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Network effects, Compatibility and the Environment: The Case of Hydrogen Powered Cars

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Abstract

The paper addresses the problem of entry barriers for a new technology – hydrogen powered cars or cars with fuel cell engines – if the network of its filling stations is missing or thin. We use Hotelling's model of product differentiation to characterize a situation where an incumbent firm produces the old technology, compatible with the existing network of filling stations, and an entrant, who cannot use this network for its products. We assume that the entrant has to invest in remodeling existing filling stations for making them compatible. This, however, raises his costs. In the intertemporal setting of our model, the Hotelling pricing rule for exhaustible resources encourages the entrant to invest in compatibility because the price of gasoline will rise in the long run to the price of the backstop technology - fuel cells. Depending on the cost of compatibility, our model indicates three possible outcomes. Either, the costs of compatibility are too high and governmental support is required. Or the incumbent bears losses in initial periods by waiting for profits in later periods when full compatibility of the network is reached. Or the entrant benefits from the fact that the price of oil reaches the price of the backstop technology (full cells) rather soon.

Keywords: Network effects, compatibility, environmental concern, price competition, lock-in effect, automobiles.

JEL classification: L 11, L 15, L 62, Q 42.

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1. Introduction

According to ambitious plans by the European Union, fossil fuel is supposed to be substituted by hydrogen by the end of the year 2100. The dream is a nearly emission free automobile which would contribute to achieve a stricter CO_2 emission target and would also reduce the dependency of Europe from oil imports. The fuel-cell technique which uses the possibility to produce electricity and heat by the controlled oxyhydrogen explosion of hydrogen with oxygen plays a central role here. Some cities already have electricity driven busses or fuel-cell power stations. However, a widespread infrastructure for hydrogen power in Europe is far off and would require huge investment costs. This network externality restrains consumers to buy hydrogen-powered cars. The consumption externality is generated through an indirect effect because an agent, purchasing a car, is concerned about the number of other agents purchasing similar cars because the units of the complementary good, hydrogen equipped gas-stations, increases with the number of hydrogen powered cars that will be sold. The number of those cars being sold will depend on the service network which, in turn, will increase if more cars have been sold. Network externalities arise out of the complementarity of different network pieces. The value of a good increases as more of the complementary good is provided (sold), and vice versa. Sales will be initially delayed or blocked by consumers' awareness of the thin network of service stations offering natural gas. The feature of this market is that cars with different engines may use the same network, however only a few petrol stations are equipped with natural gas pump posts, a fact that reduces the willingness to purchase such a natural gas driven motor vehicle.

The interest in a new kind of fuel for cars or in a new technology (fuel cells) arises from the concern about global warming and the scarcity of fossil fuel. CO_2 emissions could be (partly drastically) reduced by gas-driven cars (natural gas, methane, compressed natural gas (CNG)), by hydrogen powered cars or by a fuel-cell engine system. The fuel-cells are the technology for the distant future. They convert natural gas, methanol or hydrogen fuel into electricity without combustion. When the fuel is hydrogen, then water vapour is the only by-product from the fuel cell itself.[2] Whenever natural gas or methanol is consumed as a fuel,

[1] I am grateful to Jörg Gutsche and Peter Hasfeld for very helpful comments.

[2] But one must consider how the hydrogen gets produced. If it is produced from natural gas (as most hydrogen is) then carbon dioxide is released to the atmosphere in the production of the hydrogen.

CO_2 is released. It will take about eight years before fuel-cell powered cars are available commercially, and maybe another eight years before they become affordable due to mass production. The true time period will depend, however, on the consumption externality in terms of the network of service and filling stations. Hydrogen powered cars are even more environmentally friendly than gas powered cars, but driving with hydrogen is more expensive than with gasoline, given the current price of gasoline. Whereas a gasoline powered car emits 160 gr. CO_2 per km, a hydrogen driven car would emit only 35 gr. CO_2 per km if hydrogen is produced from a non-exhaustible resource.[3] It is filled as a liquid at a temperature of –253° C in a special tank. Similarly to the gas powered engine, it is not the technique that is the problem, but it is the network. This problem can be diminished by producing bi-fuel cars which are based on a combustion engine powered by gasoline as well as by natural gas or hydrogen. The disadvantage of these cars is the reduction of space for the backseats and the trunk, which is needed for the two tank fillings.

The purpose of this paper is to investigate the relationship between an incumbent firm and an entrant with respect to network compatibility and pricing decisions. For this purpose we consider a duopoly where firm *H* has developed a car with a hydrogen power driven engine which it wants to introduce into the market. There is already an incumbent, firm *G*, supplying cars with the familiar gasoline driven engine. The non-compatibility of filling stations to the new technology is of disadvantage to firm *H*, called the entrant, since it gives owners of gasoline powered cars large network benefits but none for the potential owner of a car with the new technology (lock-in effect). The two competing firms have chosen already simultaneously and independently from each other their technology, that is their locations at either end on the unit interval in a horizontal product differentiation dimension. We model a repeated two stage game where in a first stage firm *H* invests in the network while firm *G* does not need to invest into the existing network as it already is compatible. In a second stage both firms compete in prices. The firms repeat this game every five years when consumers replace their cars by a new model. In this dynamic setting the prices are the control variables and the size of the network is the stock variable. Support for the entrant comes from environmental concern of the consumers and from rising gasoline prices. We are interested to see whether the entrant can overcome the look-in effect and can conquer market shares.

In order to relate our findings to the existing literature, we should point out that there are two types of product differentiation – horizontal product differentiation within the same quality group and vertical product differentiation in terms of different quality levels.

[3] CO_2 emissions from the construction of a plant is included in the figure.

Horizontal product differentiation emphasizes the fact that the supply of a product variant (within this quality group) does not satisfy completely some or many consumers. It could therefore be a profit maximizing strategy to offer modifications of a standard product which is closer to the preferences of some customers. Under vertical product differentiation firms choose a high or low quality class in the product space. There is a price-quality competition with a trade-off in higher prices for better quality or a lower price for the lower quality. In either of these product differentiation models the firms will choose distinct characteristics or qualities because as those become close, price competition between the increasingly similar products reduces the firms' profit. In vertical product differentiation models with environmental background, the focus of environmental policy is often on minimum quality standards (see, e.g. Crampes and Hollander, 1995; Ronnen, 1991; Motta and Thisse (1999)). Greaker (2003) presents a partial trade model with one domestic and one foreign firm where products are differentiated along both environmental quality (vertical differentiation) and taste/eco-label (horizontal differentiation). He focuses on horizontal differentiation by assuming that the taste parameter is relatively more important for the consumer than the environmental performance of the product. In his model, as in our approach, the two firms are located at each end of the $0-1$ product line. We follow Greaker by also assuming that horizontal differentiation (environmental concern) dominates vertical differentiation (horse power).

There is a substantial amount of literature on network externalities.[4] Katz and Shapiro (1985) consider a model of static oligopolistic competition with network externalities. Consumers form exogenous expectations on the network size of the competing firms on the market (as they will do in our model). Then firms determine their prices on which consumers base their purchase decision. Farrell and Saloner (1986) analyze the incentives for adopting a new technology that is incompatible with the installed base. In an equilibrium the outcome depends on the size of the installed base when the new technology is introduced, it depends on how quickly the network benefits of the new technology are realized, and on the relative superiority of the new technology. Our results conform with their results although we use a different model.

Papers focusing also on compatibility decisions of oligopolistic firms where network externality interact with other quality dimensions, which the firms can control, are

[4] See for example Katz and Shapiro (1985, 1986, 1992, 1994), and Farrell and Saloner (1985, 1986), and especially the book by Shy (2001) devoted to this topic. These authors do not use, however, the models of horizontal or vertical product differentiation. Surveys of the literature on the network externalities and market performance are found in Katz and Shapiro (1994); Besen and Farrell (1994); Economides (1996); Matutes and Régibeau (1996).

Belleflamme (1998), Bental and Spiegel (1995), de Palma and Leruth (1996) and Baake and Boom (2001). In Belleflamme (1998) a two stage game is analyzed where the firms choose first to adopt one of two network technologies. Once firms have adopted one or the other technology, they compete on the market with a marginal cost that depends on the size of the network to which they belong. Bental and Spiegel consider a model in which the quality of a good is identified by the number of consumers who consume that good. Using the preference specification of vertical quality differentiation, it is shown that the largest network produced will be the most expensive one and used by the richest consumers. De Palma and Leruth study the endogenous decision of firms to agree to the same standard. Standardization is the result of a two-stage game: at the first stage, firms play in compatibility and at the second stage, they play in quantity. Baake and Boom focus on compatibility decisions of two firms where network externalities interact with other quality dimensions which the firms can control. Network size is not the only vertical dimension but consumers' willingness to pay increases in both the product's inherent quality and the size of the network. The example, given in the paper for network externality and incompatibility are personal computers with different qualities using the same operating system and having the same network externality from sharing the same pool of available software products. Compatibility can be achieved by an adapter. With an adapter, the network size comprises the consumers of both products. The endogenous adapter decision is similar to our network of gasoline stations where after their remodeling (adapter) the network can be used by both engines.

Another strand of the literature on network economics utilizes an approach sometimes referred to as the supporting services approach. Software packages, for example, are regarded as supporting services for the hardware. The literature utilizing the supporting services approach includes Chou and Shy (1990, 1993) and Church and Gandal (1992 a, b, 1993 and 1996). Like in our car engine case, in many instances supporting services are incompatible across brands. Since a hydrogen powered car must be gas station compatible, we can not utilize these models for our case because they compare equilibrium profits and welfare under compatibility and incompatibility.

Finally, a model similar in spirit is Grilo, Shy and Thisse (2001). They examine a model of horizontal product differentiation that introduces a consumption externality. Unlike the present paper, Grilo et al. exploit a quadratic externality function. This allows them to consider both positive and negative consumption externalities.

The paper is organized as follows. In section 2 we present the model of two firms competing for customers when network effects are present and the entrant has to invest in achieving compatibility to the existing network. Section 3 presents the dynamic structure of the game and section 4 characterizes a steady state situation where the entrant has no incentive anymore to invest in compatibility. Some or all petrol stations have been remodeled and cars can refuel either gasoline or hydrogen at those stations. Section 5 concludes.

2. Network effect and compatibility

We consider a market with an incumbent and a potential entrant or "sponsor"[5] where competition is in market shares using price and investment in compatibility as instruments. The incumbent benefits from the existing network and from the lock-in effect for the owner of gasoline cars which generates external effects. Decisions from the past favor the established technology although a new technology would lead to a socially preferable equilibrium. In the first stage of our non-cooperative repeated game the incumbent trusts on the lock-in effect and on his well-extended network whereas the sponsor has to invest in compatibility to remodel the gas-stations. Since this increases the entrant's cost, he needs support for his costly investment by environmental concern in the society and by a steadily raising gasoline price. In the second stage of the game firms compete in prices taking into account the size of their installed base. Our model differs from standard models of horizontal product differentiation because of the introduction of a network externality (lock-in) and because of the aspect that the complementary good (gasoline) for the product of the incumbent will be exhausted in the near future. Compatibility of the new product with the installed base of the incumbent is not a meaningful strategy for the producer of an engine as it is for a PC producer offering IBM compatible PCs which can use the standard software.[6] In our model the entrant has to invest money in remodeling the existing network to make it compatible to hydrogen powered cars.[7]

In our model of horizontal product differentiation each consumer buys one unit of the product every five years. There is a continuum of consumers uniformly distributed over

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[5] Katz and Shapiro (1986) call a firm a sponsor of a technology if it controls the property rights to a given technology. In that case the firm will be willing to invest into the network or in the form of penetration pricing to establish the technology because then there is the prospect of profits in later periods.

[6] See Pfähler and Wiese (1998) for a game in the degrees of compatibility. For a survey on compatibility and network effects see Wiese (1997).

[7] Since by assumption firms offer characteristics at each end of the 0-1 product line, we therefore exclude the strategy to produce cars with bi-fuel engines, having two tanks. This strategy, observable in reality, is a way to become compatible with the installed base. It would eliminate the advantage of the installed base for the incumbent but raises the cost of production and in addition reduces the capacity of the trunk compartment.

Hotelling's [0,1] interval. The net-utility of consumer $\theta \in [0,1]$, who buys a unit of good *G* (gasoline powered car) in period *t*, indexed in five-year intervals, is:[8]

(1) $$v_G(\theta) = u - \tau \cdot \theta^2 - p_G + \gamma\, n_G - \alpha_G\, q_G\,.$$

The net-utility of consumer $\theta \in [0,1]$, who buys a unit of good *H* (hydrogen powered car) in period *t* is:

(2) $$v_H(\theta) = u - \tau(1-\theta)^2 - p_H + \gamma\, n_H - \alpha_H\, q_H\,.$$

Good *G* are cars driven by gasoline and good *H* are cars driven by hydrogen power or by fuel-cells. By assumption, firm *G* produces at zero on the 0–1 Hotelling line, and firm *H* at one. The locations are fixed, i.e. the firms produce either gasoline powered cars or hydrogen powered cars (no bi-fuel engine). The meaning of the variables is:

u	- intrinsic utility
$\tau \cdot \theta^2$	- transportation costs, i.e. costs for not getting the preferred characteristics
n_i	- size of network *i*, $i = G, H$
γ	- weight of the size of the network for a monetary network effect $\gamma\, n_i$
q_G	- current gasoline price
q_H	- price for hydrogen fuel (backstop technology)
α_G, α_H	- consumption coefficients, expressing quantity aspects and environmental concern $(\alpha_G > \alpha_H)$

Horizontal product differentiation in our car model means that if both firms charge the same price, demand for both goods is positive if there are no overwhelming differences in terms of environmental concern and network effects. Some motorists prefer the established, matured technology, the high horse power and the driving dynamics while others favor the new technology because of the stillness in running, the low noise gauge of the engine, the jerk-free

[8] We consider only those consumers who buy a car every five years. This is the target group for the automobile producers. We assume that owners of cars older than five years will buy used cars older than five years when replacing their cars.

start and the more comfortable stop and go driving, and its excellent environmental performance. There is no difference in quality, i.e. we do not consider aspects of vertical product differentiation. The main characteristic of the consumer, described by $\theta \in [0,1]$ is environmental concern, which is the reason for the different willingness to pay. Some consumers consider gasoline powered cars as environmentally unfriendly, while others do not care about an environmentally friendly technology like fuel cells. The difference of the net-utilities in (1) and (2) shows the possibilities, firms have to attract customers:

$$\underbrace{v_G(\theta) - v_H(\theta)}_{\text{difference in utility}} = \underbrace{p_H - p_G}_{\text{price effect}} + \underbrace{2\,\tau\left(1/2 - \theta\right)}_{\text{product differentiation}} + \underbrace{\gamma\left(n_G - n_H\right)}_{\text{network effect}} + \underbrace{\alpha_H\, q_H - \alpha_G\, q_G}_{\text{fuel price and environmental concern}}$$

Favorable for the incumbent is (i) a higher price of the entrant, (ii) its product differentiation in terms of attracting consumers to the left of $1/2$ on the [0,1] line, (iii) a larger network effect and (iv) a relatively high price for hydrogen and a relatively low environmental concern (α_H relative to α_G).[9]

Next, we define the size of the network in its general form in terms of a share as:

$$(3) \qquad \underbrace{n_i = \theta_i^e}_{\text{direct}} + \underbrace{\frac{IB_i}{IB_G + IB_H}}_{\text{indirect}} + \underbrace{s_i\left(IB_i\right)\left(\theta_j^e + \frac{IB_j}{IB_G + IB_H}\right)}_{\text{compatibility}}$$

where θ_i^e is the expected market share of cars of type *i*, IB_i is the installed base and $s_i\left(IB_i\right)$ is the degree of compatibility of network $j \neq i$ for network *i*. The larger s_i, the more enforces the network size of firm *j* the network size of firm *i*. IB_G are the gas stations with gasoline pumps only, IB_H are the remodeled gasoline stations, i.e. those with pumps for gasoline as well as for hydrogen (natural gas, etc.). The sum of the gas stations, $SIB = IB_G + IB_H$, is constant, i.e. new gas stations will not be built, only existing gas stations IB_G will be remodeled to become compatible. The network effect can be characterized as direct, if the customers are interested that other consumers purchase or use the same good. A network

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9 Horizontal product differentiation implies that the difference in utility should be negative for consumer θ close to 1 and positive for consumer θ close to zero if $p_H = p_G$. This property requires $\gamma\left(n_G - n_H\right) + \alpha_H\, q_H - \alpha_G\, q_G < \tau$.

effect is called indirect, if a complementary good becomes better and/or cheaper to the extent the network good diffuses. A network good can fail because there are no – or not enough complementary goods on the market.

We will simplify (3) by first neglecting the direct effect, i.e. the expected market share of cars. Consumers will not expect a better network for *H*-cars because they see more *H*-cars on the road, i.e. $\theta_i^e = 0, i = G, H$. Since firm *G* does not need any gas stations that have been remodeled to also sell hydrogen, the degree of compatibility of the *H*-stations for its network equals one, i.e. $s_G\left(IB_G\right)=1$. From the point of view of firm *G* the total network *SIB* is compatible for gasoline cars, that is, the entire installed base contributes to firm *G*'s network size, regardless whether the gas stations sell gasoline only or gasoline as well as hydrogen. Therefore, (3) simplifies to $n_G = 1$. For firm *H*, the portion of the installed base in the network size in (3) is the portion of gas stations remodeled in the past to also sell hydrogen. (2. term in (3)). Firm *H* invests in the installed base IB_H in order to increase the compatibility of gas stations with hydrogen; i.e. $s_H\left(IB_H\right)>0$ and $s'_H > 0$. A higher s_H means that a higher portion of former non-compatible *G*-stations can now be made compatible. Therefore, if we normalize *SIB* to one for convenience, (3) will become:

(4) $$n_H = IB_H + s_H\left(IB_H\right)\cdot IB_G\,.$$

If $s_H = 1$, then follows $n_H = 1$ analogously to $n_G = 1$.[10] A specification for $s_H\left(IB_H\right)$ could be $s_H\left(IB_H\right) = IB_H$ or $s_H\left(IB_H\right) = a\cdot IB_H + \left(1-a\right)IB_H^2$. In both cases is $s_H = 0$ for $IB_H = 0$ and $s_H = 1$ for $IB_H = SIB = 1$. A degree of compatibility of $s_H = 1$ means that all former *G*-stations ($IB_G\left(0\right)$ at the beginning) have been remodeled as $G + H$ stations.

We are interested in finding a consumer $\theta \in [0,1]$ who is indifferent at prices p_G, p_H to purchase from firm *G* (to the left of θ) or from firm *H* (to the right of θ). From $v_G\left(\theta\right) = v_H\left(1-\theta\right)$, we can solve for θ. It is

(5) $$\tilde{\theta} = \frac{\tau + p_H - p_G - \gamma\left(-1 + IB_H + s_H\ IB_G\right) - \alpha_G\ q_G + \alpha_H\ q_H}{2\,\tau}\,.$$

[10] One should consider that the number of $IB_G{}'s$, the gas stations not yet remodeled, declines over time when IB_H increases. If all $IB_G{}'s$ have been remodeled, it is $IB_G = 0$ and $n_H = IB_H$.

Since all consumers θ with $\theta < \tilde{\theta}$ buy good *G*, the demand function of firm *G* is $x_G\left(p_G, p_H, IB_G, IB_H, q_G, q_H\right) = \tilde{\theta}$. Because of $x_G + x_H = 1$, the demand function of firm *H* is

(6)
$$x_H\left(p_G, p_H, IB_G, IB_H, q_G, q_H\right) = 1 - \tilde{\theta} = \frac{\tau + p_G - p_H + \gamma\left(-1 + IB_H + s_H\ IB_G\right) + \alpha_G\ q_G - \alpha_H\ q_H}{2\,\tau}$$

Firm *G* maximizes profit with respect to price:

$$\max_{p_G}\ \pi_G = \left(p_G - c_G\right) \cdot x_G\ \left(p_G, p_H, IB_G, IB_H, q_G, q_H\right).$$

Since firm *G* does not need to invest in compatibility, its maximization problem is not an intertemporal one but is the same in each period, given price p_H and its installed base SIB. Firm *H* maximizes profit with respect to price and to investment in compatibility:

$$\max_{p_H, I_H}\quad \pi_H = \int_0^T e^{-rt}\left\{\left(p_H - c_H\right) \cdot x_H\left(p_G, p_H, IB_G, IB_H, q_G, q_H\right) - C\left(I_H\right)\right\} dt$$

s.t. $\dot{IB}_H = I_H$ with $IB_G = 1 - IB_H$ and *r* as the discount rate. We assume that the remodeling department operates under decreasing returns to scale.[11] This is a realistic assumption because firm *H* needs special equipment and specially trained workers which it can not use anymore after remodeling is finished.

Next we simplify (5) and (6) by defining:

(7)
$$\Delta\, n^{IB} := 1 - IB_H - s_H\ IB_G$$

(8)
$$\Delta\, q := \frac{\left(\alpha_H\ q_H - \alpha_G\ q_G\right)}{\gamma}.$$

Then the market shares are[12]:

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[11] If e.g. $C\left(I_H\right) = \delta\ I_H^2$, this implies that marginal investment costs increases with the number of investment projects, I_H, of remodeling existing gasoline stations.

[12] See Pfähler and Wiese (1998) for a model with network effects, installed bases and competition in prices and degrees of compatibilities (Chapter L).

(5') $$x_G = \frac{1}{2} + \frac{1}{2\tau}\left[p_H - p_G + \gamma\left(\Delta n^{IB} + \Delta q\right)\right]$$

(6') $$x_H = \frac{1}{2} - \frac{1}{2\tau}\left[p_H - p_G + \gamma\left(\Delta n^{IB} + \Delta q\right)\right].$$

The firm has a "natural share of customers" of $1/2$. It consists of the customers $\theta \leq 1/2$ which purchase from firm *G* because it is closer to their preferences. If prices are identical and there is no network effect, then demand is equal to this natural share of customers. If $p_G < p_H$, then more customers purchase good *G* not because of the nearness to their preferences but because of the favorable price.

Under profit maximization, the FOCs with respect to p_G and p_H lead to a Nash equilibrium of the simultaneous price competition in every period *t*.

(9) $$p_G^N(t) = \frac{1}{3}\left[c_H + 2\,c_G + 3\tau + \gamma\left(\Delta n^{IB}(t) + \Delta q(t)\right)\right]$$

(10) $$p_H^N = (t)\frac{1}{3}\left[c_G + 2\,c_H + 3\tau - \gamma\left(\Delta n^{IB}(t) + \Delta q(t)\right)\right].$$

Inserting (9) and (10) in (5') and (6') results in the market shares $x_G^N\left(IB_H, q_H, q_G\right)$ and $1 - x_G^N = x_H^N$:

(11) $$x_G^N(t) = \frac{1}{2} + \frac{1}{6\,\tau}\left[c_H - c_G + \gamma\left(\Delta n^{IB}(t) + \Delta q(t)\right)\right]$$

(12) $$x_H^N(t) = \frac{1}{2} - \frac{1}{6\,\tau}\left[c_H - c_G + \gamma\left(\Delta n^{IB}(t) + \Delta q(t)\right)\right].$$

There is a sequence of Nash equilibriums over time which firm *H* can influence in its favor by investing in compatibility, $I_H(t)$, to raise its installed base, $IB_H(t)$. Firm *G*, however, need not by definition invest in its network. The main motivation for the sponsor is its perfect information on the increasing price path $p_G(t)$ of gasoline. We assume that it increases over time according to the Hotelling price rule of a resource, i.e. $q_G(0)\,e^{rt}$ where *r* is the discount rate. It means that for a firm to be indifferent between extracting the resource in the current period and a future period, the price must rise at the discount rate. An optimal exploitation path for crude oil is based on this Hotelling price rule. If the oil resource is exhausted in *T*, $q_G(T)$ must be equal to the price of the backstop-technology hydrogen, i.e. $q_H = q_G(0)e^{rT} = q_G(T)$, and no consumer buys from firm *G* anymore.

Profit for firm *H* is:

$$\pi_H(I_H, IB_H, \Delta q) = \left[p_H^N(IB_H(t), \Delta q(t)) - c_H \right] \cdot x_H^N(t) - C(I_H(t))$$

where $IB_G(t)$ has been replaced by $IB_G(t) = 1 - IB_H(t)$. With a fixed price of the backstop technology, q_H, the difference in the cost aspect of the complementary good declines in time according to (8), i.e. $\gamma \cdot \Delta q(t) = (\alpha_H\, q_H - \alpha_G\, q_G(0)e^{rt})$. The intertemporal problem of firm *H* is then to control the outcome on the path of Nash equilibria by choosing an optimal investment path $IH(t)$:

(13) $$\max_{I_H} \int_0^T e^{-rt}\, \pi_H(I_H, IB_H, \Delta q)\, dt$$

s.t. $\dot{IB}_H = I_H$.

The Hamilton function for (13) is

$$H = \pi_H(\quad) + \mu \cdot I_H .$$

The maximum principle yields

(13a) $$H_{I_H} = \frac{\partial \pi_H}{\partial I_H} + \mu = 0 .$$

The portfolio balance condition for μ is:

(13b) $$\dot{\mu} = r\mu - H_{IB_H} = r\mu - \pi_{IB_H} .$$

I_H is the control variable, IB_H the state variable and μ is the co-state variable or shadow value of the installed base.

At time *T,* the gas price $q_G\,(T) = q_G\,(0)e^{rT}$ has reached the price of the backstop technology q_H and the oil field is exhausted. In that case, x_G should be zero and there are only cars run by hydrogen; investment in IB_H should be finished by then. For such a steady state in *T* it is $\dot{IB}_H = 0$, i.e. $I_H(T) = 0$, and $\dot{\mu} = 0$, i.e. $\mu\,(T) = \frac{1}{r}\,\pi_{IB_H}$. Prior to *T*, the time path of the solutions $I_H\,(t)$ and $IB_H\,(t)$ are functions of q_H and $q_G\,(t)$,

(14) $$I_H\left(t\right) = I_H\left(q_H, q_G(t)\right) \quad , \qquad IB_H\left(t\right) = IB_H\left(q_H, q_G(t)\right).$$

The firms need to know the values of q_H, $q_G\left(0\right)$ and *T*. The price q_H of the backstop technology is defined such that the market share of the old technology is zero when the price of its complementary good gasoline, $q_G\,(T)$, reaches q_H, i.e. $x_G\left(q_H, q_G\left(0\right)e^{rT}\right) = 0$. That means that firm *H* must know the price path of gasoline, $q_G(t)$. Under complete information, however, it knows the strategy of the oil industry, which, under profit maximizing behavior, will have exploited its oil field exactly then, when the market share for cars run by gas has reached zero. That means that at time *T,* the oil price must have reached the level q_H, meaning the highest possible price when the oil field is exploited afterwards. Therefore, $q_G(T)$ at $x_G = 0$ must be equal to the price of the backstop technology,

(15) $$q_H = q_G\left(0\right)e^{rT} = q_G\left(T\right).$$

Once we have derived $q_G(T)$ from the condition $x_G\left(q_H, q_G\left(0\right)e^{rT}\right) = 0$, assuming $q_G(T) = q_H$, its solution $q_G(T) = q\left(c_H, c_G, \alpha_H, \alpha_G\right)$ defines by (15) the price of the backstop technology.

3. The dynamic structure of the game

We will postpone the explicit solution of the optimization problem and will at first clarify the dynamic structure of the game. In every period there is a compatibility decision by means of I_H, where we have assumed that firm *G* need not to invest in compatibility, because the sponsor (firm *H*) has to remodel the existing installed base which is compatible for gasoline powered cars anyhow. Knowing the Nash price equilibria, depending on IH_H and IB_H, firm *H* can determine an optimal path for its compatibility decision. Before we prove our results, we characterize in Table 1 three points in time with their corresponding values of the variables. Depending on the price of the backstop technology, a steady state could be reached at a time T^*, prior to *T*, with all gasoline stations remodeled[13] and the firms share the market. The middle part of Table 1 characterizes this situation. After a while (at *T*), the gasoline price has reached the price of the backstop technology and environmentally friendly hydrogen powered cars become more attractive to the consumers. This outcome is characterizes in the lower part of Table 1.

[13] We will prove in the next section, that in a steady state $IB_G = 0$, i.e. there are no gas-stations anymore which serve only gasoline.

$t < T^*$: firm H invests in compatibility				
$IB_G(t) > 0$:	$s_G = 1$	$I_G = 0$	$p_G(t)$	$\pi_G > 0$
$IB_H(t) > 0$:	$s_H\left(IB_H\right) > 0$	$I_H(t) > 0$	$p_H(t)$	π_H ?
$q_G(t) < q_H$		↙ ↘	$x_G(t) > 0$,	$x_H(t) > 0$
	$s_H\left(IB_H\left(t+1\right)\right)$	$\dot{IB}_H\left(t+1\right)$		
$t = T^*$: full compatibility; both firms share the market				
$IB_G(T^*) = 0$:	$s_G = 1$	$I_G = 0$	$p_G(T^*) > c_G$	$\pi_G > 0$
$IB_H(T^*) = SIB = 1$	$s_H\left(SIB\right) = 1$	$I_H(T^*) = 0$	$p_H(T^*) > c_H$	$\pi_H > 0$
$q_G(T^*) < q_H$			$x_G(T^*) > 0$,	$x_H(T^*) > 0$
$t = T > T^*$: oil price reaches the price q_H; market exit of firm G				
$IB_G(T) = 0$:	$s_G = 1$	$I_G = 0$	$p_G(T) = c_G$	$\pi_G = 0$
$IB_H(T) = SIB = 1$	$s_H\left(SIB\right) = 1$	$I_H(T) = 0$	$p_H(T) > c_H$	$\pi_H > 0$
$q_G(T) = q_H$			$x_G(T) = 0$,	$x_H(T) = 1$

Table 1: Steady state periods with crude oil $\left(T\right)$ and without $\left(T^*\right)$.

We continue by examining the point in time T where the oil resource is exhausted. For that purpose we need a resource constraint, a price of gasoline $q_G(0)\,e^{rT}$ where $x_G = 0$, and the Hotelling path satisfying $q_H = q_G\left(0\right)e^{rT}$. These three conditions permit us to determine T, q_H, and $q_G\left(0\right)$. If a steady state occurs in T, it is $I_H(T) = 0$ and therefore $C\left(I_H\right) = 0$. For the installed base we assume that in T: $IB_G = 0$ (The proof will follow later). Then $\Delta\, n^{IB} = 0$ in (7). For $\Delta\, q$ follows according to (8):

$$(16) \qquad \Delta\, q = q_H \frac{\left(\alpha_H - \alpha_G\right)}{\gamma}.$$

Using these results we obtain from (11) with $x_G^N = 0$

(17) $$q_G(0)\,e^{rT} = \frac{3\tau + \left(c_H - c_G\right)}{\alpha_G - \alpha_H}.$$

We assume that $\alpha_G > \alpha_H$, i.e. if prices q_H and $q_G\,e^{rT}$ are equal, the α_G-effect of cars run by gasoline has a stronger negative effect on utility v_i in (1) and (2) than the environmentally friendly α_H-effect. This is how we consider the negative externality of fossil fuels in the utility function. If τ in (17) is high (strong preferences for the good at either end of the product line), the demand for cars run by gasoline will be zero only when the gasoline price has reached a high level $q_G(T)$. If $\alpha_G - \alpha_H$ is high (strong negative externality of cars run by gasoline), the demand for gasoline cars will be zero already when the gasoline price has reached a relatively low level $q_G(T)$. Condition (17) can be used to solve for $q_G(0)$ as a function of *T*. Now only a value for *T* is needed. This value can be determined from the finiteness of the resource crude oil given the value of $q_G\left(0\right)$, i.e.

(18) $$\int_0^T \rho \cdot x_G\left(IB_H(t), q_G(0)\,e^{rt}\right)dt = S_0$$

where S_0 is the initial oil stock and ρ is a coefficient which transforms the portion of cars run by gasoline into gasoline consumption. Here we must bear in mind that (17) and (18) form a simultaneous system for the determination of *T* and $q_G\left(0\right)$. Figure 1 clarifies our considerations. The shaded area represents the initial stock of oil. It is exactly exhausted at the same time (*T*) when the price of oil reaches the price of the industrial substitute.

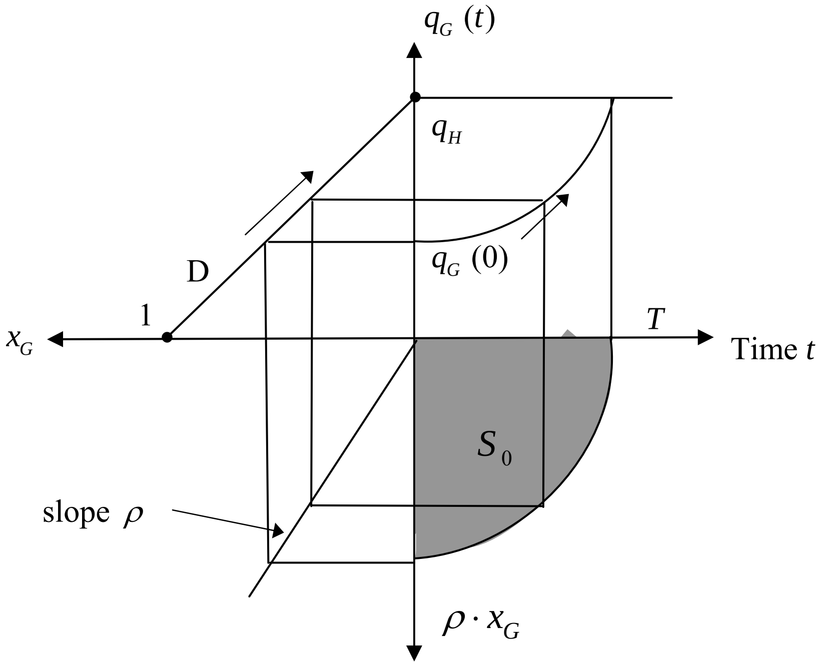

Fig. 1: Market share x_G of firm G and oil extraction $\rho \cdot x_G$ until the gasoline price q_G approaches the price of the backstop technology, q_H.

4. Analysis of the steady state

The open questions we haven't answered yet are (i) whether the sponsor will invest in compatibility $\left(I_H(t) > 0\right)$ although he will bear a loss in the first periods, and (ii) whether he will invest until full compatibility is reached $\left(IB_H = SIB\right)$, i.e. all gas stations provide hydrogen. For that purpose we first write the profit function in the variables $I_H, IB_H, \Delta q$ by using the prices and market shares in (9) to (12):

(19) $$\pi_G = \frac{1}{18\,\tau}\left[3\,\tau - \left(c_G - c_H\right) + \gamma\left(\Delta\, n^{IB} + \Delta\, q\right)\right]^2$$

(20) $$\pi_H = \frac{1}{18\,\tau}\left[3\,\tau + \left(c_G - c_H\right) - \gamma\left(\Delta\, n^{IB} + \Delta\, q\right)\right]^2 - C\left(I_H\right).$$

It is

(21) $$\frac{\partial\,\pi_H}{\partial\, I_H} = -C'\left(I_H\right) < 0\,,$$

i.e. in each particular period remodeling I_H gas stations reduces profit. The positive effect will come later once the compatibility is established. The maximizing principle postulates according to (13a):

(13a) $$\frac{\partial H}{\partial I_H} = \frac{\partial \pi_H}{\partial I_H} + \mu = 0 \, .$$

As the installed base changes by $\dot{IB}_H = I_H$, it is $I_H\left(T\right) = 0$ in the steady state (period T). With $C'\left(I_H\right) = 0$ at $I_H\,(T) = 0$,[14] it is $\frac{\partial \pi_H\left(T\right)}{\partial I_H} = 0$ (see (20)); therefore $\mu\left(T\right) = 0$ from (13a). Hence (see (13b)), we have to solve the equation $\frac{\partial \pi_{IB_H}}{\partial IB_H} = 0$ in order to determine $IB_H\left(T\right)$. According to (20) and (7) we obtain

(22) $$\frac{\partial \pi_H}{\partial IB_H} = \frac{1}{18\tau} \cdot 2\left[\cdot\right]\left(-\gamma \cdot \left(-1 - s'_H \; IB_G + s_H\right)\right) = 0$$

where $IB_G = 1 - IB_H$ accounts for the last term. As $\left[\cdot\right] \neq 0$, which is the term in brackets in (20), the FOC for IB_H is

$$\left(-1 - s'_H \cdot IB_G + s_H\right) = 0 \, .$$

With our assumption of $s_H\left(IB_H\right) = IB_H$, the FOC is:

$$-1 - IB_G + IB_H = 0 \, .$$

Since $IB_G = 1 - IB_H$, we obtain $IB_H^* = 1$, i.e. $s_H^*\left(IB_H\right) = 1$ and $IB_H^* = SIB = 1$. All gas stations in the steady state provide gasoline as well as natural gas, and firm *H* has invested in the

[14] e.g. $C\left(I_H\right) = \delta \cdot I_H^2$.

network of gasoline stations to achieve full compatibility.[15]

In Table 1 we distinguished two periods of a steady state. We had assumed a time period T^* where a steady state is reached but the path of the gasoline price is still below q_H; i.e. $q_H > q_G(0)e^{rt}$ for $t \geq T^*$. In period $T > T^*$, we assumed that $q_H = q_G(0)e^{rT}$ and oil is exhausted. This then implies a steady state with $x_G = 0$ and permits to solve for $q(0), T$ and q_H. Figure 2 shows the situation where a steady state is reached but the exploited stock of oil S^* is still below the proven reserves S_0.

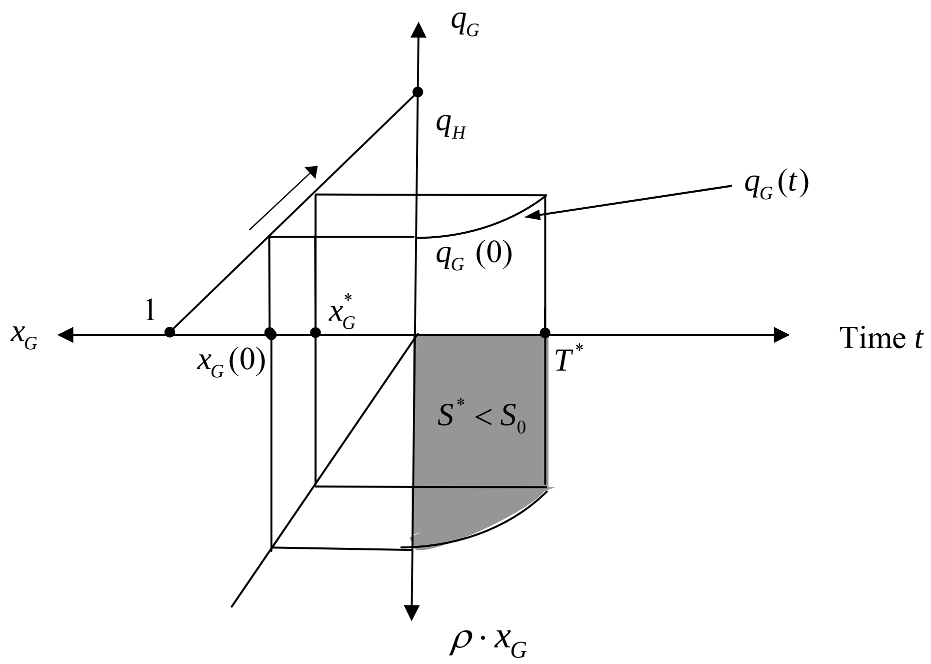

Fig. 2: A steady state is reached (full compatibility) before the stock of proven oil reserves is exhausted.

In the steady state with oil still available $\left(I_H^*(t) = 0, IB_H^*(t) = 1, IB_G^*(t) = 0\right.$ and $\Delta n^{IB} = 0$ for $\left. T^* \leq t < T\right)$, the market shares follow from (11) and (12)

$$x_G^*(t) = \frac{1}{2} + \frac{1}{6\tau}\left[c_H - c_G + \alpha_H\, q_H - \alpha_G\, q_G(t)\right],$$

[15] If we had employed the specification $s_H(IB_H) = a\, IB_H + (1-a)(IB_H)^2$, the result for s_H^* would also have been $s_H^* = 1$ with $IB_H^* = 1$.

$$x_H^*(t) = \frac{1}{2} - \frac{1}{6\tau}\left[c_H - c_G + \alpha_H\, q_H - \alpha_G\, q_G(t)\right] \qquad \text{for} \qquad T^* \le t < T\,.$$

$x_G^*(t)$ declines in $q_G(t)$ and hence $x_H^*(t)$ increases in $q_G(t)$ until $t = T$. Prices are

$$p_G^*(t) = \frac{1}{3}\left[\left(c_H + 2\,c_G\right) + 3\tau + \alpha_H\, q_H - \alpha_G\, q_G(t)\right],$$

$$p_H^*(t) = \frac{1}{3}\left[\left(c_G + 2\,c_H\right) + 3\tau - \alpha_H\, q_H + \alpha_G\, q_G(t)\right] \qquad \text{for} \qquad T^* \le t < T\,.$$

$p_G^*\,(t)$ declines in $q_G(t)$ and $p_H^*\,(t)$ increases in $q_G(t)$ until $t = T$ is reached. Profits are

$$\pi_G^*(t) = \frac{1}{18\,\tau}\left[3\tau + \left(\alpha_H\, q_H - \alpha_G\, q_G(t)\right)\right]^2$$

$$\pi_H^*(t) = \frac{1}{18\,\tau}\left[3\tau - \left(\alpha_H\, q_H - \alpha_G\, q_G(t)\right)\right]^2$$

with $\pi_G^*(t)$ declining in $q_G(t)$ and $\pi_H^*(t)$ increasing in $q_G(t)$ for $T^* \le t < T$.

In $T\left(\ge T^*\right)$ we assume that the gasoline price has reached the price of the backstop technology, i.e. $q_H = q_G\left(0\right)e^{rT}$. For a backstop technology, the demand for the former substitute is zero ($x_G^* = 0$), hence $q_G(T)$ as in (17). The prices are

$$p_G = c_G \quad , \qquad\qquad p_H = c_H + 2\tau$$

and profits are

$$\pi_G = 0\,, \qquad\qquad \pi_H = 2\,\tau\,.$$

T follows from (18), i.e.

$$\rho \int_0^{T^*} x_G(t)\,dt + \rho \int_{T^*}^{T} x_G(t)\,dt = S_0$$ [16]

With $T(q_G(0))$ as solution from this resource availability restriction, we obtain $q_G(0)$ from (17).

We finally can do a phase diagram analysis in order to evaluate the slopes of the equations of motion near the steady state. For that purpose we choose $\dot{I}_H$ and $\dot{IB}_H$ as our two equations of motion.[17] The dynamics of IB_H and I_H around the steady state is presented in Figure 3.

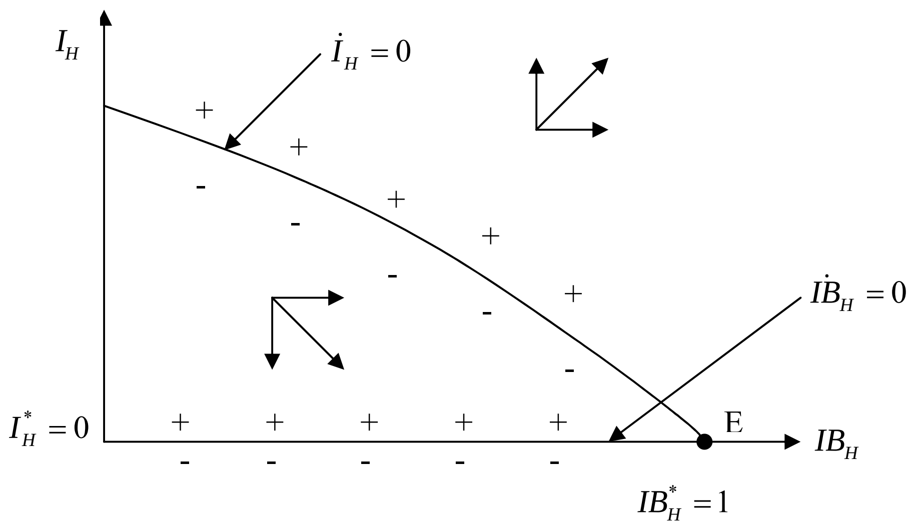

Fig. 3: The dynamics towards the steady state *E*.

As shown in Fig. 3, the control variable I_H is used to guide IB_H from $IB_H(0) = 0$ to $IB^*_H = 1$ (solved for) in an optimal way.

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[16] With $x_G(t)$ from (11) we obtain

$$\rho \int_0^{T^*} \left[\frac{1}{2} + \frac{1}{6\cdot\tau}\left[c_H - c_G + \gamma\left(\Delta n^{IB}(t) + \Delta q(t)\right)\right] dt + \rho \int_{T^*}^{T} \left[\frac{1}{2} + \frac{\gamma}{6\cdot\tau}\left[c_H - c_G + \gamma\,\Delta q(t)\right]\right] dt\right] = S_0 .$$

Using $\Delta n^{IB}(T^*) = 0$ for $T > T^*$ and by integrating some parts, the condition is

$$\frac{T^*}{2} + \frac{1}{6\tau}\int_0^{T^*}(c_H - c_G)\,dt + \frac{\gamma}{6\tau}\int_0^{T^*}\Delta n^{IB}(t)\,dt + \frac{1}{6\tau}\left[\alpha_H\, q_H\, T^* - \frac{\alpha_G}{r}\left(q_H - q_G(0)\right)\right] = \frac{S_0}{\rho} .$$

[17] For details see the Appendix.

5. Conclusion

Our model has shown that there need not be a market failure if a new technology lacks a network. First, we have shown that although profit π_H of the entrant will decline in I_H in each period, π_H will at the same time increase in IB_H in each period. If the network effect γ is strong enough, then even in the short run the positive effect from $\frac{\partial \pi_H}{\partial IB_H} > 0$ contributes more to profit than the negative effect from $\frac{\partial \pi_H}{\partial I_H} < 0$ reduces profit (for a proof, the difference between the two partial derivatives has to be calculated). Second, our model has pointed out the crucial role of the price path of gasoline, of environmental concern ($\alpha_G > \alpha_H$) and of the price of the backstop technology on network size and price competition. Especially the price path of gasoline forces the incumbent to lower its price p_G and permits the entrant at the same time to raise his price. Nevertheless, the incumbent will lose market shares and the entrant will benefit from that. However, in case unit production costs c_H including costs of remodeling existing gasoline stations $\left(C\left(I_H\right)\right)$ are high compared to the production costs c_G of the incumbent, then the entrant might bear a loss over several periods until the Hotelling price path of oil might help him to make positive profit after some years (note that in (20) $\Delta q > 0$ decline in *t*). If the costs of remodeling the installed base IB_G are high, then there will be market failure in spite of the Hotelling price path for gasoline. Market failure can occur if in the first period $\pi_H(1) < 0$, that is, the sponsor makes a loss. In case it makes a profit in later periods, loss in the beginning is not a problem. However, if the loss is rather high and the firm cannot get a loan from the banks, then the new technology will not be introduced. According to π_H (1) from (20), firm *H* will make a loss in the first period[18] if (*i*) the investment costs for achieving compatibility are high, (*ii*) the network effect $\gamma \cdot \Delta n^{IB}$ is high, (*iii*) unit cost c_H are much higher than c_G, or (*iv*) the price of hydrogen is relatively high (Δq is large). In such a case the government could pay a subsidy such that $\pi_H(1) = 0$ in the first period. As Δq declines in *t* and Δn^{IB} declines in I_H, the loss situation improves from period to period. If there is no subsidy program, cooperative policy between the car producers, the energy companies and the government is required to prevent a situation where the society runs out of oil and huge investment costs emerge all of a sudden to remodel

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[18] That is if $3\tau + \left(c_G - c_H\right) - \gamma\left(\Delta n^{IB}(1) + \Delta q(1)\right) < 3\sqrt{C\left(I_H(1)\right) \cdot 2\tau}$.

gasoline stations as well as automobile engines. In view of the exhaustion of crude oil within the next decades, the investment in compatibility of existing gasoline stations should be a profit maximizing strategy for the motor vehicle industry.

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Appendix:

The Phase Diagram Analysis

From π_H in (20), (13a) becomes

$$-C'(I_H)+\mu=0$$

or

$$-C''(I_H)\dot{I}_H+\dot{\mu}=0\,.$$

With $\dot{\mu}$ from (13b) we obtain

$$-C''(J_H)\dot{I}_H+r\,C'(I_H)-\frac{\partial\,\pi_H}{\partial\,IB_H}=0$$

that is

(23) $$\dot{I}_H=\frac{1}{C''(I_H)}\left(r\,C'(I_H)-\frac{\partial\,\pi_H}{\partial\,IB_H}\right)$$

as one equation of motion. The other one is

(24) $$I\dot{B}_H=I_H$$

To show the motions of IB_H and I_H off the steady state $\left(IB_H^*,I_H^*\right)$, we first determine for the steady state $\left(\dot{I}_H=d\,\dot{I}_H=I\dot{B}_H=d\,I\dot{B}_H=0\right)$ the slope of the $\dot{I}_H=0$ equation and of the $I\dot{B}_H=0$ equation (the proof of the signs are given after equation (28).

(25) $$I\dot{B}_H=0: \qquad d\,I_H=0$$

(26) $$\dot{I}_H=0 \qquad \frac{d\,I_H}{d\,IB_H}<0$$

For the motions off the steady state we obtain for the $I\dot{B}_H=0$ equation

(27) $$\frac{\partial\, \dot{IB}_H}{\partial\, I_H} = 1 \;, \qquad \frac{\partial\, \dot{IB}_H}{\partial\, IB_H} = 0$$

and for the $\dot{I}_H = 0$ equation

(28) $$\frac{\partial\, \dot{I}_H}{I_H} = r \;, \qquad \frac{\partial\, \dot{I}_H}{\partial\, IB_H} > 0\,.$$

In order to evaluate the slopes of the two equations of motion (23) and (24) near the steady state we differentiate each of them with respect to each of the included variables. For example, for the $IB_H(IB_H, I_H)$ equation,

$$d\, \dot{IB}_H = d\, I_H$$

and for the $I_H\left(I_H, IB_H\right)$ equation, using $C\left(I_H\right) = \delta\, I_H^2$, $s_H = IB_H$, and π_H in (20)

$$d\, \dot{I}_H = r\, I_H + \frac{\gamma}{18 \cdot \delta}\Big(\left(-2 + 2IB_H\right)\left(\cdot\right) + \left[\;\;\right] \cdot 2\Big) d\, IB_H$$

with $\left[\cdot\right]$ as the term in brackets in (20) and $\left(\cdot\right)$ as the terms in parentheses in (22).
Next we verify the slope of the $\dot{I}_H = 0$ equation at the steady state:

(26') $$\frac{d\, I_H}{d\, IB_H} = -\frac{\gamma}{9 \cdot \delta}\left[\;\;\right] < 0 \qquad \text{as } \left[\;\;\right] > 0$$

and $\left(\cdot\right) = 0$ at the steady state.
In order to verify (27) and (28), the motions off the steady state, we obtain for the $\dot{IB}_H = 0$ equation the partial derivatives given in (27), and for the $\dot{I}_H = 0$ equation

(28') $$\frac{\partial\, \dot{I}_H}{\partial\, I_H} = r \qquad \frac{d\, \dot{I}_H}{d\, IB_H} = \frac{\gamma}{9 \cdot \delta}\left(-\frac{\gamma}{\tau}\left(-1 + IB_H\right)^2 + \left[\;\;\right]\right)$$

which is positive as $IB_H^* \approx 1$ in the neighbourhood of the steady state.