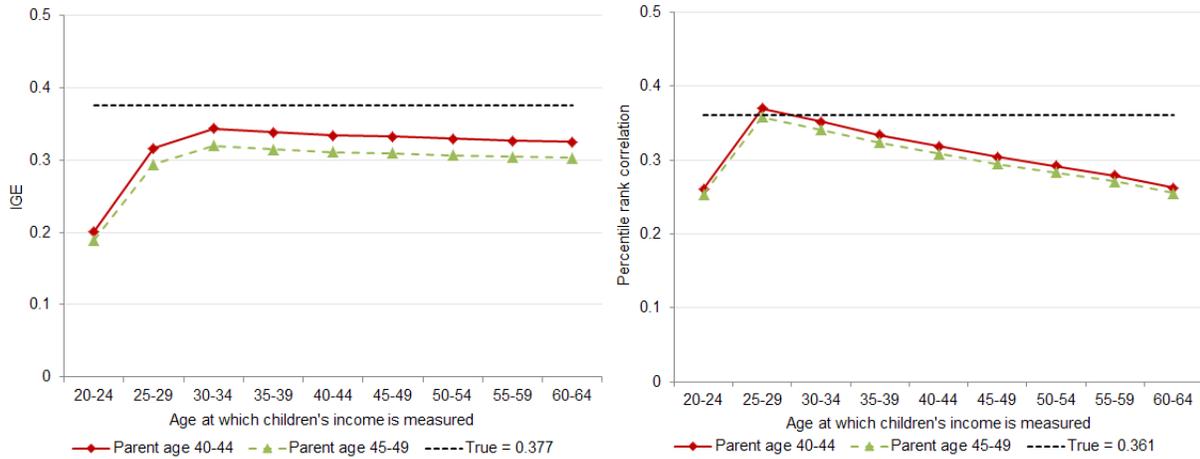
Notes: I simulate 100,000 pairs of a parent and a child for the model-generated data. The log-log slope estimate is obtained from a univariate regression equation where the dependent variable is the child’s log income and the independent variable is the parent’s log income. The rank-rank slope estimate is obtained from an equivalent regression equation replacing log transformation with the percentile rank.

which is greater than the estimate of 0.320 using proxy income. This is in line with findings in the empirical studies noting that the short-term income (even multi-year averages) may not represent the permanent income because of persistent transitory shocks.<sup>16</sup> The bias is noticeably smaller in the estimate of the rank-rank slope using proxy income instead of lifetime income (0.341 and 0.361).

It is important to note that the degree of approximation using the proxy income depends on the point at which income is measured, as can be seen in Figure 1. In this figure, I plot the estimates of the IGE (left panel) and the rank correlation (right panel) by varying the age at which children’s income is measured while holding constant the age at which parents’ income is measured at 40-44 (red solid) or at 45-49 (green dashed). As documented in the literature (Solon, 1999; Haider and Solon, 2006), there is serious attenuation bias in the IGE estimates when children’s income is measured too early. The left panel shows that the IGE estimate when children’s income is measured in the early 20’s is less than half the true value using the lifetime income (black dotted line). The IGE estimates become stable once the children’s age is over 30. The rank correlation estimates show similar patterns with the two key differences. First, the absolute magnitude of the attenuation bias is smaller. Second, the rank correlation moderately declines with the age at which children’s income is measured. This pattern is also present in Chetty et al. (2014a)’s Figure III using the SOI (the Statistics of Income) sample. Appendix A provides this so-called lifecycle bias in the intergenerational mobility estimates for every combination of the parent and child ages.

<sup>16</sup> Another issue, which I do not consider in this paper, is the classical measurement error leading to attenuation bias when parents’ income is measured with error. Solon (1992) suggests using the multi-year averages to mitigate the errors-in-variables bias. This widely known issue is not considered because the model-generated data are accurately measured without measurement error.

Figure 1: Lifecycle bias by age of child



Notes: In both panels, I vary the age at which children’s income is measured while holding the age at which parents’ income is measured constant at 40-44 (red solid line) or at 45-49 (green dashed line). The left panel shows the IGE estimates and the right panel shows the rank correlation estimates. The black dotted lines show the corresponding estimates using the actual lifetime income.

### Quintile transition matrix:

The third measure I consider is the income quintile transition matrix. The  $(a, b)$  element of the matrix gives the conditional probability that a child’s lifetime income is in the  $a$ -th quintile of his generation’s distribution, given that his parent’s income is in the  $b$ -th quintile of her own generation’s distribution. This provides a richer description of how economic status is transmitted across generations than do the first two measures. It is important to emphasize that calibration targets do not include any elements in the income quintile transition matrix.

Table 6 compares the transition matrix obtained from U.S. data (Chetty et al. 2014) to the transition matrices using the model-generated data. Three features are worth noting in the transition matrix from U.S. data. First, it shows that the observed positive correlation of income across generations (0.341) is not simply due to the intergenerational poverty trap but is also due to the rich families that sustain their economic status intergenerationally. Specifically, both the probability of children staying in the bottom (34 percent) and that of top quintile (37 percent) are substantially higher than the other diagonal elements (22 – 24 percent). Second, there is quite a bit of mobility in the middle of the income distribution. For instance, children born into the third quintile parents are almost equally likely to be located in any income quintiles (18 – 22 percent).

Table 6: Income transition matrices: U.S. data vs. Model economy

Parent quintile	U.S. data					Model									
	Chetty et al (2014a)					Proxy income					Lifetime income				
	Child quintile					Child quintile					Child quintile				
	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th	1st	2nd	3rd	4th	5th
1st	.34	.28	.18	.12	.08	.35	.23	.18	.14	.10	.36	.23	.17	.14	.09
2nd	.24	.24	.22	.18	.12	.26	.23	.20	.18	.14	.26	.23	.20	.17	.13
3rd	.18	.20	.22	.22	.18	.20	.22	.21	.20	.17	.19	.22	.22	.20	.17
4th	.13	.16	.21	.24	.25	.13	.19	.22	.22	.23	.13	.19	.22	.23	.23
5th	.11	.12	.17	.24	.37	.05	.14	.19	.26	.36	.06	.12	.19	.25	.38

Third, both upward mobility, measured by the probability of moving from the bottom quintile to the top quintile, and downward mobility, measured by the probability of moving from the top quintile to the bottom quintile, are quite low (7.5 percent and 10.9 percent, respectively).

Overall, the model successfully accounts for the features in the U.S. income quintile transition matrix despite the fact the calibration only targets the correlation of percentile income regarding intergenerational mobility. In particular, the model reproduces the salient features of high probabilities of staying in the bottom quintile (35.3 percent using proxy income and 36.1 percent using lifetime income) and in the top quintile (36.0 percent using proxy income and 37.6 percent using lifetime income). The model also predicts a substantial degree of mobility in the middle distribution. For instance, children born into the third quintile parents are almost equally likely to end up with any quintiles (17 – 22 percent). Finally, the degree of upward mobility and that of downward mobility are also low in the model although the model somewhat underpredicts downward mobility (5.3 percent using proxy income and 5.9 percent using lifetime income compared to 10.9 percent in U.S. data).

## 4 Sources of intergenerational mobility

In this section, I assess the quantitative importance of various channels in explaining intergenerational income persistence, and inspect the mechanisms through which each channel affects mobility. To evaluate the relative contribution of the key elements to the observed intergenerational income persistence, I consider special cases alongside the baseline specification. I first focus on the role

of the parental time investment, the key channel of interest in this paper. Then, I examine other channels including college education, parental financial transfers, and pre-birth factors.

#### 4.1 Parental time investment channel

The first two columns in Table 7 summarize the intergenerational income persistence estimates (IGE and the rank correlation using lifetime income). The next three columns report the summary statistics on the key endogenous channels in my model: (i) the average parental time investment (hours per week); (ii) average parental transfers relative to the five-year GDP per capita; and (iii) the fraction with a college degree.

In the first counterfactual case, denoted by (2) in Table 7, I eliminate this heterogeneity in the quantity of parental time investment by imposing that all parents invest the same amount of time at its average from the baseline specification (i.e.,  $h = \bar{h}$ ). This is motivated by the substantial educational gradient in parental time investment; the ratio between the amount of time investment spent by college-educated parents and that spent by less-educated parents is 1.35 in the baseline specification. As a result, both measures of intergenerational persistence, the IGE and the rank correlation of income, decrease significantly by roughly 20 percent. Note that, despite the homogeneous amount of time investment, the quality of parental time investment still varies depending on the parent's human capital (capturing parenting skills, the amount of useful information, etc.) and the level of child's initial human capital endowment, thereby transmitting human capital across generations. In the second counterfactual case, denoted by (3), I impose that all parents spend zero hours of time investment. This essentially shuts down the overall parental time investment channel since both the quantity and quality of parental time investment are not allowed to be transmitted to the next generation. Compared to the above case (2), the intergenerational persistence estimates decrease further (nearly 20 percent more). Overall, the parental time investment channel accounts for approximately 40 percent of the observed intergenerational persistence of lifetime income.

It is worth noting that this quantitative significance of the parental time investment channel is obtained in the presence of the parental financial transfer channel, a competing intergenerational transmission channel. In the model, parental time investment (human capital investment) and financial transfers (physical capital investment) are the two channels that can substitute each

Table 7: Importance of heterogeneity in quantity and quality of parental time investment

	IGE	rank corr	$\bar{h}$ (hrs/wk)	Average IVT/Y	College (%)
(1) Baseline	0.377	0.361	6.3	0.047	30.5
(2) Homogenous time investment at $\bar{h}$	0.298	0.280	6.3	0.064	30.4
(3) No parental time investment	0.219	0.201	0.0	0.082	30.5

Notes: The mobility estimates are obtained using lifetime income from 100,000 pairs of a parent and a child. Row (2) (homogenous time investment) exogenously imposes that all parents spend the same time investment at the mean value from the baseline model ( $h = \bar{h}$ ). Row (3) (no parental time investment) sets  $h = 0$  for all parents.

other.<sup>17</sup> As can be seen in Table 7, although parents increase financial transfers substantially in the above counterfactual exercises where I impose restrictions on parental time investment in human capital, intergenerational persistence estimates drops quite significantly. This suggests that the early parental time investment channel is quantitatively much more significant as a source of intergenerational income persistence.

To better understand the mechanism through which the parental time investment channel affects intergenerational association, I characterize properties of the optimal parental time investment decision in equilibrium. A simple and intuitive way is to run a linear regression:

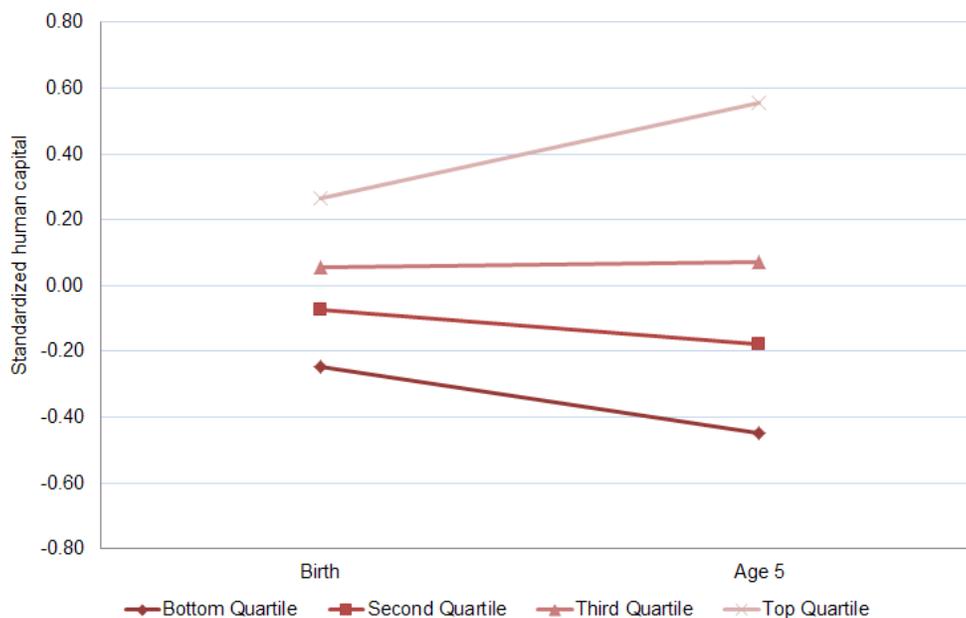
$$E[\log(h) | \log(\theta), \log(\theta_0), \log(a)] = \beta_0 + \underset{(0.95)}{\beta_1} \log(\theta) + \underset{(-0.76)}{\beta_2} \log(\theta_0) + \underset{(0.0072)}{\beta_3} \log(a) \quad (18)$$

where  $\theta$  is parent's human capital,  $\theta_0$  denotes child's human capital endowment, and  $a$  is parent's assets using the simulated data. Note that, in the model, these regressors are parent's state variables, implying that coefficients  $\beta_i$  can describe how the average behavior of parental time investment changes with respect to changes in those variables according to the theory. I report coefficient estimates right below each parameter. Standard errors are omitted since they are very small.

The estimated  $\beta_1$  is 0.95, implying that, holding child's human capital constant, parents increase their time investment as their own human capital increases. Despite their higher opportunity costs of time, it is higher human capital parents who invest more time in their children. Thus,

<sup>17</sup>Krebs (2003) shows that, in a general equilibrium environment, uninsurable idiosyncratic shocks to persistent human capital tend to be positively related to physical capital investment, but negatively related to human capital investment.

Figure 2: Mean standardized human capital at birth and age 5, by quartile of permanent family income



this property of the human capital investment behavior acts as a mechanism that amplifies the intergenerational correlation of human capital. To quantitatively visualize this amplification, I plot how dispersion of human capital, by parent’s permanent income, changes in early periods. Figure 2 shows the means of normalized human capital (such that each period human capital has mean zero with a unit standard deviation) by the quartile of permanent family income in different ages, following Carneiro and Heckman (2003), Cunha and Heckman (2007) and Cunha (2013). It clearly demonstrates that the parental time investment channel amplifies the gap across the whole quartiles. Note that as long as children’s initial human capital endowment is not strongly correlated with their parent’s human capital, this amplification mechanism would generate substantial mobility during this period.<sup>18</sup> Figure 2 reveals that amplification is biased toward the top quartile, consistent with Figure 2 in Cunha (2013).<sup>19</sup> This indicates that this amplification mechanism could be responsible for the feature in the income quintile transition matrix in the U.S. that the probability of staying in the top quintile is greater than the high probability of staying in the bottom quintile.

<sup>18</sup>For instance, consider a case in which children’s human capital endowment is highly correlated with their parent’s human capital. In this case, amplification would preserve the children’s rank and thus generate little mobility. In the baseline model, this percentile rank correlation is positive (0.27) but still far from 1.

<sup>19</sup>Recall that calibration only targets the change of the ratio between the conditional mean of the second quartile and that of the third quartile between birth and age 5, which is silent about asymmetry.

In contrast to  $\beta_1$ , the estimate of  $\beta_2$  is negative. This means that the amount of parental time investment tend to decrease with their child's human capital at birth, holding the parent's human capital constant. This may seem puzzling since productivity of parental time investment does increase with children's human capital endowment. In fact, a key force that drives this tendency is the dynastic smoothing of the marginal value of human capital, analogous to the consumption smoothing of the infinitely-lived households in a standard dynamic model. The difference here is that parents choose to invest in their child's human capital, which in turn affects her future lifetime consumption and leisure.

To see this more concretely, note that optimal parental time investment is characterized by

$$-\frac{\partial U(\frac{c}{q}, 1 - n - h)}{\partial h} = \beta^4 \eta \frac{\partial V_{j=1}(\theta_c, a_c)}{\partial \theta_c} \frac{\partial \theta_c}{\partial \theta_0'} \alpha_1 \theta^{\alpha_1} h^{\alpha_1 - 1} \theta_0^{\alpha_2}. \quad (19)$$

where the left-hand side represents the marginal cost of  $h$  (which is the negative marginal utility of leisure) and the right-hand side summarizes the marginal benefit of  $h$ . The marginal benefit has two components. First, additional time would develop the child's ability further ( $\alpha_1 \theta^{\alpha_1} h^{\alpha_1 - 1} \theta_0^{\alpha_2}$ ), which in turn would increase the initial adult human capital ( $\frac{\partial \theta_c}{\partial \theta_0'}$ ). This positive marginal product of  $h$  in terms of adult human capital is captured by the whole second half terms ( $\frac{\partial \theta_c}{\partial \theta_0'} \alpha_1 \theta^{\alpha_1} h^{\alpha_1 - 1} \theta_0^{\alpha_2}$ ). However, what is valued by parents is not the level of child's future human capital per se, but the lifetime utility which their child would enjoy. Therefore, the first half terms ( $\beta^4 \eta \frac{\partial V_{j=1}(\theta_c, a_c)}{\partial \theta_c}$ ) translate the marginal product of human capital into the present-value marginal utility which parents actually care about. Note that, although the marginal cost of  $h$  (left-hand side) is independent of the child's initial endowment  $\theta_0$ , the marginal benefit of  $h$  (right-hand side) may shift up or down with respect to  $\theta_0$ , depending on the relative size of the two effects: (i) it may go up since the human capital technology implies that marginal return on  $h$  increases with  $\theta_0$ ; and (ii) it may go down because of diminishing marginal utility. In the baseline model, that the second effect dominates the first effect.<sup>20</sup>

In sum, parents' dynastic smoothing motives imply that, when a child's human capital en-

---

<sup>20</sup>This result is somewhat in line with Bernal (2008)'s empirical finding that mothers compensate less able children by spending more time with them despite lower returns. The compensating nature of the parental investment (not necessarily time investment) can be also seen in another context where parents care about inequality among multiple children (Behrman, Pollak, and Taubman, 1982). Their setting is different from the current setting where parents compensate their child who is born with a lower endowment relative to *themselves*, not among siblings.

downment turns out to be far from her parent’s socioeconomic status captured by human capital, parental time investment plays a role as a channel that moves the child closer to her parent’s status. It is important to note that parents’ dynastic smoothing motives, interacting with the amplification mechanism, give rise to even more mobility when parents’ human capital and their child’s initial endowment is not strongly correlated, as in the baseline model (0.27 of percentile rank correlation). For example, when a high human capital parent’s child is born with a relatively low human capital endowment, then the dynastic smoothing motives imply a even greater amount of parental time investment. Hence, both high quantity and high quantity of parental time investment would move this relatively low human capital child towards his parent’s high human capital status. A corollary is that there are unlucky children born with a relative high human capital endowment into low human capital families who would actually move down the ladder despite their potentials. This can justify policies that attempt to improve parental time investment in disadvantaged families.

Finally, the estimated  $\beta_3$  is also positive, suggesting that more asset holdings do increase parental time investment. It is worth noting, however, that its magnitude is substantially smaller than  $\beta_1$ . This has some implications for effective policy designed to increase intergenerational mobility, which I consider in Section 5.

## 4.2 Inspecting other channels

I now move on to the other key human capital investment channel, namely college education. To quantitatively measure the importance of college education in accounting for intergenerational income persistence, I shut down the college channel by imposing that there is no benefit of college education ( $\Delta = 0$ ). Row (2) in Table 8 shows that both intergenerational persistence estimates decrease roughly by 10 percent. This suggests that college education does provide mobility but to a lesser extent, compared to the parental time investment channel.

In the model, the discrete decision rule for college education features threshold-based behavior. More specifically, holding other things constant, the college decision rule is to complete college if his or her human capital is above some threshold level. The reason is that the return to college, which is accumulated over the life cycle, increases with their pre-college human capital level. On the other hand, given a college fixed cost draw, the marginal opportunity cost of going to the college

Table 8: Inspecting other channels as sources of intergenerational persistence

	IGE	rank corr	$\bar{h}$ (hrs/wk)	Average IVT/Y	College (%)
(1) Baseline	0.377	0.361	6.3	0.047	30.5
(2) No college education	0.341	0.326	6.1	0.024	0.0
(3) No parental financial transfer	0.383	0.367	6.6	0.0	29.2
(4) No pre-birth human capital transmission	0.224	0.211	5.9	0.087	29.1

Notes: The mobility estimates are obtained using lifetime income from 100,000 pairs of a parent and a child. Row (2) sets  $\Delta = 0$  to remove the incentive to go to college. Row (3) sets  $a_h = a_l = 0$ . Row (4) sets  $a_0 = 0$  so that children’s endowment of human capital is independent of their parent’s human capital.

(i.e., foregone earnings) is relatively small because young households tend to have lower wages.<sup>21</sup> This property of the college decision rule leads to positive selection in equilibrium, meaning that a better prepared student is more likely to complete college education.

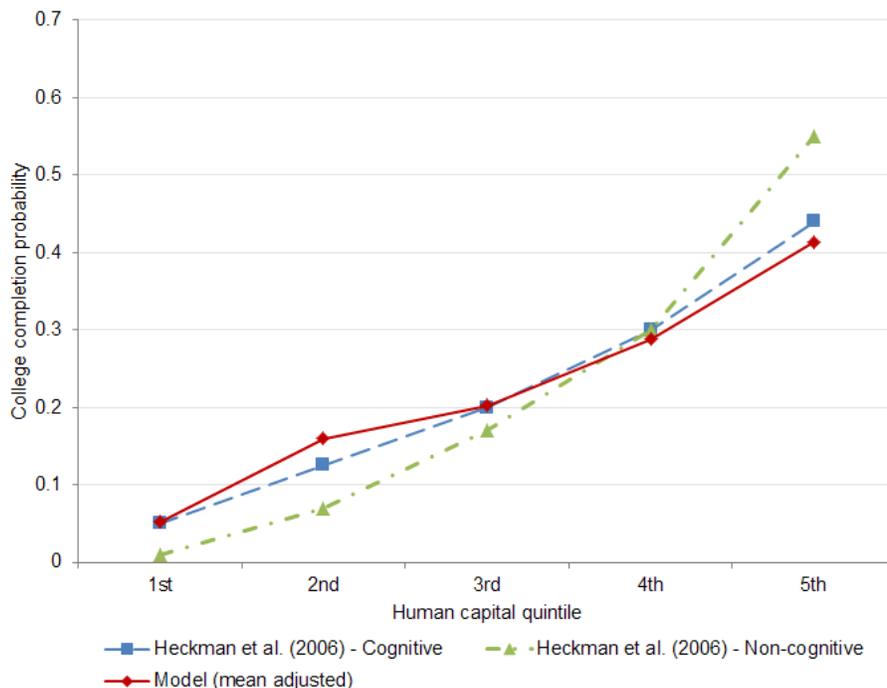
To visualize the quantitative importance of college readiness that exists in the model, in Figure 3, I present college completion probabilities by the quintiles of the equilibrium pre-college human capital distribution. The data counterparts are college completion probabilities by both cognitive factors and non-cognitive factors in Heckman, Stixrud and Urzua (2006).<sup>22</sup> Figure 3 shows that high human capital students are more likely to complete college, indicating positive selection into college both in the model and the data. Since the slope captures the strength of positive-selection, it is worth checking if the model produces a reasonable degree of the selection. Indeed, although not directly targeted in the calibration procedure, the degree of positive selection in the model is very close to that in the data (especially of cognitive factors).

A consequence of this property is that the college channel tends to endogenously sort out those who have relatively higher human capital, amplifying differences in pre-college human capital. In Figure 4, I plot dispersion of human capital, measured by Gini index, at different ages. In line with the finding by Restuccia and Urrutia (2004), there is a significant jump in dispersion of human capital during the college education period. Note that a big increase is more likely when

<sup>21</sup>In reality, merit-based scholarship could make the college cost smaller for the people with higher ability. This would strengthen the importance of human capital in deciding whether to go to college. The effect of need-based scholarship may work in the other direction; however, this effect is less clear since children’s ability is not perfectly correlated with parental income, which is typically a criterion for such scholarship.

<sup>22</sup>The data set in Heckman et al. (2006) has a lower unconditional college completion rate. To focus on the slope rather than the level, I adjust the model-implied college completion probability curve by the magnitude of the difference in the unconditional college completion rate.

Figure 3: College completion rates by human capital quintiles

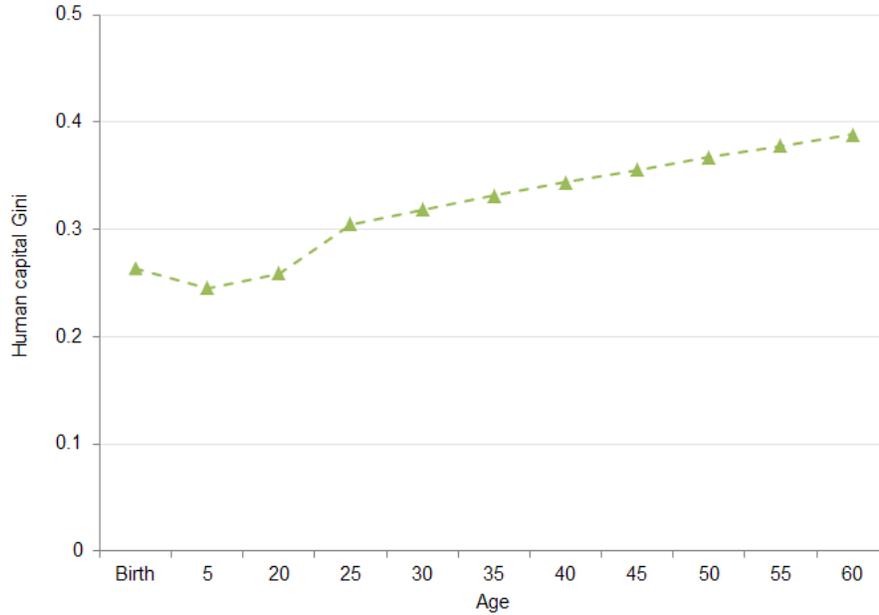


amplification is accompanied with little rank reversals. In fact, this is in sharp contrast to the parental time investment channel, which also features amplification but provides a great deal of mobility. All of the above results suggest that college education would not serve an effective means of raising mobility.

In this model, parents can also transfer money to their child when she becomes independent. An important role of this transfer is to provide financial help for their child’s college decision. In the third row of Table 8, I shut down the inter-vivos transfer channel by setting  $\alpha_h = a_l = 0$ . First of all, as expected, the college completion rate goes down because private financial help is missing. A surprising result is that intergenerational persistence becomes higher in this counterfactual case. This is partially due to the fact that parents, who are not allowed to transfer money to their children, choose to invest more time in young children instead. Given that the parental time investment channel has a strong power of strengthening intergenerational association, this substitution towards parental time investment actually raise intergenerational persistence.

Finally, the bottom row of Table 8 shows the case in which I set  $\alpha_0 = 0$ , making the stochastic endowment of children’s human capital to become independent of their parent’s human capital.

Figure 4: Changes in human capital Gini over the lifecycle



This shuts down all pre-birth factors capturing not only genetic transmission (nature) but also any nurtural factors (e.g., prenatal investment) that could affect the child’s initial endowment of human capital. Unsurprisingly, closing the pre-birth factors reduces intergenerational persistence quite significantly. This result is not surprising since nature, which could account for a significant portion of the pre-birth factors in the model, is often found to be quite important in explaining intergenerational persistence of education (e.g., Plug and Vijverberg, 2003; see also Sacerdote, 2010 for a survey on the evidence by psychologists and behavioral geneticists as well). What is worth pointing out instead is that the parental time investment channel alone has quantitatively similar impacts on intergenerational income persistence as does the pre-birth factors.

## 5 Policy experiments

In this section, the baseline model economy is used to study various policies that can be considered as tools to influence intergenerational mobility. I consider universal policies that can avoid stigmatization (Heckman, 2008). It is important to note that the objective of these exercises is not to provide accurate quantitative predictions of each policy. Instead, the main objective is to

examine and illustrate desirable properties of effective policies that can increase intergenerational mobility. It is also worth noting that it would be more ideal to examine how policies would change the degree of equality of opportunity instead of the observed intergenerational mobility. However, equality of opportunity is not straightforward to define and is difficult to measure. Alternatively, this section provides other measures of absolute mobility such as changes in aggregate output and welfare gains along with the intergenerational mobility estimates, a measure of relative mobility. That way, we could better evaluate policy changes that raise relative mobility by examining whether those changes are accompanied by overall absolute improvement.

The first set of tools intends to provide easier access to college. Next, I consider a set of hypothetical policies that are designed to change the opportunity cost of the parental time investment in order to raise the quantity of such investments. Lastly, I consider policies that approximate the universal preschool program in order to improve both quantity and quality of parental time investment. Note that results for the policy experiments in this section represent long-run general equilibrium effects in which prices adjust to clear the markets so that human capital investment and savings interact with each other through changes in prices in the presence of persistent risky human capital and risk-free assets (Krebs, 2003). All of the policies are designed not to exceed the amount of government surplus to satisfy the government budget constraint.

## 5.1 Providing easier access to college

I consider two ways to provide easier access to college. The first is to relax borrowing limits up to \$5,000 in  $j = 1$  (the college education stage). The second is to lower the mean of the college cost distribution  $m_\xi$  by \$5,000 in the same first period. I explore the equilibrium effects on intergenerational persistence (IGE and percentile rank correlation), aggregate human capital investment (average parental time investment and college completion rates), the ratio of average parental transfers to five-year GDP per capita and aggregate output per capita. For illustration, assuming that the annual GDP per capita in the baseline model is \$50,000, a value close to nominal US GDP per capita in 2011, all dollar values are approximately in 2011 U.S. dollar. I also report welfare gains for the first adult generation, measured by the percentage change in consumption during the lifetime of the first adult generation in the baseline model that makes them indifferent to living in the alternative economy. It is worth noting that the consumption compensations, or

Table 9: Long-run effects of providing easier access to college

	(1)	(2)	(3)
	Baseline	Borrowing limit relaxed	Lower college cost
IGE	0.377	0.377	0.377
Rank correlation	0.361	0.362	0.363
Mean parental time investment (hours/week)	6.3	6.3	6.4
College fraction (%)	30.5	31.8	32.1
Mean parental transfers to output ratio	0.047	0.046	0.048
Aggregate output changes (%)	-	+0.17	+0.50
First adult generation's welfare gains			
including utility from future generations (%)	-	+0.30	+0.55
excluding utility from future generations (%)	-	+0.24	+0.45

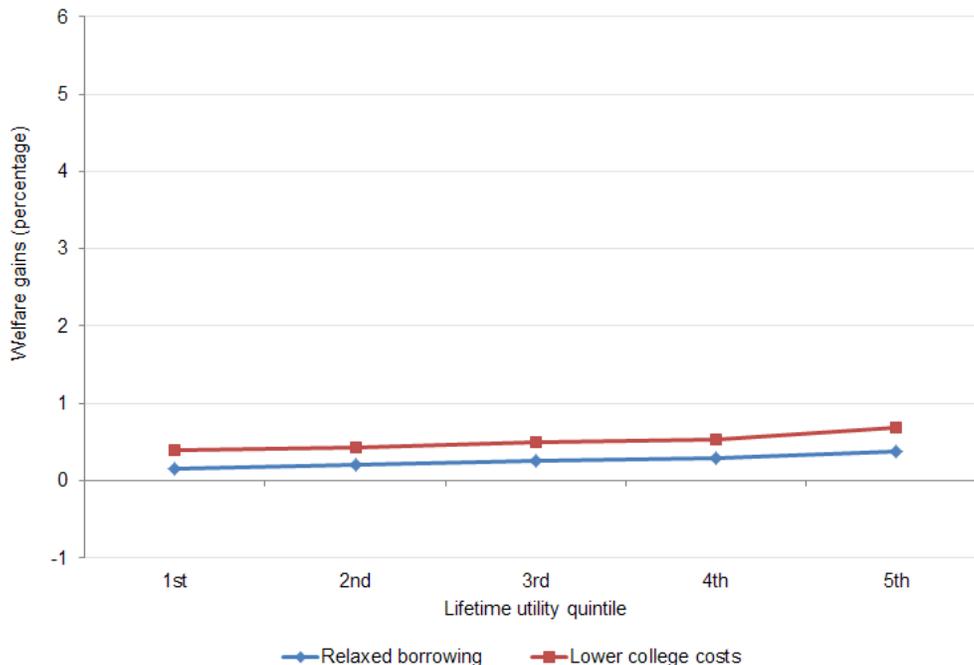
Notes: The unit of parental time investment is converted to hours per week. Output gains are measured by the percentage change in the five-year GDP per capita (Y) relative to the baseline specification. Welfare gains are measured by the percentage change in the first adult generation's consumption during their lifetime which makes them to be indifferent to living in an alternative economy. Each household is weighted by the utilitarian social welfare function. When computing consumption compensations, the last row controls for the continuation values as in the baseline models.

welfare gains, are based on the expected utility from the perspective of those whose adult life has not begun yet. Thus, when the stationary distribution changes, the welfare gains could reflect changes driven by their higher initial adult human capital and assets. I also compute their welfare gains while controlling for the values derived from the future generations in order to isolate the required consumption gains of the first generation with respect to the changes in their own lifetime utilities.

Table 9 summarizes the results. In both cases, the college completion rates do increase quite significantly (roughly by 1.5 percentage points). However, a greater fraction of college-educated population does not imply that intergenerational mobility improves. In fact, both measures of intergenerational persistence slightly increase. The main reason is positive ability self-selection into college, discussed in the previous section. That is, the marginal students tend to have lower returns to college than those who are already in college. On average, this makes those marginal college graduates to accumulate more of human capital only up to the point less than those who already choose to complete college. This is consistent with one of the main findings in the previous section that the contribution of the college channel to intergenerational persistence is relatively small.

Both policies have positive impacts on the average welfare and aggregate output, driven by more

Figure 5: Welfare gains of providing easier access to college, by quintiles



college-educated labor forces. However, the magnitude of welfare gains may differ across different income groups. In Figure 5, I plot welfare gains by the quintile of the lifetime utility, measuring the percentage increase in consumption of the households in the  $x$ -th quintile of the baseline economy in order them to be indifferent to living in the  $x$ -th quintile of the alternative economies. It is not surprising to find that those who have higher pre-college human capital enjoy greater welfare gains. The positive slope of these gains is in line with the above result that facilitating college education does not increase intergenerational mobility.

## 5.2 Increasing quantity of parental time investment

The analysis in Section 4 has called for the policies that improve parental time investment in children born into disadvantaged families since low quantity and quality of parental time investment could be important sources of hindering mobility and aggregate efficiency. To quantitatively examine such policies, I first move on to the policies that aim to increase the amount of parental time investment. I consider three kinds of exercises that can lower the opportunity cost of parental time investment. First, I consider a simple form of subsidy: a lump-sum transfer to the parents who live in period

Table 10: Long-run effects of improving quantity of parental time investment

	(1)	(2)	(3)	(4)
	Baseline	Cash Transfer	Time subsidy	Earnings Tax
IGE	0.377	0.377	0.366	0.387
Rank correlation	0.361	0.362	0.351	0.371
Mean parental time investment (hours/week)	6.3	6.4	8.2	8.2
College fraction (%)	30.5	30.1	30.5	29.5
Mean parental transfers to output ratio	0.047	0.044	0.046	0.047
Aggregate output changes (%)	-	+0.04	+2.59	+1.83
First adult generation's welfare gains				
including utility from future generations (%)	-	+1.93	+4.15	+1.36
excluding utility from future generations (%)	-	+1.57	+3.36	+1.09

Notes: The amount of lump-sum transfer is chosen to be approximately \$200 per month. The flat subsidy for parental time investment  $s_h$  is set to be approximately \$1 per hour. The labor tax is raised to induce the same increase in the average parental time investment as in the case of time investment subsidy. Welfare gains are measured by consumption compensations. See Table 9 for the details.

$j = 2$ . The amount is chosen to be \$200 per month.<sup>23</sup> Second, I consider a flat subsidy  $s_h$  that is paid proportional to parental time investment  $h$ . In other words, the resource constraint in period  $j = 2$  becomes

$$c + a' \leq (1 - \tau_w)w\theta n + [1 + (1 - \tau_a)r]a + \omega + s_h h \quad (20)$$

An important feature of this policy is that, unlike individual wage, this flat subsidy  $s_h$  is independent of the parent's human capital. Therefore, the same amount  $s_h$  matters more for low human capital parents. In practice, since time spent by parents at home is hardly observable, this policy should be thought of as an approximation of subsidies to child care centers in which parents themselves must present and interact with their children (e.g., a version of Gymboree Play and Music classes). Lastly, I consider an increase in earnings tax rate  $\tau_w$  only in period  $j = 2$ . As shown below, this policy serves as a benchmark that can also raise the amount of parental time investment by decreasing the marginal benefit of market work, but has completely different impacts on intergenerational mobility.

<sup>23</sup>For example, as of 2013, South Korea introduced a policy that provides a lump sum transfer to the parents who have a child of age 5 or less. In U.S. dollars, the amount ranges roughly from \$100-\$400 per month depending on the age of the child and the use of day-care centers. Importantly, the subsidy is independent of the parents' income level. Also, Germany also provides child benefits (around 200 euros per month) to all residents, depending on the number of children. However, German government does provide extra amounts to low income families.

Table 10 summarizes the results. The second column shows that the sizeable lumpsum transfers barely change most statistics including intergenerational mobility.<sup>24</sup> Note that, in the baseline economy, higher assets cause parents to invest more time in their child ( $\beta_3 > 0$  in Eq (14)). That is, income effect from lump-sum transfers could reduce hours of work, which in turn could increase parental time investment indirectly. The above result shows that this indirect channel is quantitatively insignificant.

The third column shows the case in which a flat subsidy  $s_h$  is set roughly to one dollar per hour. It does increase the average parental time investment quite significantly by 1.9 hours per week, and both the IGE and percentile rank correlation fall noticeably by 3 percent. A beneficial by-product due to an increase in average parental time investment is a 2.6 percent rise in aggregate output and even greater welfare gains, driven by higher quality of average human capital in the labor force. In the fourth column, the earnings tax in period 2 is raised up to the level at which we achieve the same amount of increases in the mean parental time investment. However, its impact on intergenerational persistence is actually opposite, demonstrating aggregate statistics such as the mean of human capital investment is not sufficient to predict changes in intergenerational mobility.

Why do the increases in average time investment have opposite effects on intergenerational mobility? Figure 6 demonstrates the key to understanding this difference. It plots the conditional mean of parental time investment by parent's human capital quintiles in the following three cases: (i) the baseline model; (ii) parental time investment subsidy; and (iii) higher earnings tax. Although the effect of (ii) and (iii) on the unconditional mean of parental time investment is the same (see Table 10), they have different distributional implications. In particular, an increase in the amount of parental time investment is much more pronounced at the bottom quintile in the case of the flat time investment subsidy. Since lower human capital parents have lower opportunity costs of parental time investment (i.e., lower wage), they increase time investment in their children sharply with respect to the same amount of the monetary incentive  $s_h$ . This helps those children born into low human capital parents that on average invest less time than high human capital parents. Figure 7, which again plots disaggregated welfare gains, clearly shows that welfare gains are disproportionately larger for the lower quintiles, particularly in the case of the flat time investment subsidy.

---

<sup>24</sup>The exception is the positive welfare gains, which should be interpreted with caution since this may simply represent the government surplus being not used in the baseline model.

Figure 6: Parental time investment by parent's human capital quintiles: baseline vs policies related to parental time investment

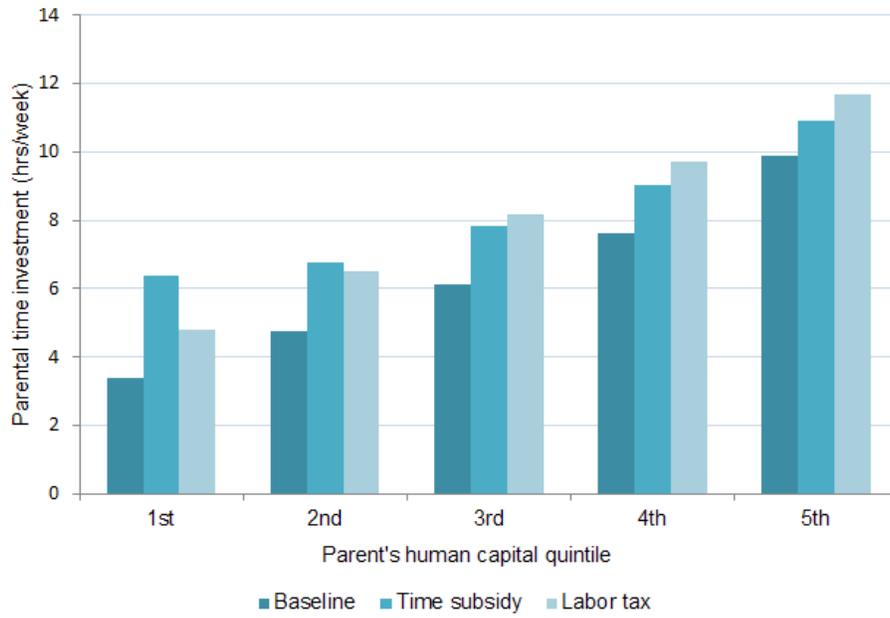
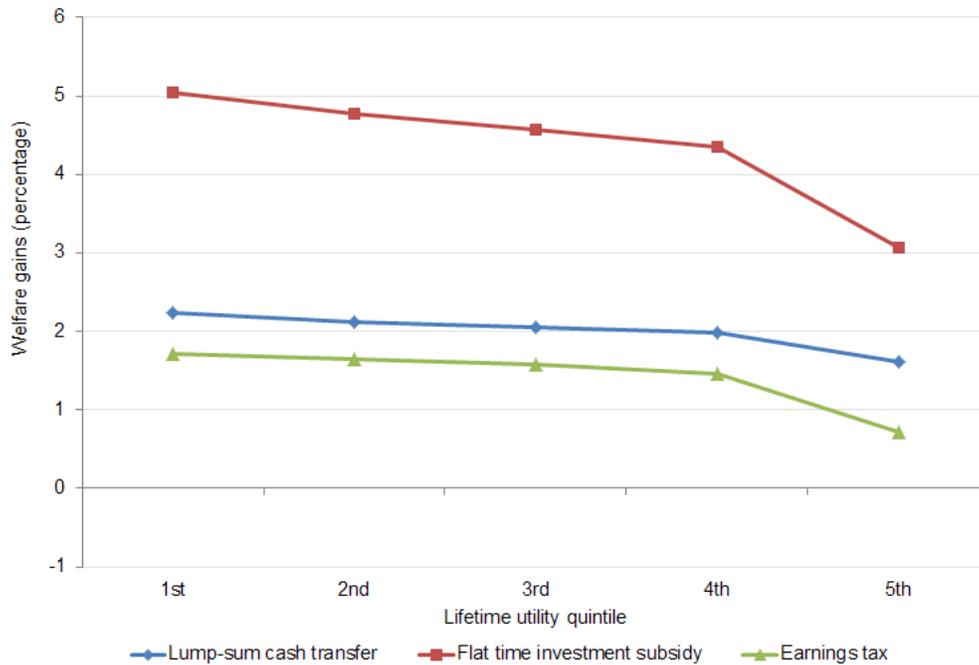


Figure 7: Welfare gains of improving quantity of parental time investment, by quintiles



### 5.3 Universal preschool program

The flat time investment subsidy in the previous exercise is useful in demonstrating the importance of improving the amount of time investment in children from disadvantaged families. In fact, those children lack the other margin as well (i.e., the quality of such investments), of which the quantitative importance is as significant as the quantity margin (see Table 7 in Section 5). This subsection broadens the kind of policies that can also improve time investment along the quality margin as well.

For illustration, I consider a simple exercise that approximates a universal preschool program. More specifically, I assume that human capital accumulation in  $j = 1$  (birth to age 5) follows

$$\theta'_0 = \theta_c + (\theta h)^{\alpha_1} \theta_0^{\alpha_2} + (\tilde{\theta} \tilde{h})^{\alpha_1} \theta_0^{\alpha_2} \quad (21)$$

The only difference here is the addition of the last term which captures the amount of extra human capital development from the universal preschool.<sup>25</sup> As children spend the same time with common teachers in such preschools, the last term replaces the parent's human capital and the amount of time investment with a common level  $\tilde{\theta}$  and a common level of  $\tilde{h}$ , respectively. For the quality of time investment, I consider two values  $\tilde{\theta} \in \{\tilde{\theta}_m, \tilde{\theta}_h\}$  where  $\tilde{\theta}_m$  is mapped to approximately \$13 hourly wage, which is slightly greater than half of the economy-wide mean wage. This is very close to the mean hourly wage of childcare workers and preschool workers (except special education teachers and teacher assistants) in the U.S., according to the Occupational Employment Statistics survey of the Bureau of Labor Statistics. For  $\tilde{\theta}_h$ , I increase the value of  $\tilde{\theta}_m$  by 50 percent to examine the effect of the quality improvement. Lastly, I set  $\tilde{h}$  as the amount of the rise in the mean parental time investment in the case of flat time subsidy (1.9 hours per week) from Section 5.2. This will serve as a benchmark in this subsection. It is important to note that the amount of parental time investment is still determined by parents who understand that there are now universal preschools. In computing a new equilibrium, I account for the effective labor supplied to the preschool program using the staff-child ratio of 6 (i.e.,  $(\frac{\mu_2}{6}) \int \tilde{\theta} \tilde{h} d\pi_j$  is subtracted from labor supply).<sup>26</sup>

<sup>25</sup>Baker, Gruber and Milligan (2008) find that the introduction of the recent universal child care in Quebec had some negative impacts on the children's outcomes in terms of non-cognitive skills. The nature of the universal preschool in this section is somewhat different in that it intends to provide shorter but more educationally-focused child care.

<sup>26</sup>One could also assume that there is a separate competitive preschool sector with a technology which is linear

Table 11: Long-run effects of universal preschool programs

	(1)	(2)	(3)	(4)
	Baseline	Time subsidy	Universal Preschool	Universal Preschool (high quality)
IGE	0.377	0.366	0.362	0.358
Rank correlation	0.361	0.351	0.347	0.342
Mean parental time investment: $\bar{h}$ (hours/week)	6.3	8.2	5.9	5.8
Mean total time investment: $\bar{h} + \tilde{h}$ (hours/week)	6.3	8.2	7.7	7.6
Mean parental transfers to output ratio	0.047	0.046	0.045	0.044
College fraction (%)	30.5	30.5	30.3	30.1
Aggregate output changes (%)	-	+2.59	+6.31	+8.19
First adult generation's welfare gains				
including utility from future generations (%)	-	+4.15	+9.63	+12.55
excluding utility from future generations (%)	-	+3.36	+7.71	+10.01

Notes: Each column is obtained using the following specifications: (1) baseline; (2) flat time investment subsidy; (3) universal preschool; universal preschool of high quality (50 percent higher hourly wage). Welfare gains are measured by consumption compensations. See Table 9 for the details.

Column (3) in Table 11 shows that the universal preschool program can decrease intergenerational persistence even more than the case of the flat time subsidy (Column (2)). Column (4) shows that, if the quality of the time improves by 50 percent ( $\tilde{\theta}_h$ ), intergenerational income persistence decreases even further. These changes in intergenerational persistence are mainly driven by the two forces. The first direct effect is due to the fact that the additional human capital accumulation is composed of the same teacher quality and the same amount of time inputs. Such publicly provided time investments are particularly beneficial to high human capital children born into low human capital parents because these children's high human capital endowment can effectively complement such public time investments. Second, the public time investments actually crowd out the amount of parental time investment as can be seen from reductions in the total time investment ( $\bar{h} + \tilde{h}$ ) in Columns (3) and (4) relative to Column (2). This partially loosens the intergenerational transmission of human capital in early periods, substituted out by public  $\tilde{h}$ , thereby reducing intergenerational persistence.

Finally, as a consequence of extra human capital investment, we can see that the universal

---

in labor only. Then, the above exercise of changing teacher wages can be viewed as the exercise of changing the productivity of the preschool sector.

preschool program increases aggregate output and welfare substantially. These large gains are possible since the same effective time investment  $\tilde{\theta}\tilde{h}$  from the universal preschool complements high human capital children regardless of their family background. It is important to note that, since  $\tilde{h}$  was chosen to make comparisons with the time investment subsidy case, the absolute levels of improvement in aggregate output and welfare should be not taken as the key messages. Instead, the takeaway should be that improvement in the quality of time investment in early periods is an important characteristic of desirable policies not only from the equality point of view but also from the efficiency perspectives as well. This is because those children with high human capital endowments born into low human capital parents have potentials to constitute an efficient workforce if they receive a fair amount of quality time investments in early periods.

## 6 Conclusion

I have investigated parental time investment in children prior to formal schooling as a channel through which human capital is transmitted intergenerationally in a general equilibrium, incomplete markets framework that endogenously generates changes in lifetime inequality across generations. I show that quite a large portion, nearly 40 percent, of intergenerational income persistence present in the baseline model is accounted for by the parental time investment channel that transmits human capital from parents to children. Importantly, around 20 percent of intergenerational persistence of income is accounted for by differences in the amount of time investments chosen by parents. I find that two fundamental forces of the parental time investment channel are that (i) it is the parents with higher human capital who invest more time in their children, amplifying the gaps in early ages; and (ii) parental time investment tends to be negatively related to children's initial human capital endowment (dynastic smoothing). The key policy implication is that able children born into low human capital families are particularly adversely affected by the parental time investment channel since these children receive lower quantity and quality of parental time investment, hindering their chance of higher education and thus upward mobility.

The policy experiments I examine in this paper show that facilitating college directly is not an effective way of raising intergenerational income mobility. Because of positive self-selection into college, increases in the college completion rate tend to benefit better-off families more. In

contrast, I find that intergenerational income mobility can be raised by lowering opportunity cost of the parental time investment. In particular, a flat subsidy proportional to time invested in children provides an incentive for low human capital parents to increase the amount of parental time investment because the subsidy has a stronger effect on parents with lower opportunity costs of time. I also consider the effects of universal preschool policies, which can reduce discrepancies in the quality of time investments in early periods. I find that this policy delivers even greater intergenerational mobility because the universal preschool benefits children with high human capital endowments, independent of their family backgrounds.

The above results provide guidance in designing actual policies to promote intergenerational mobility. For instance, a government policy which encourages parents to bring their young children to subsidized child play centers has a potential to benefit children who lack parental time investment in early periods. Furthermore, it is important to note that the above exercises abstract from the spill-over effects of such centers to parental time investment. For instance, if parents can (i) learn parenting while watching how other parents spend time with their children or (ii) directly share valuable information regarding early education while spending time in such centers, they could potentially increase their parenting quality at home as well. These spill-over effects would strengthen the main idea that improving parental time investment in young children at both quantity and quality margins would provide a better equality of opportunity for their adult life.

## References

- Abbott, Brant, Giovanni Gallipoli, Costas Meghir, and Giovanni L. Violante. 2013. "Education Policy and Intergenerational Transfers in Equilibrium" NBER Working Paper No. 18782
- Aiyagari, S. Rao, Jeremy Greenwood, and Ananth Seshadri. 2002. "Efficient Investment in Children." *Journal of Economic Theory* 102 (2): 290-321.
- Babcock, Philip and Mindy Marks. 2011. "The Falling Time Cost of College: Evidence from Half a Century of Time use Data." *Review of Economics and Statistics* 93 (2): 468-478.
- Baker, Michael, Jonathan Gruber, and Kevin Milligan. 2008. "Universal Child Care, Maternal Labor Supply, and Family Well-Being." *Journal of Political Economy* 116 (4): 709-745.

- Becker, Gary S. and Nigel Tomes. 1986. "Human Capital and the Rise and Fall of Families." *Journal of Labor Economics* 4 (3): S1-39.
- Behrman, Jere Richard, Robert A. Pollak, and Paul Taubman. 1982. "Parental Preferences and Provision for Progeny." *Journal of Political Economy* 90 (1): 52-73.
- Bernal, Raquel. 2008. "The Effect of Maternal Employment and Child Care on Children's Cognitive Development\*." *International Economic Review* 49 (4): 1173-1209.
- Black, Sandra E. and Paul J. Devereux. 2011. "Recent Developments in Intergenerational Mobility." *Handbook of Labor Economics* 4: 1487-1541.
- Blau, David M. 1999. "The Effect of Income on Child Development." *Review of Economics and Statistics* 81 (2): 261-276.
- Blau, David M. and Janet Currie. 2006. "Pre-School, Day Care, and After-School Care: Who's Minding the Kids?" *Handbook of the Economics of Education* 2: 1163-1278.
- Carneiro, Pedro and James. J. Heckman. 2003. "Human Capital Policy." In *Inequality in America: What Role for Human Capital Policies?*, edited by James J. Heckman, Alan B. Krueger and Benjamin M. Friedman, 77. Cambridge, MA: MIT Press.
- Casanova, Maria. 2013. "Revisiting the Hump-Shaped Wage Profile." Working paper.
- Caucutt, Elizabeth M., and Lance Lochner. 2012. "Early and Late Human Capital Investments, Borrowing Constraints, and the Family," NBER Working Paper No. 18493.
- Chetty, Raj, Nathaniel Hendren, Patrick Kline, and Emmanuel Saez. 2014a. "Where is the Land of Opportunity? the Geography of Intergenerational Mobility in the United States" NBER Working Paper No. 19843.
- Chetty, Raj, Nathaniel Hendren, Patrick Kline, Emmanuel Saez, and Nicholas Turner. 2014b. "Is the United States Still a Land of Opportunity? Recent Trends in Intergenerational Mobility." NBER Working Paper No. 19844
- Cunha, Flavio. 2013. "Investments in Children When Markets Are Incomplete." Working paper.
- Cunha, Flavio and James J. Heckman. 2007. "The Technology of Skill Formation." *The American Economic Review* 97 (2): 31-47.

- . 2008. "Formulating, Identifying and Estimating the Technology of Cognitive and Noncognitive Skill Formation." *Journal of Human Resources* 43 (4): 738-782.
- Cunha, Flavio, James J. Heckman, and Susanne M. Schennach. 2010. "Estimating the Technology of Cognitive and Noncognitive Skill Formation." *Econometrica* 78 (3): 883-931.
- Dahl, Gordon B. and Lance Lochner. 2012. "The Impact of Family Income on Child Achievement: Evidence from the Earned Income Tax Credit." *American Economic Review* 102 (5): 1927-1956.
- Del Boca, Daniela, Christopher Flinn, and Matthew Wiswall. 2014. "Household Choices and Child Development." *The Review of Economic Studies* 81 (1): 137-185.
- De Nardi, Mariacristina. 2004. "Wealth Inequality and Intergenerational Links." *The Review of Economic Studies* 71 (3): 743-768.
- Domeij, David and Jonathan Heathcote. 2004. "On the Distributional Effects of Reducing Capital Taxes." *International Economic Review* 45 (2): 523-554.
- Erosa, Andres and Tatyana Koreshkova. 2007. "Progressive Taxation in a Dynastic Model of Human Capital." *Journal of Monetary Economics* 54 (3): 667-685.
- Fernandez, Raquel and Richard Rogerson. 1998. "Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform." *American Economic Review* 88 (4): 813-833.
- Guryan, Jonathan, Erik Hurst, and Melissa Kearney. 2008. "Parental Education and Parental Time with Children." *The Journal of Economic Perspectives* 22 (3): 23-16A.
- Haider, Steven J. and Gary Solon. 2006. "Life-Cycle Variation in the Association between Current and Lifetime Earnings." *American Economic Review* 96 (4): 1308-1320.
- Heathcote, Jonathan, Fabrizio Perri, and Giovanni L. Violante. 2010. "Unequal we Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006." *Review of Economic Dynamics* 13 (1): 15-51.
- Heckman, James J. 2008. "Schools, Skills, and Synapses." *Economic Inquiry* 46 (3): 289-324.
- Heckman, James J. and Stefano Mosso. 2014. "The Economics of Human Development and Social Mobility." NBER Working Paper No. 19925.

- Heckman, James, Rodrigo Pinto, and Peter Savelyev. 2013. "Understanding the Mechanisms through which an Influential Early Childhood Program Boosted Adult Outcomes." *The American Economic Review* 103 (6): 1-35.
- Heckman, James J., Jora Stixrud, and Sergio S. Urzua. 2006. "The Effects of Cognitive and Noncognitive Abilities on Labor Market Outcomes and Social Behavior." *Journal of Labor Economics* 24 (3): 411-482.
- Herrington, Christopher M. 2015. "Public Education Financing, Earnings Inequality, and Intergenerational Mobility." Working paper.
- Holter, Hans Aasnes. 2014. "Accounting for Cross-Country Differences in Intergenerational Earnings Persistence: The Impact of Taxation and Public Education Expenditure." Available at SSRN 1978261.
- Huggett, Mark, Gustavo Ventura, and Amir Yaron. 2011. "Sources of Lifetime Inequality." *The American Economic Review* 101 (7): 2923-2954.
- Johnson, Matthew T. 2013. "Borrowing Constraints, College Enrollment, and Delayed Entry." *Journal of Labor Economics* 31 (4): 669-725.
- Keane, Michael P. and Kenneth I. Wolpin. 1997. "The Career Decisions of Young Men." *Journal of Political Economy* 105 (3): 473-522.
- Knudsen, Eric I., James J. Heckman, Judy L. Cameron, and Jack P. Shonkoff. 2006. "Economic, Neurobiological, and Behavioral Perspectives on Building America's Future Workforce." *Proceedings of the National Academy of Sciences of the United States of America* 103 (27): 10155-10162
- Krebs, Tom. 2003. "Human Capital Risk and Economic Growth." *The Quarterly Journal of Economics* 118 (2): 709-744.
- Lee, Chul-In and Gary Solon. 2009. "Trends in Intergenerational Income Mobility." *The Review of Economics and Statistics* 91 (4): 766-772.
- Lee, Sang Yoon Tim and Ananth Seshadri. 2014. "On the Intergenerational Transmission of Economic Status." Working paper.

- Mazumder, Bhashkar. 2005. "Fortunate Sons: New Estimates of Intergenerational Mobility in the United States using Social Security Earnings Data." *Review of Economics and Statistics* 87 (2): 235-255
- Plug, Erik and Wim Vijverberg. 2003. "Schooling, Family Background, and Adoption: Is it Nature Or is it Nurture?" *Journal of Political Economy* 111 (3): 611-641.
- Ramey, Garey and Valerie A. Ramey. 2010. "The Rug Rat Race." *Brookings Papers on Economic Activity* 41 (1): 129-199.
- Restuccia, Diego and Carlos Urrutia. 2004. "Intergenerational Persistence of Earnings: The Role of Early and College Education." *The American Economic Review* 94 (5): 1354-1378.
- Rupert, Peter and Giulio Zanella. 2012. "Revisiting Wage, Earnings, and Hours Profiles." Working paper.
- Sacerdote, Bruce. 2007. "How Large are the Effects from Changes in Family Environment? A Study of Korean American Adoptees." *The Quarterly Journal of Economics* 122 (1): 119-157.
- . 2010. "Nature and Nurture Effects on children's Outcomes: What have we Learned from Studies of Twins and Adoptees." *Handbook of Social Economics* 1: 1-30.
- Solon, Gary. 1992. "Intergenerational Income Mobility in the United States." *American Economic Review* 82 (3): 393-408.
- Solon, Gary. 1999. "Intergenerational Mobility in the Labor Market." *Handbook of Labor Economics* 3: 1761-1800.

Table A1: IGE estimates: life-cycle bias

		Child's age								
		20	25	30	35	40	45	50	55	60
Parent's age	20	.18	.28	.30	.30	.30	.30	.29	.29	.29
	25	.33	.49	.53	.53	.52	.52	.51	.51	.51
	30	.23	.36	.40	.39	.38	.38	.38	.38	.37
	35	.21	.34	.37	.36	.36	.36	.35	.35	.35
	40	.20	.32	.34	.34	.33	.33	.33	.33	.33
	45	.19	.29	.32	.31	.31	.31	.31	.30	.30
	50	.18	.28	.30	.29	.29	.29	.29	.28	.28
	55	.16	.26	.28	.27	.27	.27	.27	.26	.26
	60	.15	.23	.25	.25	.24	.24	.24	.24	.24

Notes: Ages denote the first age of the five year period at which lifetime income is measured.

## A Lifecycle bias in the intergenerational mobility estimates

Table A1 reports the IGE estimates by varying the point (for both parents and children) at which the proxy income is measured. Although the true IGE, estimated by using the discounted lifetime income, is 0.377 as reported in Table 5, the estimates vary quite significantly depending on the timing at which income is measured. There are several systematic patterns regarding the lifecycle bias. First, regardless of when the parent's income is measured, the IGE estimates are seriously downward-biased if the child's income is measured early in their life. For instance, when the child's income is measured at 20-24, the IGE estimates are close to half the true IGE even if the parent's income is measured at old ages. This is consistent with prior empirical research that points out attenuation bias when children's income is measured at early ages (Solon, 1999; and Haider and Solon, 2006). On the other hand, holding the parent's age fixed, the IGE estimates become insensitive to a point at which child's income is measured as long as it is measured at the age of 30 or above.

In Table A2, I perform the same exercise with a rank-rank specification instead of a log-log specification. Overall, we can see the right-skewed inverse U shape in the rank-rank slope as a function of the timing at which child's (or parent's) income is measured: that is, (i) there is attenuation bias when either child's income or parent's income is measured at early ages; and (ii) the slope estimates rise sharply and then decrease gradually as the measured-age rises. A notable difference compared to the IGE estimates is that, holding the parent's measured income fixed, the

Table A2: Percentile rank correlation estimates: life-cycle bias

		Child's age								
		20	25	30	35	40	45	50	55	60
Parent's age	20	.22	.30	.29	.27	.26	.25	.24	.23	.21
	25	.35	.48	.45	.43	.41	.39	.38	.36	.34
	30	.28	.40	.38	.36	.34	.33	.31	.30	.28
	35	.27	.38	.36	.35	.33	.31	.30	.29	.27
	40	.26	.37	.35	.33	.32	.30	.29	.28	.26
	45	.25	.36	.34	.32	.31	.30	.28	.27	.26
	50	.24	.35	.33	.31	.30	.28	.27	.26	.25
	55	.23	.33	.32	.30	.29	.27	.26	.25	.24
	60	.22	.32	.30	.29	.28	.26	.25	.24	.23

Notes: Ages denote the first age of the five year period at which lifetime income is measured.

rank-rank slope estimates rise sharply in the early 20's and then gradually decrease whereas the IGE estimates are virtually flat with respect to child's age after they sharply rise in the 20's. These findings are in line with Chetty et al. (2014a) who show that their rank-rank slope estimates rise steeply in the early 20's and then steadily decrease after age 30.

## B Determining parameters using simulation

A vector of parameter values  $\hat{\Theta} = (B, \beta, \eta, \alpha_1, \alpha_2, \alpha_0, \sigma_\zeta, \sigma_z, m_\xi, \sigma_\xi, \Delta, a_h, \omega, m)$  in Table 3 is jointly determined using simulation. More specifically, define  $M_m(\Theta)$  as the  $m$ -th target statistic obtained from the model-generated data with the set of parameters  $\Theta$ ; and  $D_m$  as the same  $i$ -th target statistics obtained from data, as defined in Table 4. Then  $\hat{\Theta}$  is the minimizer of the objective function:  $\sum_{m=1}^{14} [\log(M_m(\Theta)/D_m)]^2$ . I use the downhill simplex method to solve this minimization problem.

## C Data

Statistics regarding time-use are computed using the 2003-2012 waves of the American Time Use Survey (ATUS). To compute average hours worked and the fraction holds a college degree, I consider both men and women and include those whose age is greater than or equal to 20 and less than 65. To construct a variable of parental time investment in the child's human capital, I focus on the

educational activities that requires the existence of both a parent and a child in a common space. Such categories include reading to/with children, playing with children, doing arts and crafts with children, playing sports with children, talking with/listening to children, looking after children as a primary activity, caring for and helping children, doing homework, doing home schooling, and other related educational activities. As the focus of this paper is time spent in children before age 5, the time investment variable is computed using households whose youngest child is less than five years old. For the time investment variable, I further restrict the sample to the households who are not enrolled in school and whose age is between the age of 21 and 55 (inclusive), as in Guryan, Hurst and Kearney (2008). For all statistics reported, the ATUS statistical weights are used.

Note that the parental time investment variable does not include the activity of physical care for children, which accounts for quite a large portion of time. However, it is interesting to note that, even with the definition of the parental time including the physical care activities, I also find a similar size of the positive educational gradient and it is robust to the parental gender as well. Furthermore, I also broaden the definition of the parental time investment to include some activities that have educational aspects but do not necessarily require the direct/active contact between a parent and a child. Such activities are organizing and planning for children, attending children's events, waiting for/with children, picking up/dropping off children, attending meetings and school conferences for children, waiting associated with children's education. The inclusion of such educational activities that could have indirect impacts on the children's human capital development increases the mean by 14 percent but barely changes the educational gradient. The time-diary survey also reports secondary activities and part of them may also include childcare. However, since the childcare time recorded as secondary activities is expected to be less active and the same hours may not be effective as an input to the human capital function, I do not consider the time of childcare recorded as secondary activities, and only focus on childcare activities reported as a main activity.